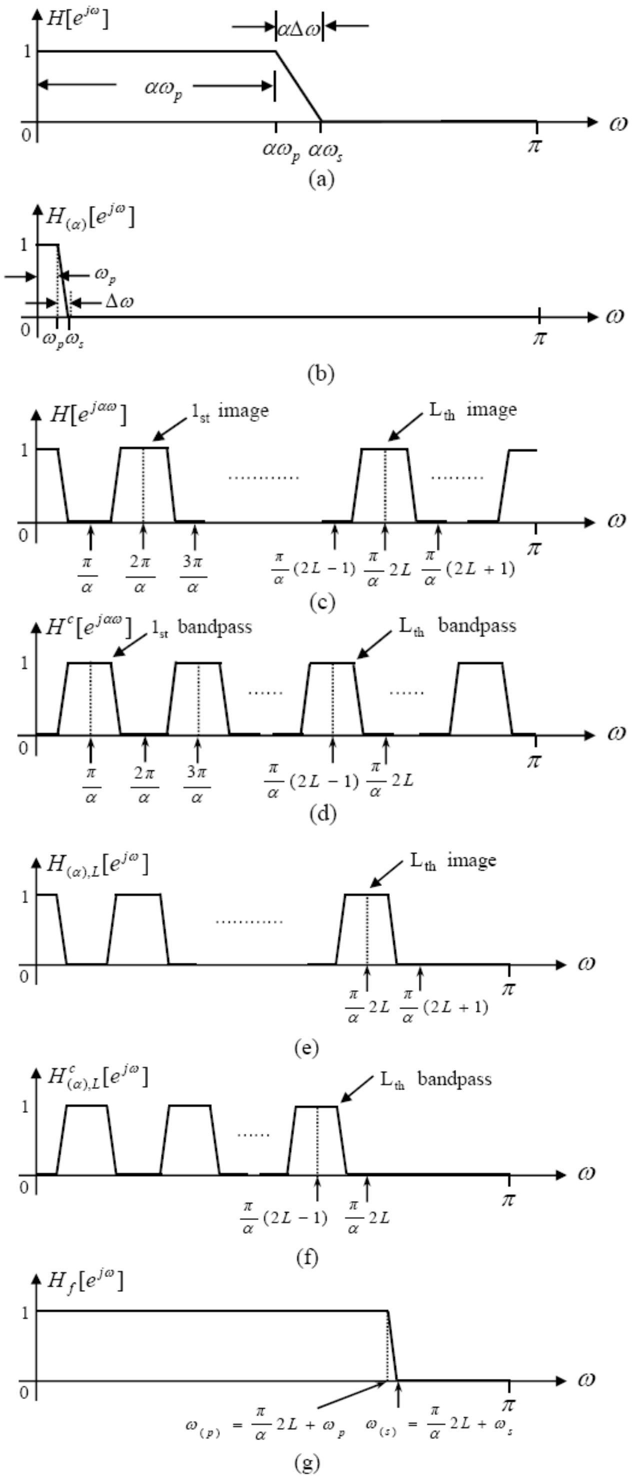

Fig. 1 Filter design procedure: Gain responses.

In (4), $*$ indicates the linear convolution operation, and $h_u[n]$ is obtained by inserting ($\alpha - 1$) zeros between adjacent samples of $h[n]$. Note that $H[z^{\alpha}]$ corresponds to the Z-transform of $h_u[n]$, and (4) can be implemented based on the conventional IFIR filter structure [2]. In particular, conventional computationally efficient filter design methods (e.g., IFIR and FRM) are very effective in the design of sharp FIR filters with fixed coefficients. However, it may not be easy to utilize conventional design methods further for the design of reconfigurable FIR filters allowing a great flexibility to the filter design. To solve the problem, generalized sampling kernels, derived by revising the sampling kernel of [5, 6], and the complementary filter concept [3] were employed in this paper for the design of a wide-band FIR filter with sharp transition. The convolution expression (4) can be used to design a computationally efficient FIR filter with computational complexity similar to or a bit more than those conventional approaches. In particular, since $sinc(n/\alpha)$ in (2) (i.e., an ideal low-pass filter) is of doubly infinite length, it should be modified for the design of a sharp linear-phase FIR filter of finite order. Accordingly, one of widely used adjustable window functions whose impulse responses are of finite length [7] is employed in this paper as a lowpass filter. In the next section, a more detail procedure related to the adjustable window functions and generalized sampling kernels are discussed.

