able to deliver a power of 1,5 Kw each, a central of vapor to control dampness inside the channel and a ventilator carried by an asynchronous engine to control the emanating air during the drying operation.

The control of the heating resistors, of the central vapor and the ventilation speed is ensured by a programmable logic controller linked to a computer carrying a program of control and supervision.

The blower is a multivariable system with the heating power, the vapor quantity and the ventilation speed like inputs and temperature, and dampness like outputs (Fig.1).

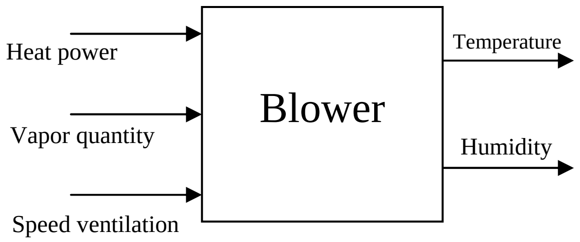

Fig 1 Inputs and outputs of the plant

We are limiting ourselves, in this paper, to control temperature by action on the heating power and ventilation speed.

B. Modeling of the blower

Preliminary experiments have demonstrated that the evolution of temperature is non linear in relation with the heating power and the ventilation speed. The idea is to apprehend the non linear behavior of this system through a group of local models either linear or affine [1]. Each local model characterizes the system's behavior in a particular functioning zone. Local models are then aggregated by specific interpolation laws [14], [11], [10], [17].

The blower, non linear discrete time system, can be represented by the following discrete multimodel

$$\begin{cases} x(k+1) = \sum_{j=1}^{N} \mu_j(\xi_V) \sum_{i=1}^{M} \mu_{j,i}(\xi_T)(A_{j,i}\,x(k) + \\ \qquad + B_{j,i}\,u(k) + d_{j,i}) \\ y(k) = \sum_{j=1}^{N} \mu_j(\xi_V) \sum_{i=1}^{M} \mu_{j,i}(\xi_T)\, C_{j,i}\, x(k) \end{cases} \tag{1}$$

Where $x(k) \in \Re^{nx1}$ is the state vector, $u(k) \in \Re^{m}$ is the heating power, $y(k) \in \Re^{p}$ output vector.

$A_{j,i} \in \Re^{nxn}$, $B_{j,i} \in \Re^{nxm}$, $C_{j,i} \in \Re^{pxn}$ and $d_{j,i} \in \Re^{nx1}$ are respectively the state matrix, input matrix, output matrix and a vector depending on the operating point of local model (j,i) depending on the temperature and speed ventilation. ξ_T is the vector decision depending on the measurable states, temperature in this case. ξ_V is the vector decision depending on the inputs of plant and the speed of ventilation.

The activating functions $\mu_j(\xi_V)$, $j = 1...N$, have the following properties:

$$\begin{cases} \sum_{i=1}^{N} \mu_j(\xi_V) = 1 \\ 0 \le \mu_j(\xi_V) \le 1 \;\forall\; j = 1...N, \end{cases} \tag{2}$$

These functions are known in real-time and used to establish a command law by varying the ventilation speed.

The activating functions $\mu_{j,i}(\xi_T), i = 1...M$, have the following properties:

$$\begin{cases} \sum_{i=1}^{M} \mu_{j,i}(\xi_T) = 1 \;\forall\; j = 1...N, \\ 0 \le \mu_{j,i}(\xi_T) \le 1 \;\forall\; i = 1...M,\, et \;\forall\; j = 1...N, \end{cases} \tag{3}$$

These functions are known in real-time but can't be directly modified by the command because they are depending on the internal non measurable states.

The number of models N and M is depending on the complexity of the non linear system, the desired precision and the structure of activating functions [2].

III. Construction and stabilization of multiobserver

A. Global control law

When certain components of the state vector are non measurable and the system is observable, it's possible to construct a multiobserver associated to a multimodel described by (4) as follows:

$$\begin{cases} \hat{x}(k+1) = \sum_{j=1}^{N} \mu_j(\xi_V) \sum_{i=1}^{M} \mu_{j,i}(\xi_T)(A_{j,i}\hat{x}(k) + B_{j,i}u(k) \\ \qquad + d_{j,i} + L_{j,i}C_{j,i}(x(k) - \hat{x}(k))) \\ \hat{y}(k) = \sum_{j=1}^{N} \mu_j(\xi_V) \sum_{i=1}^{M} \mu_{j,i}(\xi_T)\, C_{j,i}\,\hat{x}(k) \end{cases} \tag{4}$$

