



$$\rho(\tau) = \frac{\sigma^2}{2} [(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}]. \quad (4)$$

The normalized ACF of FGN is given by

$$R(\tau) = \frac{1}{2} [(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}]. \quad (5)$$

Below, we use  $R(k)$  ( $k$  is integer) to indicate the ACF of FGN in the discrete case.

**Note 2.** A series is of LRD if its ACF is non-summable and it is of short-range dependence (SRD) if its ACF is summable [2].  
□

**Note 3.** FGN for  $H \in (0.5, 1)$  is of LRD while it is of SRD for  $H \in (0, 0.5)$ . □

### B. $H$ Estimation

The parameter  $H$  plays a vital role in time series with LRD. There are various methods for  $H$  estimation, see e.g., [1, 2]. This paper uses the method introduced in [8].

A series measured in practice is of finite length. In fact, it is of finite length when numeric computation is involved. Let  $R$  be the ACF of a traffic series  $s(i)$  with LRD. Then, for  $H \in (0.5, 1)$ ,

$$R(k) = \frac{E\{[s(i+k) - \mu][s(i) - \mu]\}}{\sigma^2} \sim ck^{2H-2} (k \rightarrow \infty),$$

where  $c > 0$  is a constant,  $\mu = E(s)$ , where  $E$  is the mean operator. Without losing generality, the maximum possible length of  $R$  is assumed to be  $N$ . Define the norm of  $R$  as the inner product given by

$$\|r\| = \sqrt{\langle r, r \rangle} = \sqrt{\sum_{k=0}^{N-1} |r|^2}, N \in I. \quad (6)$$

Then, the following

$$l_N^2 = \left\{ r; \sqrt{\sum_{k=0}^{N-1} |r|^2} < \infty \right\} \quad (7)$$

is a Hilbert space [8,16].

Define the set  $\mathcal{E}$  as

$$\mathcal{E} = \{R; R(k) = 0.5[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}], H \in (0.5, 1), (k = 0, 1, \dots, N-1)\}, \quad (8)$$

Then,

$$\mathcal{E} \subseteq l_N^2. \quad (9)$$

According to the theorem of existence of a unique minimizing element in Hilbert space [8,16], for an ACF of a real series  $r \in l_N^2$ , there exists a unique  $R \in \mathcal{E}$  such that

$$\|r - R\| = \inf_{a \in \mathcal{E}} \|r - a\|. \quad (10)$$

Let

$$J(H) = \frac{1}{N} \sum_k [r(k; H) - R(k; H)]^2. \quad (11)$$

Then, minimizing  $J(H)$  yields an estimate

$$H_0 = \arg \min J(H). \quad (12)$$

The value of  $J(H_0)$  is the minimum mean square error, which is denoted by  $M^2(R) = E[(r - R)^2]$ .

The research thought is stated like this. By investigating the  $M^2(R)$ s of 28 real traces, we may experimentally observe the

error order of magnitude by the curve fitting of ACF modeling. Moreover, we may explore the possible cause that produces errors by using FGN model.

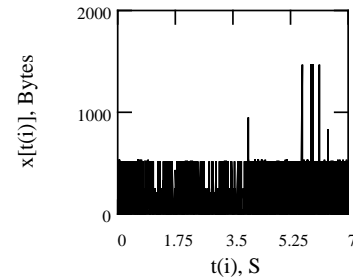
## III. EXPERIMENTAL INVESTIGATIONS

### A. Real Data Used

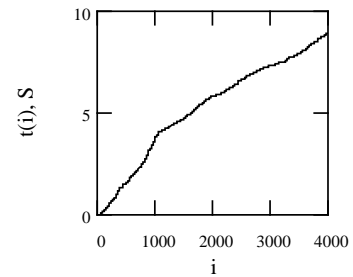
Real data used in this paper consist of 28 series. They are 6 series of TCP traffic (Table 1), 10 series of UDP traffic (Tables 2 and 3), 6 of IP traffic (Table 4), and 6 of OTHER traffic (Table 5). The series with the prefix DEC were measured at Digital Equipment Corporation, those with Lbl were recorded at the Lawrence Berkeley Laboratory, and the series with NUS were collected at the National Univeristy of Singapore. In Tables 1-5, the first column stands for series name, the second for record date, and the third for series length. We denote  $R(k)$  by  $R(k; H)$  for facilitating the illustrations in what follows.

