For the URVs there are some parametric uncertainties in the dynamic model (2), and certain parameters are generally unknown. Hence, parameter estimation is necessary in case of model-based control. For this purpose it is assumed that the equations of motion (2) are linear according to a parameter vector $\mathbf{p}$, i.e. [8]:

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g}(\boldsymbol{\eta}) \cong \mathbf{Y}(\boldsymbol{\eta}, \mathbf{v}, \dot{\mathbf{v}})\mathbf{p} = \boldsymbol{\tau} \qquad (3)$$

where $\mathbf{Y}(\boldsymbol{\eta}, \mathbf{v}, \dot{\mathbf{v}})$ is a known matrix function of measured signals, usually referred as the regressor matrix (dimension $n \times r$), and $\mathbf{p}$ is a vector of uncertain or unknown parameters.

Let estimates of the matrices $\mathbf{M}$, $\mathbf{C}(\mathbf{v})$, $\mathbf{D}(\mathbf{v})$ and the vector $\mathbf{g}(\boldsymbol{\eta})$ be described as $\hat{\mathbf{M}}$, $\hat{\mathbf{C}}(\mathbf{v})$, $\hat{\mathbf{D}}(\mathbf{v})$ and $\hat{\mathbf{g}}(\boldsymbol{\eta})$. If model parameters are known with some accuracy the following nonlinear control law can be applied [5], [8]:

$$\begin{aligned} \boldsymbol{\tau} &= \hat{\mathbf{M}}\dot{\mathbf{u}} + \hat{\mathbf{C}}(\mathbf{v})\mathbf{u} + \hat{\mathbf{D}}(\mathbf{v})\mathbf{u} + \hat{\mathbf{g}}(\boldsymbol{\eta}) - \mathbf{K}_D\mathbf{s} = \\ &= \mathbf{Y}(\boldsymbol{\eta}, \mathbf{v}, \mathbf{u}, \dot{\mathbf{u}})\hat{\mathbf{p}} - \mathbf{K}_D\mathbf{s} \end{aligned} \qquad (4)$$

where:

$\mathbf{M}$ – inertia matrix (including added mass);

$\mathbf{K}_D$ – positive definite diagonal matrix;

$\mathbf{s} = \mathbf{e}_2 + \boldsymbol{\Lambda}\mathbf{e}_1$;

$\mathbf{e}_1 = \boldsymbol{\eta} - \boldsymbol{\eta}_d$;

$\mathbf{e}_2 = \mathbf{v} - \mathbf{v}_d$;

$\mathbf{u} = \mathbf{v}_d - \boldsymbol{\Lambda}\mathbf{e}_1$;

$\boldsymbol{\Lambda}$ – positive definite weighting matrix.

Choosing the parameter update law as [4], [8]:

$$\dot{\hat{\mathbf{p}}} = -\boldsymbol{\Gamma}\mathbf{Y}^T(\boldsymbol{\eta}, \mathbf{v}, \mathbf{u}, \dot{\mathbf{u}})\mathbf{s} \qquad (5)$$

where $\boldsymbol{\Gamma}$ is a positive definite symmetric matrix, stability of the control system and convergence $\mathbf{s}$ to zero is guaranteed.

A block diagram of the control system with parameter adaptation law shows Fig. 1.

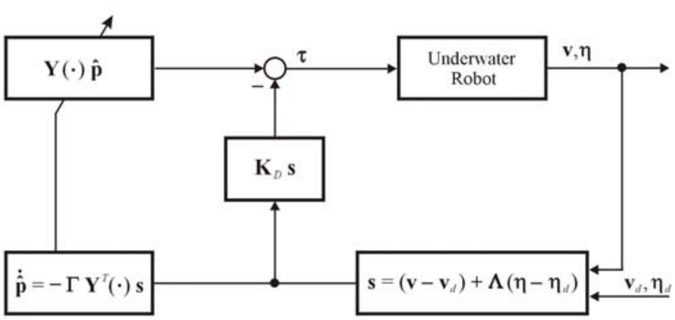


Fig. 1 Diagram showing the parameter adaptation law

III. Simulation study

A main task of the designed tracking control system is to minimize distance of attitude of the robot's centre of gravity to the desired trajectory under assumptions:

1. the robot can move with varying linear velocities u, v, w and angular velocity r;
2. its velocities u, v, w, r and coordinates of position x, y, z and heading ψ are measurable;
3. the desired trajectory is given by means of set of way-points $\{(x_{di}, y_{di}, z_{di})\}$;
4. segments of the reference trajectory between two successive way-points are defined as smooth and bounded curves;
5. the command signal $\boldsymbol{\tau}$ consists of four components: $\tau_1 = \tau_X = X$, $\tau_2 = \tau_Y = Y$, $\tau_3 = \tau_Z = Z$ and $\tau_4 = \tau_N = N$ calculated from the control law (4).