Where $\hat{x}(k)$ and $\hat{y}(k)$ design respectively the estimation produced by the observer of the state vector and the estimation of the output; $L_{j,i}$ design the gain of the observation matrix associated to the model (j,i). We define the state error observation as follows:

$$\varepsilon(k) = x(k) - \hat{x}(k) \tag{5}$$

The command *u(k)*, in case of dynamic output feedback, based on the multiobserver (4), is :

$$u(k) = K(k)\hat{x}(k) + \bar{N}(\infty)(y_c) \tag{6}$$

Taking into account (4), (5) and (6) the augmented equation of the system is then:

$$\begin{cases} x(k+1) = \sum_{i=1}^{N} \mu_j(\xi_v) \sum_{i=1}^{M} \mu_{j,i}(\xi_T)((A_{j,i} + B_{j,i} K_{j,i})x(k) \\ \qquad -B_{j,i} K_{j,i} \varepsilon(k) + d_{j,i}) \\ \varepsilon(k+1) = \sum_{i=1}^{N} \mu_j(\xi_v) \sum_{i=1}^{M} \mu_{j,i}(\xi)(A_{j,i} - L_{j,i} C_{j,i})\varepsilon(k) \end{cases} \quad (7)$$

Equality (7) can be written also as follows:

$$\begin{pmatrix} x(k+1) \\ \varepsilon(k+1) \end{pmatrix} = \sum_{j=1}^{N} \mu_j(\xi_v) \sum_{i=1}^{M} \mu_{j,i}(\xi_T) \quad (8)$$
$$\left(\begin{pmatrix} A_{j,i} + B_{j,i} K_{j,i} & -B_{j,i} K_{j,i} \\ 0 & A_{j,i} - L_{j,i} C_{j,i} \end{pmatrix} \begin{pmatrix} x(k) \\ \varepsilon(k) \end{pmatrix} + \begin{pmatrix} d_{j,i} \\ 0 \end{pmatrix} \right)$$

Since the matrix of augmented system is a superior triangular, the instantaneous characteristics values of augmented system are those of matrices $\sum_{j=1}^{N} \mu_j(\xi_v) \sum_{i=1}^{M} \mu_{j,i}(\xi_T)(A_{j,i} + B_{j,i} K_{j,i})$ and $\sum_{j=1}^{N} \mu_j(\xi_v) \sum_{i=1}^{M} \mu_{j,i}(\xi_T)(A_{j,i} - L_{j,i} C_{j,i})$.

It's important to notice that the calculation of the gains *K(k)* and *L(k)* poses on the use of following relations:

$$\sum_{i=1}^{M} \mu_{j,i}(\xi_T) B_{j,i} K(k) = \sum_{i=1}^{M} \mu_{j,i}(\xi_T) B_{j,i} K_{j,i} \quad (9)$$

$$\sum_{i=1}^{M} \mu_{j,i}(\xi_T) L(k) C_{j,i} = \sum_{i=1}^{M} \mu_{j,i}(\xi_T) L_{j,i} C_{j,i} \quad (10)$$

B. Stabilization of the multiobserver

B.1 polyquadratic Stabilization

In this part we start with citing the theorem of Daafouz and Bernussou [9] about the polyquadratic stability of the uncertain dynamic system.

An uncertain dynamic system can be described as follows:

$$x(k+1) = A(\xi(k))x(k) \quad (11)$$

With $x \in \Re^n$ is the state vector, $\xi \in \Xi \subset \Re^p$ is unknown parameter, time variable but bounded. The dynamic matrix A is written as follows:

$$A(\xi(k)) = \sum_{i=1}^{N} \xi_i(k) A_i, \qquad \xi_i(k) \geq 0, \quad \sum_{i=1}^{N} \xi_i(k) = 1. \quad (12)$$

Theorem 1 (Daafouz and Bernussou, 2001):

There exists a Lyapunov function ,with a polytopic structure, whose difference is negative definite proving asymptotic stability of the system (11) if and only if there exist N symmetric matrices $S_1 \ldots S_N$ and N matrices $G_1 \ldots G_N$ satisfying :

$$\begin{pmatrix} G_i + G_i^T - S_i & G_i^T A_i^T \\ A_i G_i & S_j \end{pmatrix} > 0 \quad (13)$$
$$\forall\, i = 1, \ldots, N \ \ et \ \ \forall\, j = 1, \ldots, N.$$

The PDLF is given by :

$$P(k) = \sum_{i=1}^{N} \xi_i(k) S_i^{-1} \quad (14)$$

B.2 Poles Localization

In case of multimodel where state matrix is variable in each period of sampling, the use of command by poles localization guarantees the desired performances of different controllers [21] and observers [3].