### B. Demonstrations

Demonstration with DEC-pkt-1.TCP: The series  $x[t(i)]$  of DEC-pkt-1.TCP is indicated in Fig. 1 (a) and timestamp series  $t(i)$  is in Fig. 1 (b). The interarrival series  $s(i)$  is in Fig. 2. The measured ACF of  $s(i)$  is shown in Fig. 3 (a). Minimizing  $J(H)$  yields  $H_0 = 0.923$  with  $M^2(R) = 2.264 \times 10^{-3}$ . Therefore, the modeled ACF  $R(k)$  of  $s(i)$  of DEC-pkt-1.TCP using FGN is indicated in Fig. 3 (b). Fig. 3(c) shows the fitting the data. By eye, one sees that FGN does not satisfactorily fits the ACF of  $s(i)$  of DEC-pkt-1.TCP for short-term lags.



(a)



(b)

Fig. 1. Real traffic series DEC-pkt-1.TCP. (a). series in packet size  $x[t(i)]$ . (b) Timestamp series  $t(i)$ .

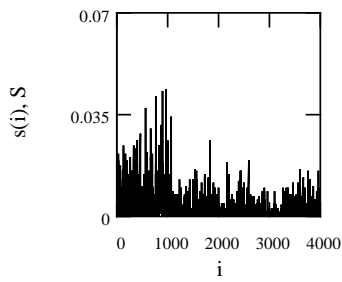
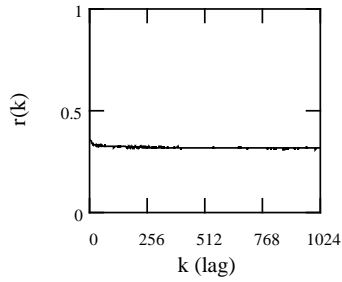
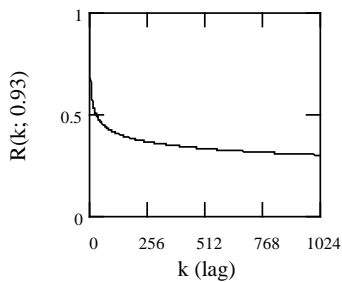


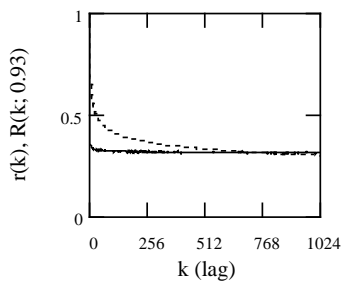
Fig. 2. Interarrival series  $s(i)$  of DEC-pkt-1.TCP.



(a)



(b)



(c) —  $r(k)$ , .....  $R(k)$ .

Fig. 3. Modeling procedure. (a).  $r(k)$ : Measured ACF of  $s(i)$  of DEC-pkt-1.TCP. (b).  $R(k)$ : Modeled ACF based on FGN. (c). Fitting the data.

Demonstration with DEC-pkt-1.UDP: Real series  $t(i)$  for DEC-pkt-1.UDP is shown in Fig. 4 and  $s(i)$  in Fig. 5, respectively. The measured ACF of  $s(i)$  is shown in Fig. 6 (a). Minimizing  $J$  yields  $H_0 = 0.945$  with  $M^2(R) = 6.09 \times 10^{-3}$ . Fig. 6 (b) indicates the modeled ACF using FGN and Fig. 6 (c) shows the fitting the data of ACF of  $s(i)$  of DEC-pkt-1.UDP based on FGN.

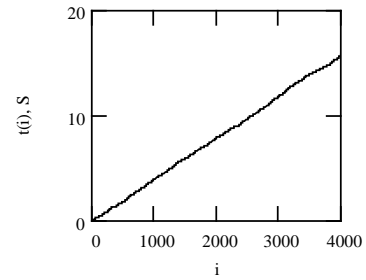


Fig. 4. Real series  $t(i)$  for DEC-pkt-1.UDP.