A structure of the proposed control system is depicted in Fig. 2.

To validate the performance of the developed nonlinear control law, simulations results using the MATLAB/Simulink environment are presented below. The model of the vehicle basis on a real construction of an underwater robot called "Coral" designed and built for the Polish Navy. The URV is an open frame robot controllable in four degrees of freedom, being 1.5 m long and having a propulsion system consisting of six thrusters. Displacement in horizontal plane is done by means of four ones which generate force up to ±750 N assuring speed up to ±1.2 m/s and ±0.6 m/s in x and y direction, consequently. All parameters of the robot's dynamics are presented in the Appendix.

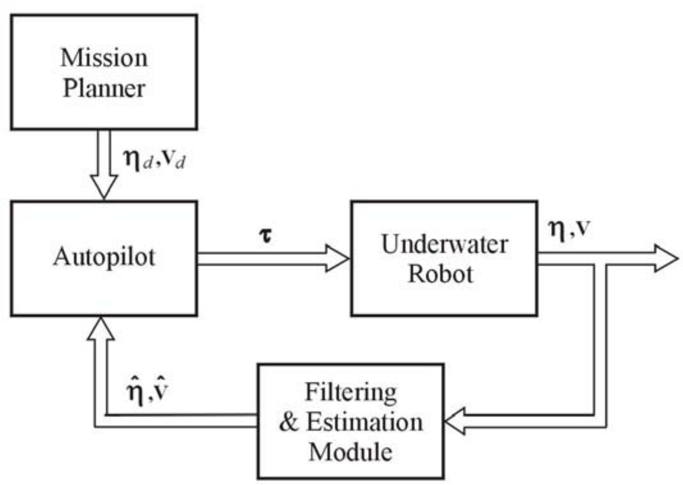


Fig. 2 Main parts of the control system

Numerical simulations have been made to confirm quality of the proposed control algorithm for the following assumptions:

1. the robot has to follow the desired trajectory beginning from (10 m, 10 m, 0 m), passing target way-points: (10 m, 10 m, -5 m), (10 m, 90 m, -5 m), (30 m, 90 m,

-5 m), (30 m, 10 m, -5 m), (60 m, 10 m, -5 m), (60 m, 90 m,-5 m), (60 m, 90 m, -15 m), (60 m, 10 m,-15 m), (30 m, 10 m, -15 m), (30 m, 90 m,-15 m), (10 m, 90 m, -15 m) and ending in (10 m, 10 m, -15 m);
2. a turning point is reached if the robot is inside of a half meter circle of acceptance;
3. the sea current interacts the robot with maximum velocity 0.3 m/s and direction 135^0;
4. dynamic equations of the robot's motion are integrated with higher frequency (18 Hz) than the rest of modules (6 Hz).

It has been assumed that the time-varying reference trajectories at the way-point i to the next way-point i+1 are generated using desired speed profiles [7], [8]. Such approach allows us to keep constant speed along certain part of the path. For these assumptions and the following initial conditions:

$$\begin{aligned} &\eta_{dk}(t_b)=\eta_0\,, && \dot\eta_{dk}(t_b)=\dot\eta_0 \\ &\eta_{dk}(t_f)=\eta_1\,, && \dot\eta_{dk}(t_f)=\dot\eta_1 \\ &\max\dot\eta_{dk}(t)=\dot\eta_{\max} \end{aligned} \tag{6}$$

where $k=\overline{1,4}$, the i^{th} segment of the trajectory in a period of time $t\in\langle t_b,t_f\rangle$ has been modelled according to the expression [8]:

$$\eta_{dk}(t)=\begin{cases} \eta_0+\dfrac{\dot\eta_{\max}-\dot\eta_0}{2t_m}t^2 & t_b\le t\le t_m \\ \dfrac{\eta_1+\eta_0-\dot\eta_{\max}(t_f-2t_m)}{2}+ & t_m<t\le t_f-t_m \\ +\dot\eta_{\max}(t-t_m) & \\ \eta_1-\dfrac{\dot\eta_{\max}-\dot\eta_1}{2t_m}(t_f-t)^2 & t_f-t_m<t\le t_f \end{cases} \tag{7}$$

where $t_m=t_f-\dfrac{\eta_1-\eta_0}{\dot\eta_{\max}}$.