An effective way of determining the dynamic characteristics of a system is to localize poles inside a disc $D_{\alpha,r}$ contained in the unit circle, α is the center of the disc and r its ray [5], which is equivalent to the fact of placing characteristic values of auxiliary system $(A - \alpha I)/r$ in the unit circle [5].

Definition 1: (Granado, 2004)

A matrix A is known as D-stable if his characteristic values belong to the disc D(α,r)

Lemma1: (Furuta et Kim 1987) :A matrix A is D-stable if and only if $\forall Q = Q^T > 0$ the following equality:

$$A^T PA - \alpha(A^T P + PA) + (\alpha^2 - r^2)P + Q = 0 \quad (15)$$

has a symmetric definite positive matrix solution P.

This is equivalent to test if the following inequality has a definite positive solution [5] [12]:

$$\left(\frac{A - \alpha I}{r}\right)^T P \left(\frac{A - \alpha I}{r}\right) - P < 0 \quad (16)$$

The belonging of characteristic values to the disc $D_{\alpha,r}$ is then equivalent to the belonging of characteristic values of the auxiliary plant $\tilde{A} = (A - \alpha I)/r$ to the unit disc [8][7][6].

The resolution of poles localization in a disc can be taken back to the stability of the auxiliary system $\tilde{A}$, we can conclude the following theorem:

Theorem 2

The error state estimation between the multimodel described by (1), and the multiobserver described by (4), is polyquadratically D-stabilizable by gains L_i, if and only if there exist a symmetric definite matrices S_i and S_j and matrices F_i and G_i which satisfies the following inequalities:

$$\begin{pmatrix} G_i + G_i^T - S_i & (\bullet)^T \\ \left(\frac{A_i^T - \alpha I}{r}\right) G_i - \frac{C_i}{r} F_i & S_j \end{pmatrix} > 0 \quad (17)$$

for $i = 1...N$ and $j = 1...N$

The observers gains are calculated by $L_i = (F_i G_i^{-1})^T$

Proof: It's possible to get the conditions of theorem 1 by substituting A_i by

$$A_i = \left(\frac{(A_i - L_i C_i)^T - \alpha I}{r}\right) \quad (18)$$

$$F_i = L_i^T G_i \quad (19)$$

Because $\left(\frac{(A_i - L_i C_i)^T - \alpha I}{r}\right)$ and $\left(\frac{(A_i - L_i C_i) - \alpha I}{r}\right)$ have the same characteristic values, the existence of G_i^{-1} is guaranteed by the following inequality:

$$G_i + G_i^T > S_i > 0 \quad (20)$$

IV. Synthesis of state feedback control law

We look to calculate the gain *K(k)*, at each period of sampling, by state feedback allowing to ensure the polyquadratic D-stability of multimodel (1). By making the following substitutions in theorem 1:

$$A_i = \left(\frac{A_i + B_i K_i - \alpha I}{r}\right) \quad (21)$$

$$R_i = K_i G_i \quad (22)$$

We can cite the following theorem:

Theorem 3:

The multimodel (1) is polyquadratically D-stabilisable by state feedback, if and only if there exists a symmetric definite matrices S_i and S_j and matrices G_i and R_i of appropriate dimensions solutions of the following *LMIs*:

$$\begin{pmatrix} G_i + G_i^T - S_i & (\bullet)^T \\ \left(\frac{A_i - \alpha I}{r}\right) G_i + \frac{B_i}{r} R_i & S_j \end{pmatrix} > 0 \quad (23)$$

for $i = 1...N$ and $j = 1...N$

The state feedback gains are calculated as follows:

$$K_i = R_i G_i^{-1} \quad (24)$$

V. The command quantification

A. The command Algorithm

The calculated command, at each period of sampling, is:

$$u(k) = K(k)(\hat{x}(k)) + \bar{N}(\infty)(y_c) \quad (25)$$

K(k) verify equality (9)

$$d(k) = \sum_{j=1}^{N} \mu_j(\xi_v) \sum_{i=1}^{M} \mu_{j,i}(\xi_T(k)) d_{j,i}) \quad (26)$$