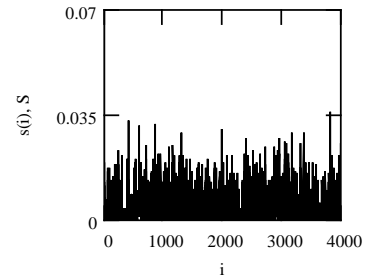
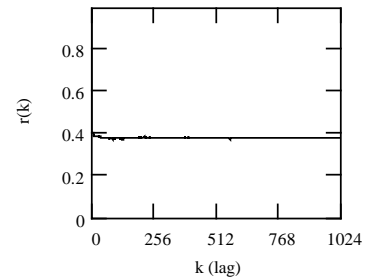
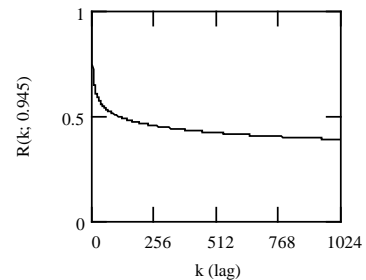


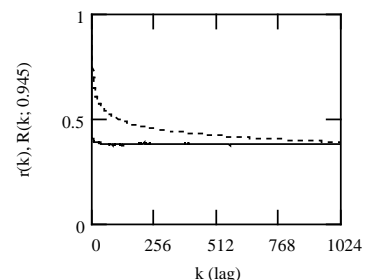
Fig. 5. Real series  $s(i)$  for DEC-pkt-1.UDP.



(a)



(b)



(c) —  $r(k)$ , .....  $R(k)$ .

Fig. 6. Modeling procedure. (a). Measured ACF of  $s(i)$  of DEC-pkt-1.UDP. (b).  $R(k)$ : Modeled ACF based on FGN. (c). Fitting the data.

Demonstration with DEC-pkt-1.IP: Timestamp series for DEC-pkt-1.IP is plotted in Fig. 7 and  $s(i)$  in Fig. 8. The measured ACF of  $s(i)$  is in Fig. 9 (a). Minimizing  $J$  yields  $H_0 = 0.958$  with  $M^2(R) = 4.133 \times 10^{-3}$ . Fig. 9 (b) indicates the modeled ACF  $R(k)$  of  $s(i)$  of DEC-pkt-1.IP using FGN and Fig. 9 (c) fitting the data.

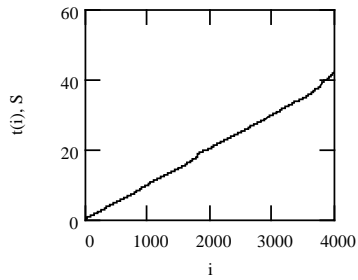


Fig. 7. Real series  $t(i)$  for DEC-pkt-1.IP.

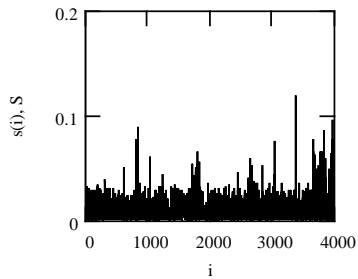
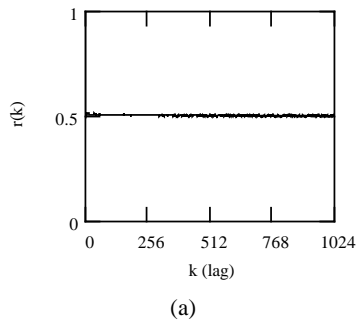
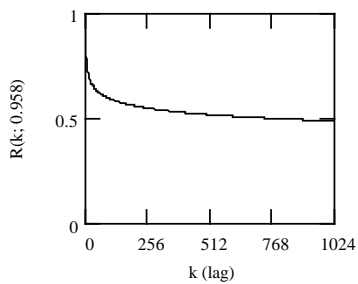


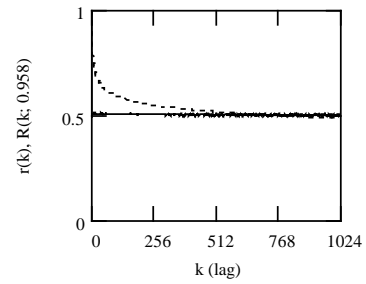
Fig. 8. Real series  $s(i)$  for DEC-pkt-1.IP.