The algorithm of control worked out basis on the simplified URV model proposed in [4], [9]:

$$\mathbf{M}_d\dot{\mathbf{v}}+\mathbf{D}_d(\mathbf{v})\mathbf{v}=\boldsymbol{\tau} \tag{8}$$

where all kinematics and dynamics cross-coupling terms are neglected. Here $\mathbf{M}_d$ and $\mathbf{D}_d$ ($\mathbf{v}$) are diagonal matrices with the diagonal elements of the inertia and damping matrices, consequently. Uncertainties in the above model are compensated in the designed control system.

The model (8) for motion of four DOF takes a form:

$$\begin{aligned} m_X\dot u+d_X|u|u&=\tau_X \\ m_Y\dot v+d_Y|v|v&=\tau_Y \\ m_Z\dot w+d_Z|w|w&=\tau_Z \\ m_N\dot r+d_N|r|r&=\tau_N \end{aligned} \tag{9}$$

Define the parameter vector $\mathbf{p}$ in a form $\mathbf{p}=[m_X \;\; d_X \;\; m_Y \;\; d_Y \;\; m_Z \;\; d_Z \;\; m_N \;\; d_N]^T$ the expression (8) can be written as:

$$\mathbf{Y}(\mathbf{v},\dot{\mathbf{v}})\mathbf{p}=\boldsymbol{\tau} \tag{10}$$

where:

$$\mathbf{Y}(\mathbf{v},\dot{\mathbf{v}})=\begin{bmatrix} \dot u & |u|u & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot v & |v|v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dot w & |w|w & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dot r & |r|r \end{bmatrix}.$$

The regulation problem has been examined under interaction of environmental disturbances, i.e. sea currents. To simulate such influence on robot's motion the current velocity V_c was assumed to be slowly varying and having a fixed direction. For computer simulations it was calculated by using the 1st order Gauss-Markov process [5]:

$$\dot V_c+\mu V_c=\omega \tag{11}$$

where ω is a Gaussian white noise, $\mu\ge 0$ is a constant and $0\le V_c(t)\le V_{c\max}$.

Results of track-keeping in the presence of external disturbances and the courses of command signals are presented in Fig. 3.

It can be seen that the proposed autopilot enhanced good tracking control of the desired trajectory in the spatial motion. The main advantage of the approach is using the simple nonlinear law to design the autopilot and its high performance for relative large sea current disturbances (comparable with resultant speed of the robot).

During simulations it was assumed that the true values of components of the vector $\mathbf{p}$ are unknown. An evaluation process started from the level of half of nominal values of mass and damping coefficients. Time histories of estimated parameters during tracking are presented in Fig. 4.

IV. Conclusion

In the paper the nonlinear control system for the underwater robot has been described. The obtained results of simulation study allows to state that the proposed algorithm with parameter adaptation law assures a high accuracy of tracking control along a predefined trajectory and shows its numerical simplicity and usefulness for practical applications.

Disturbances from the sea current were added to verify the performance and confirm correctness and robustness of the approach.

Further works are devoted to the problem of tuning of the autopilot parameters in relation to the robot's dynamics.