III. Design of a Wide-band Sharp FIR Filter using a Generalized Sampling Kernel

From (3) and Fig. 1(b), we can see that $h_{(\alpha)}[n]$ can be obtained by taking the inverse DTFT of only the lowpass part (i.e., $\omega \in [-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}]$) of $H[e^{j\alpha\omega}]$: That is,

$$h_{(\alpha)}[n] = \frac{1}{2\pi} \sum_{k=0}^{N-1} h[k] \int_{-\frac{\pi}{\alpha}}^{\frac{\pi}{\alpha}} e^{j(n-\alpha k)\omega'} d\omega' \qquad (5)$$

Furthermore, the concept in (5) can be extended to the [*single-lowpass* + *multi-bandpass*] (or multi-image as in Fig. 1(c)-(d)) case by changing the upper and lower bounds of the definite integral of (5) to include up to L-th images in addition to the lowpass part of $H[e^{j\alpha\omega}]$ as in Fig. 1(e): i.e.,

$$\begin{aligned} h_{(\alpha),L}[n] &= \frac{1}{2\pi} \sum_{k=0}^{N-1} h[k] \int_{-\frac{(2L+1)\pi}{\alpha}}^{\frac{(2L+1)\pi}{\alpha}} e^{j(n-\alpha k)\omega'} d\omega' \\ &= \sum_{k=0}^{N-1} h[k] \bullet \frac{1}{\alpha}(2L+1) sinc((\frac{n}{\alpha} - k)(2L+1)) \end{aligned} \qquad (6)$$

More specifically, we employ, as an adjustable window function [7], the raised-cosine filter with a desired roll-off factor R, whose impulse response of finite length is a low-pass filter widely used for pulse shaping in the communication fields. In addition, the following modified sampling kernel can be derived by applying the raised-cosine filter, instead of the sampling kernel $sinc((n/\alpha) - k)$ in (3): That is,

$$\overline{h}_{(\alpha)}[n] = \sum_{k=0}^{N-1} h[k] \bullet \frac{1}{\alpha} K_{\alpha}(n,k) = \frac{1}{\alpha} h_u[n] * g(\frac{n}{\alpha}) \qquad (7)$$

$$K_{\alpha}(n,k) = \frac{sin\pi(\frac{n}{\alpha} - k)}{\pi(\frac{n}{\alpha} - k)} \times \frac{cos\pi R(\frac{n}{\alpha} - k)}{1 - 4R^2(\frac{n}{\alpha} - k)^2} \qquad (8)$$

In (8), $K_{\alpha,L}(n,k)$ is a generalized sampling kernel in the "*single-lowpass* plus *multi-bandpass*" case, which reduces to the sampling kernel of (3) when $L = 0$ (Fig. 1(b)). Also, R is the roll-off factor ($0 \le R \le 1$), and the sharpness of the lowpass FIR filter can be controlled by adjusting the R. By employing (7), (6) can be revised as

$$\bar{h}_{(\alpha),L}[n] = \sum_{k=0}^{N-1} h[k] \bullet \underbrace{\frac{1}{\alpha}(2L+1)g[(\frac{n}{\alpha}-k)(2L+1)]}_{K_{\alpha,L}(n,k)} \quad (9)$$

By taking the DTFT of both sides of (9), its frequency-domain expression $H_{(\alpha),L}[e^{j\omega}]$ can be obtained (or see Fig. 1(e)) as follows:

$$H_{(\alpha),L}[e^{j\omega}] = \begin{cases} H[e^{j\alpha w}], & \omega \in [-\frac{(2L+1)\pi}{\alpha}, \frac{(2L+1)\pi}{\alpha}] \\ 0, & \omega \notin [-\frac{(2L+1)\pi}{\alpha}, \frac{(2L+1)\pi}{\alpha}] \end{cases} \quad (10)$$