(a)



(b)



(c) —  $r(k)$ , .....  $R(k)$ .

Fig. 9. Modeling procedure. (a). Measured ACF of  $s(i)$  of DEC-pkt-1.IP. (b).  $R(k)$ : Modeled ACF based on FGN. (c). Fitting the data.

Demonstration with DEC-pkt-1.OTHER: The series  $t(i)$  of DEC-pkt-1.OTHER is indicated in Fig. 10 and  $s(i)$  in Fig. 11. The measured ACF of  $s(i)$  is in Fig. 12 (a). Minimizing  $J$  yields  $H_0 = 0.937$  with  $M^2(R) = 5.038 \times 10^{-3}$ . Fig. 12 (b) indicates the modeled ACF model of  $s(i)$  of DEC-pkt-1.OTHER using FGN. Fig. 12 (c) gives the fitting the data.

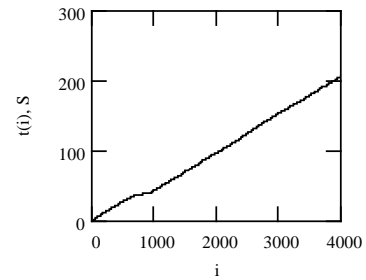


Fig. 10. Real series  $t(i)$  for DEC-pkt-1.OTHER.

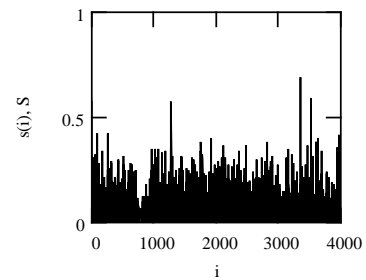
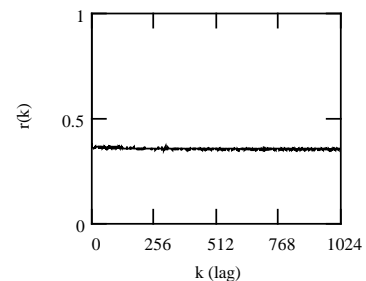


Fig. 11. Real series  $s(i)$  for DEC-pkt-1.OTHER.



(a)

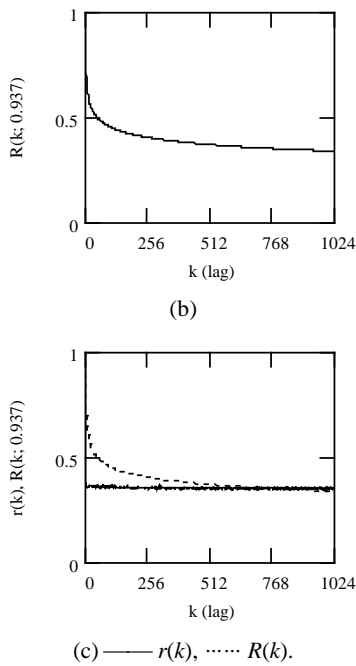


Fig. 12. Modeling procedure. (a). Measured ACF of  $s(i)$  of DEC-pkt-1.OTHER. (b).  $R(k)$ : Modeled ACF based on FGN. (c). Fitting the data.

### C. Summary

We summarize the experimental results for all 28 series in the columns 5-6 in Tables 1-4, where the fourth column stands for  $H_0$  estimate and the fifth for  $M^2(R)$ .

Table 1. Six real series of TCP traffic.