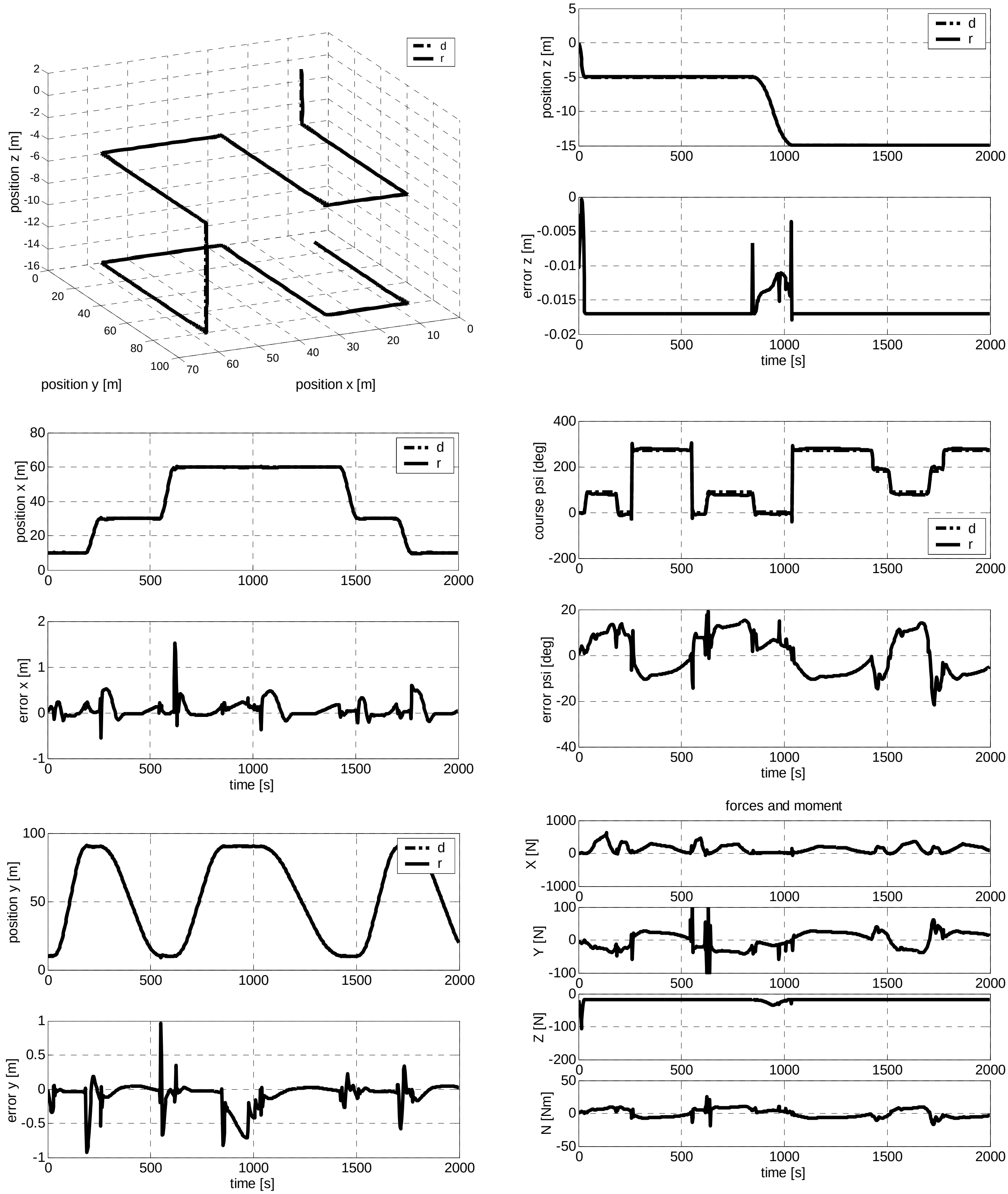
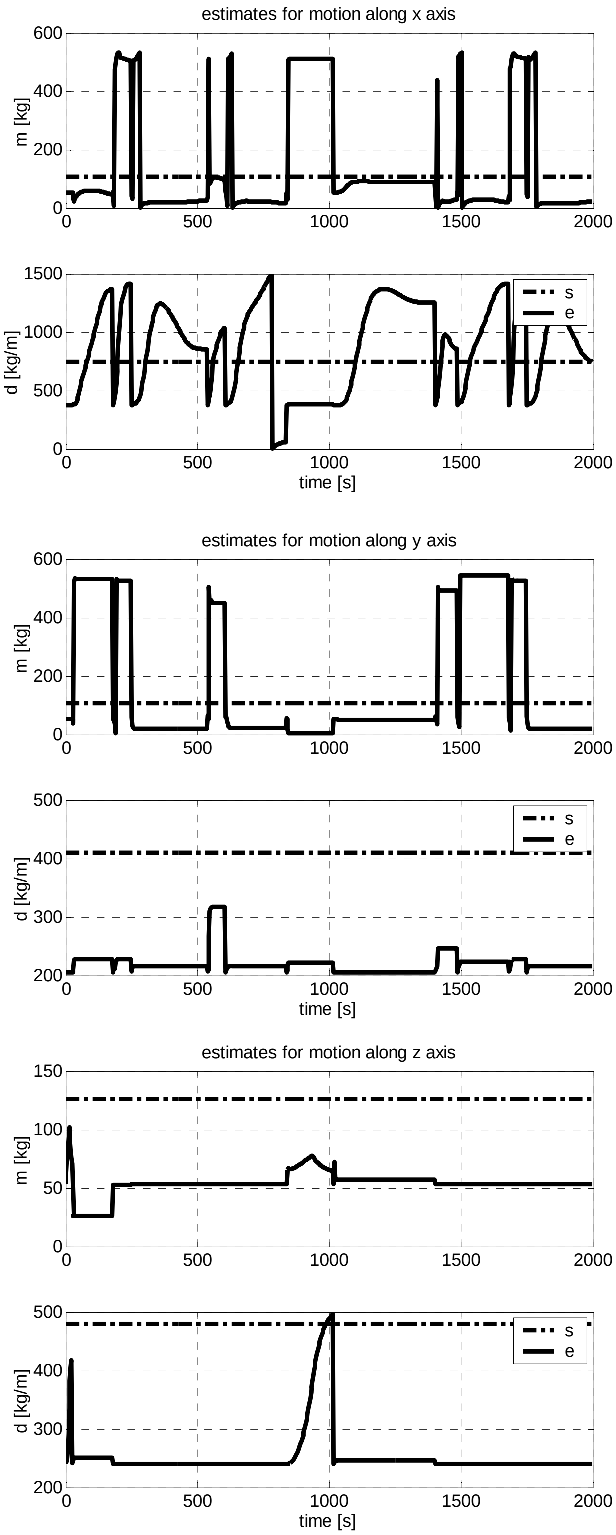