Moreover, another generalized sampling kernel $K^c_{(\alpha),L}(n,k)$ for the complementary filter $H^c[z]$ can be derived in a similar manner to (5)-(10) by considering the "*multi-bandpass*" case whose frequency response includes up to *L-th* bandpass parts of $H^c[e^{j\alpha\omega}]$ (or see Fig. 1(f)):

$$\bar{h}^c_{(\alpha),L}[n] = \sum_{k=0}^{N-1} h^c[k] \bullet \underbrace{\frac{1}{\alpha}(2L)g[(\frac{n}{\alpha}-k)(2L)]}_{K^c_{\alpha,L}(n,k)} \quad (11)$$

By taking the DTFT of both sides of (11), its frequency-domain expression $H^c_{(\alpha),L}[e^{j\omega}]$ can be obtained (or see Fig. 1(f)) as follows:

$$H^c_{(\alpha),L}[e^{j\omega}] = \begin{cases} H^c[e^{j\alpha w}], & \omega \in [-\frac{(2L)\pi}{\alpha}, \frac{(2L)\pi}{\alpha}] \\ 0, & \omega \notin [-\frac{(2L)\pi}{\alpha}, \frac{(2L)\pi}{\alpha}] \end{cases} \quad (12)$$

Accordingly, (9) and (11) can be used to design a wide-band FIR filter with sharp transition (i.e., $h_f[n]$: see Fig. 1(g)) from the following:

$$h_f[n] = \bar{h}_{(\alpha),L}[n] + \bar{h}^c_{(\alpha),L}[n] \quad (13)$$

In summary, the design procedures for a linear-phase wide-band FIR filter with sharp transition are as follows:

Step 1: Given desired filter specifications as in Fig. 1(g), an appropriate L (= number of images included as in Fig. 1(c)) should be chosen first, from which the narrow-band lowpass filter as in Fig. 1(b) can be determined and thus an optimal integer scaling factor α (as in (9)) can be calculated by using the optimal determination equation for an interpolation factor α in case of a generalized IFIR design [9, 10].

Step 2: Design a prototype model filter $h[n]$ as in Fig. 1(a), whose filter length can be estimated by the well-known formula of Kaiser [8].

Step 3: Derive $\bar{h}_{(\alpha),L}[n]$ from (9) and $\bar{h}^c_{(\alpha),L}[n]$ from (11).

Step 4: Design the desired FIR filter with sharp transition (i.e., $h_f[n]$) from (13) as in Fig. 1(g).

IV. Design Examples

A design example is presented to demonstrate the proposed design procedure. The specifications of a desired wide-band FIR filter with sharp transition [10] are given as follows: passband edge $\omega_{(p)} = 0.90\pi$, stopband edge $\omega_{(s)} = 0.92\pi$, passband ripple $\delta_{(p)} = 0.02$, stopband ripple $\delta_{(s)} = 0.001$ (i.e., minimum stopband attenuation: 60dB and peak passband ripple: 0.36dB). We can design a wide-band filter with arbitrary passband width, depending on the number of images being included (= L: also, see Fig. 1(c)-(d)). To design a wide-band FIR filter with sharp transition as in Fig. 1(g), a sharp narrow-band lowpass filter first as in Fig. 1(b) should be determined. In this design example, we choose L=2 to include up to 2nd (image as well as bandpass) parts as in Fig. 1(e) and Fig. 2(f). Then, filter specifications for a narrow-band filter as in Fig. 1(b) can be calculated from the L and from Fig. 1(d)-(f): $\omega_p = 0.09\pi$, $\omega_s = 0.11\pi$, $\delta_p = 0.02$, and $\delta_s = 0.001$. Also, the optimal scaling factor α of (9) is calculated to be 5 (rounded), and then the corresponding prototype model filter $H[e^{j\omega}]$ (as in Fig. 1(a)) is determined, where the length of the prototype model filter and that of the raised-cosine pulse are given as 53 and 27, respectively (here, the roll-off factor is 0.5). Thus, the total number of multipliers to implement the type-1 wide-band FIR filter with sharp transition (corresponding to Fig. 1(g)), designed by the proposed approach, is equal to 41 (= (53+1)/2+(27+1)/2). Note that the filter length of the raised-cosine filter with a desired roll-off factor can be determined by using the equations for adjustable window functions [7]. Furthermore, $\bar{h}_{(\alpha),L}[n]$ and $\bar{h}^c_{(\alpha),L}[n]$ can be derived from (9) and (11), and, finally, the filter coefficients and the gain response of the FIR filter $h_f[n]$ designed from (13) are shown in Table I and Fig. 2, respectively. Note that the proposed approach enables us to save about 65% of multiplications, when compared with the filter designed by the Remez-based algorithm.