Series name	Record date	Series length	$H_0$	$M^2(R)$
DEC-pkt-1.TCP	08Mar95	$3.3 \times 10^6$	0.923	$2.264 \times 10^{-3}$
DEC-pkt-2.TCP	09Mar95	$3.9 \times 10^6$	0.920	$2.282 \times 10^{-3}$
DEC-pkt-3.TCP	09Mar95	$4.3 \times 10^6$	0.925	$2.270 \times 10^{-3}$
DEC-pkt-4.TCP	09Mar95	$5.7 \times 10^6$	0.922	$2.320 \times 10^{-3}$
Lbl-pkt-4.TCP	21Jan94	862946	0.930	$2.208 \times 10^{-3}$
Lbl-pkt-5.TCP	28Jan94	710614	0.925	$2.220 \times 10^{-3}$

Table 2. Four real series of UDP traffic.

Series name	Record date	Series length	$H_0$	$M^2(R)$
NUS-1.UDP	24Mar03	$1 \times 10^6$	0.920	$3.654 \times 10^{-3}$
NUS-2.UDP	24Mar03	$1 \times 10^6$	0.915	$3.469 \times 10^{-3}$
NUS-3.UDP	26Mar03	$1 \times 10^6$	0.915	$3.415 \times 10^{-3}$
NUS-4.UDP	26Mar03	$1 \times 10^6$	0.920	$3.534 \times 10^{-3}$

Table 3. Six real series of UDP traffic.

Series name	Record date	Series length	$H_0$	$M^2(R)$
DEC-pkt-1.UDP	08Mar95	829759	0.935	$7.729 \times 10^{-3}$
DEC-pkt-2.UDP	09Mar95	805802	0.935	$2.881 \times 10^{-3}$
DEC-pkt-3.UDP	09Mar95	1035457	0.935	$2.883 \times 10^{-3}$
DEC-pkt-4.UDP	09Mar95	1187454	0.935	$2.886 \times 10^{-3}$
Lbl-pkt-4.UDP	21Jan94	33744	0.904	$2.886 \times 10^{-3}$
Lbl-pkt-5.UDP	28Jan94	69358	0.875	$2.182 \times 10^{-3}$

Table 4. Six real series of IP traffic.

Series name	Record date	Series length	$H_0$	$M^2(R)$
DEC-pkt-1.IP	08Mar95	225237	0.955	$2.416 \times 10^{-3}$
DEC-pkt-2.IP	09Mar95	335556	0.938	$2.884 \times 10^{-3}$
DEC-pkt-3.IP	09Mar95	325833	0.900	$2.517 \times 10^{-3}$
DEC-pkt-4.IP	09Mar95	511287	0.935	$2.624 \times 10^{-3}$
Lbl-pkt-4.IP	21Jan94	303055	0.890	$4.264 \times 10^{-3}$
Lbl-pkt-5.IP	28Jan94	195241	0.890	$4.312 \times 10^{-3}$

Table 5. Six real series of OTHER traffic.

Series name	Record date	Series length	$H_0$	$M^2(R)$
DEC-pkt-1.OTHER	08Mar95	74135	0.931	$2.893 \times 10^{-3}$
DEC-pkt-2.OTHER	09Mar95	78021	0.931	$3.040 \times 10^{-3}$
DEC-pkt-3.OTHER	09Mar95	105410	0.931	$2.874 \times 10^{-3}$
DEC-pkt-4.OTHER	09Mar95	92361	0.931	$2.662 \times 10^{-3}$
Lbl-pkt-4.OTHER	21Jan94	121140	0.878	$1.105 \times 10^{-3}$
Lbl-pkt-5.OTHER	28Jan94	401231	0.890	$2.012 \times 10^{-3}$

## IV. CONCLUSION

The results in Tables 1-5 suggest that  $s(i)$  of traffic (TCP, UDP, IP, OTHER) is of LRD and the modeling accuracy of ACF based on FGN is in the order of magnitude of  $10^{-3}$ . The plots in Fig. 3 (c), Fig. 6 (c), Fig. 9 (c), and Fig. 12 (c) imply that FGN may not satisfactorily fit the short-term lags of those traffic data. This might likely be the main error source with respect to the curve fitting of ACF modeling based on FGN.

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