Fig. 3 Track-keeping control under interaction of sea current disturbances (maximum velocity 0.3 m/s and direction 135^0): desired (d) and real (r) trajectories (left upper plot), *x*-, *y*-, z-position and their errors ($2^{nd} \div 4^{th}$ plots), course and its error (5^{th} plot), commands (right low plot)



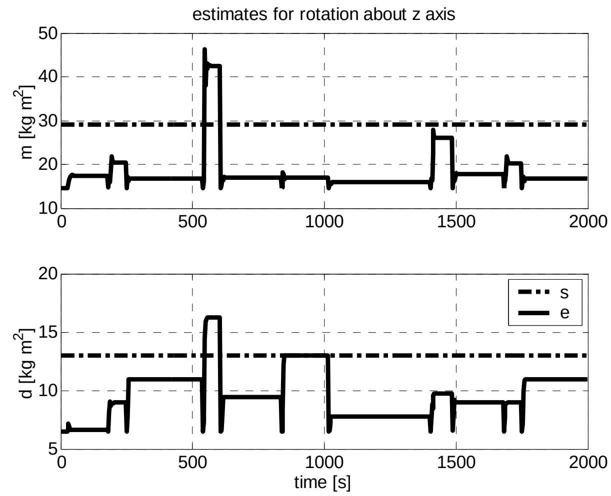


Fig. 4 Estimates of mass and damping coefficients: set value (s) and estimate value (e)

Appendix

The set of parameters used in computer simulations:

1. the URV model:

$$\mathbf{M} = diag\{99.0 \quad 108.5 \quad 126.5 \quad 8.2 \quad 32.9 \quad 29.1\}$$

$$\mathbf{D}(\mathbf{v}) = \mathbf{D} + \mathbf{D}_d(\mathbf{v})$$

where:

$$\mathbf{D} = diag\{10.0 \quad 0.0 \quad 0.0 \quad 0.2 \quad 1.9 \quad 1.6\}$$

$$\mathbf{D}_d(\mathbf{v}) = diag\left\{\begin{matrix} 227.2|u| & 405.4|v| & 478.0|w| & & & \\ & & & 3.2|p| & 14.0|q| & 12.9|r| \end{matrix}\right\}$$

$$\mathbf{C}(\mathbf{v}) = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}$$

where:

$$\mathbf{C}_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_{12} = \begin{bmatrix} 0 & 26w & -28v \\ -26w & 0 & 18u \\ 2v & -18u & 0 \end{bmatrix}$$

$$\mathbf{C}_{21} = \mathbf{C}_{12}$$

$$\mathbf{C}_{22} = \begin{bmatrix} 0 & 5r & -6q \\ -5r & 0 & p \\ 6q & -p & 0 \end{bmatrix}$$

$$\mathbf{g}(\boldsymbol{\eta}) = \begin{bmatrix} -17\sin(\theta) \\ 17\cos(\theta)\sin(\phi) \\ 17\cos(\theta)\cos(\phi) \\ -279\cos(\theta)\sin(\phi) \\ -279(\sin(\theta)+\cos(\theta)\cos(\phi)) \\ 0 \end{bmatrix}$$

2. nonlinear control law:

$$\mathbf{K}_D = diag\{100 \quad 20 \quad 50 \quad 0 \quad 0 \quad 10\}$$

$$\Gamma = diag\{200 \quad 10 \quad 20 \quad 0 \quad 0 \quad 20\}$$

$$\Lambda = diag\{50 \quad 10 \quad 20 \quad 0 \quad 0 \quad 20\}.$$

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