The static gain $\bar{N}$ is defined like by:

$$\bar{N}(\infty) = \sum_{j=1}^{N} \mu_j(\xi_v(\infty)) \sum_{i=1}^{M} \mu_{j,i}(\xi_T(\infty)) \Big[C_{j,i} (I - (A_{j,i} - B_{j,i} K(\infty)))^{-1} B_{j,i} \Big]^{-1} \quad (27)$$

This command cannot be applied directly to the plant because the available command values are quantified. So we have to choose between the nearest inferior or the nearest superior quantified commands:

$$u_{Q_{\min}} \le u(k) \le u_{Q_{\max}} \quad (28)$$

It's noted that quantified commands are multiple of 1500 Watts. For example if the generated command from the control and supervision program is of 6739 Watts, we are obliged to choose between 6000 Watts and 7500 Watts.

To compensate the quantification effects, we have chosen to action the ventilation speeds. In fact, the changes of ventilation can increase or decrease the temperature. The determination of the best ventilation speed is ensured by the minimization of the following criterion:

$$J_{(v,u)}(k) = (\hat{y}_{e_{(\xi_v,u)}}(k+1) - y_c)^T I_p (\hat{y}_{e_{(\xi_v,u)}}(k+1) - y_c) \quad (29)$$

with *v*: speed of ventilation obtained from the decision vector ξ_v .

$$u = \{u_{Q_{\min}}, u_{Q_{\max}}\} \quad (30)$$

$\hat{y}_{e_{(\xi_v,u)}}(k+1)$ is the estimation of the output, at the discrete time *k+1*, it depends on the inputs speed ventilation *v* and the command *u*:

$$\hat{y}_{e_{(\xi_v,u)}}(k+1) = \sum_{j=1}^{N} \mu_j(\xi_v) \sum_{i=1}^{M} \mu_{j,i}(\xi_T) C_{j,i} \hat{x}_{e_{(\xi_v,u)}}(k+1) \quad (31)$$

$\hat{x}_{e(\xi_v,u)}(k+1)$ is the estimated state , at the discrete time *k+1*, it depends on the inputs speed ventilation *v* and the command *u*:

$$\hat{x}_{e(\xi_v,u)}(k+1)=\sum_{j=1}^{N}\mu_j(\xi_v)\sum_{i=1}^{M}\mu_{j,i}(\xi_T)(A_{j,i}\hat{x}(k) + B_{j,i}u(k)+d_{j,i}+L_{j,i}C_{j,i}(x(k)-\hat{x}(k))) \quad (32)$$

The control algorithm is then:

step 0 : Initialisation

- calculate the gains $K_{i,j}$ and $L_{i,j}$ using *LMIs* (17) and (24)
- apply the maximum value of power heat
- apply the maximum value of speed ventilation

step 1 : Determination of the actual model and computing of *u(k)*

- measure the internal temperature of the blower
- evaluate the decision vector $\xi_T(k)$
- deduce *A*, *B*, *C* and *d*
- calculate *L(k)* using (10)
- deduce $\hat{x}(k)$ using (4)
- calculate *K(k)* using (9)
- deduce *u(k)* using (25)

step 2 : quantification problem resolution

- calculate $u_{Q_{\min}}$ *et* $u_{Q_{\max}}$
- calculate $\hat{x}_{e(\xi_v,u)}(k+1)$ using (32)
- calculate $\hat{y}_{e(\xi_v,u)}(k+1)$ using (31)
- calculate $J_{(v,u)}(k)$ using(29)
- deduce and apply $u(k)=u\left\{\min_{\xi_v,u}(J(u,v))\right\}$
- deduce and apply $\xi_v(k)=\xi_v\left\{\min_{\xi_v,u}(J(u,v))\right\}$
- back to step 1

B. Results

The application of the control law generated by the proposed algorithm allows the stabilization of temperature with an error less than two degrees in the bad cases figure (2, 3 and 4) despite the power quantification

VI. Conclusion

In this paper we have presented a modelisation of a multivariable non linear system by a multimodel. The synthesis of a multiobserver with localized poles by D-stability approach which is useful for the development of control law making a variety of the heating power and the speed of ventilation. An algorithm has been proposed to lessen the impact of quantification of control. The results were incentive for completing the control of this system taking into consideration humidity.