V. Conclusion

In this paper, we presented a new design of a computationally efficient wide-band FIR filter with sharp transition, where generalized sampling kernels are introduced and utilized along with a complementary filter concept. Also, it is shown that a closed-form expression for filter coefficients may be derived and can be further utilized for the design of reconfigurable sharp FIR filters. In addition, the proposed design approach may be further extended to the design of various kinds of linear-phase sharp FIR filter (e.g., narrow-band lowpass, narrow-band highpass, narrow-band bandpass, wide-band lowpass, wide-band highpass, wide-band bandpass, half-band, multi-band, etc.). Future researches include a systematic approach to the design of sharp FIR filters with an optimized scaling factor.

Table I. Filter coefficients of the prototype model filter $h[n]$ and the raised-cosine function $g[n]$.

n	h[n](model)	n	g[n](raised-cosine)
0, 52	0.0012	0, 26	0.0007
1, 51	-0.0006	1, 25	-0.0012
2, 50	-0.0043	2, 24	-0.0045
3, 49	-0.0039	3, 23	-0.0097
4, 48	0.0021	4, 22	-0.0154
5, 47	0.0037	5, 21	-0.0186
6, 46	-0.0034	6, 20	-0.0155
7, 45	-0.0057	7, 19	-0.0023
8, 44	0.004	8, 18	0.0225
9, 43	0.0081	9, 17	0.0573
10, 42	-0.0047	10, 16	0.0973
11, 41	-0.0112	11, 15	0.1347
12, 40	0.0054	12, 14	0.1614
13, 39	0.0153	13	0.1711
14, 38	-0.0061		
15, 37	-0.0208		
16, 36	0.0067		
17, 35	0.0284		
18, 34	-0.0072		
19, 33	-0.0399		
20, 32	0.0076		
21, 31	0.0596		
22, 30	-0.008		
23, 29	0.0082		
24, 28	0.3175		
25, 27	0.3175		
26	0.4918		

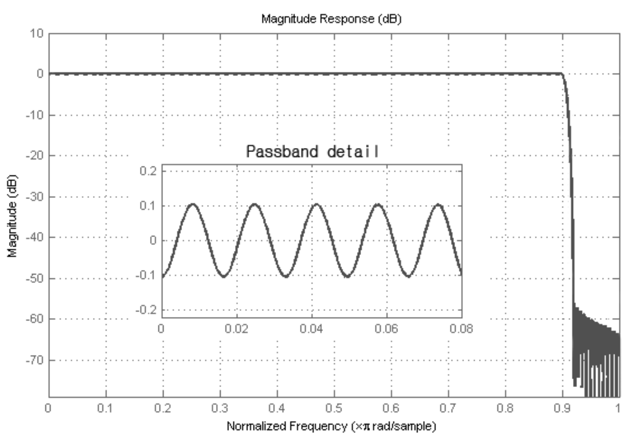

Fig. 2 Gain response of a wide-band FIR filter designed by the proposed method (also, see Fig. 1(g)).

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