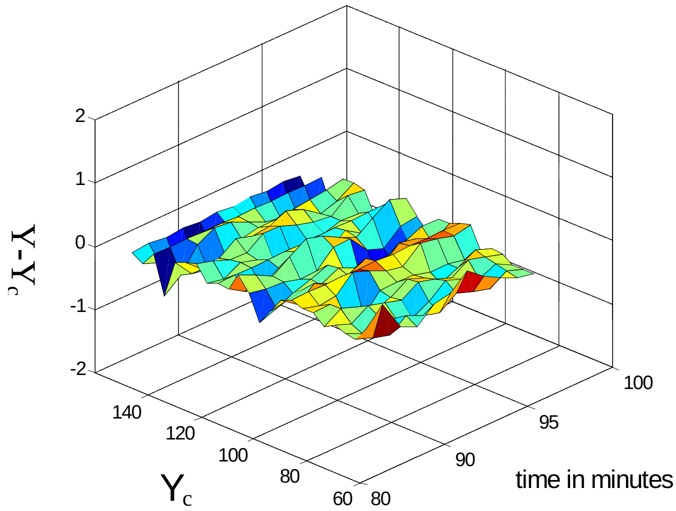

Fig.2 Evolution of errors for different Y_c

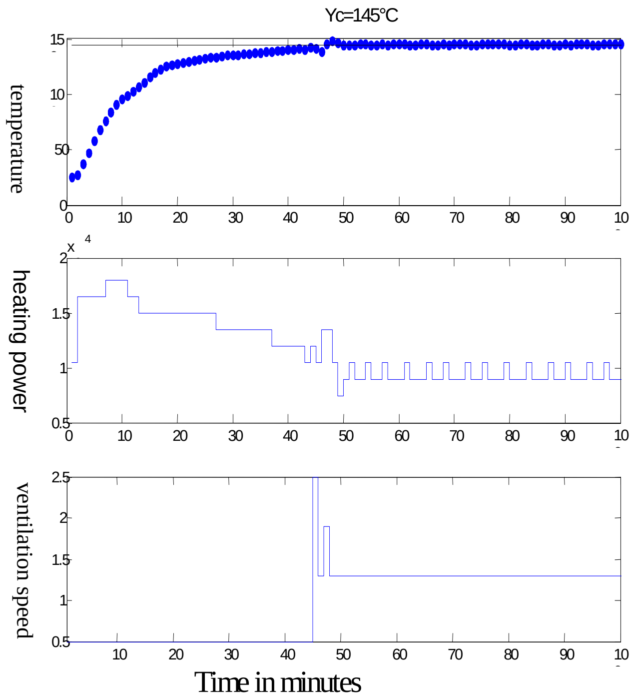

Fig.3 Evolution of temperature, heating power and ventilation speed for Y_c = 145°C

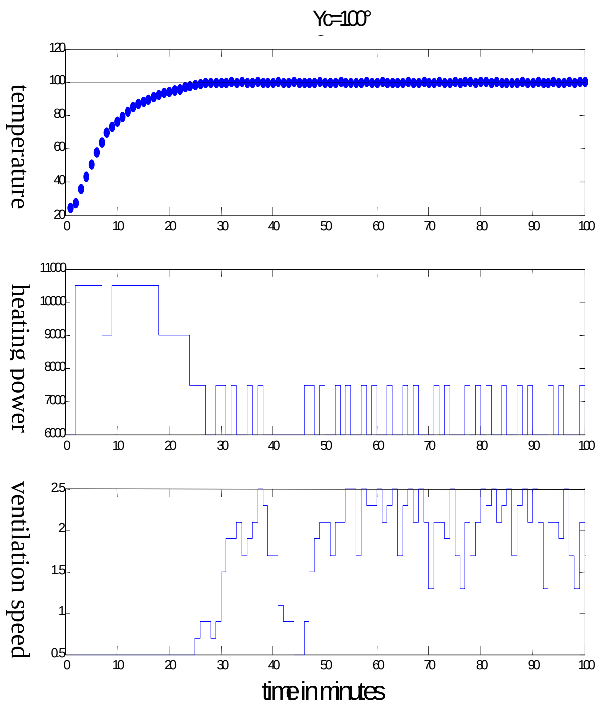

Fig.4 Evolution of temperature, heating power and ventilation speed for Y_c = 100°C

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