Modelling of a Neuron and Point Contact Transistor

Atsushi Fukasawa and Yumi Takizawa

***Abstract*—**Modelling of a neuron and Point Contact Transistor (PCT) are presented. They generate electrical pulses through operations for amplification and feedback. A p – n boundary is analyzed to form a depletion layer in electrolyte. Where, p and n stand for major charges carrying signals. Electro-physical modeling of a neuron is given by three zones and two depletion layers under the membrane. A neuron is proved to operate as an amplifier or as a pulse generator to transmit electrical signals. Electrical modeling of a neural group is given by mutual coupling among neurons. This coupling provides a neural group with sensing capability of time, space, and transition of events. It is lastly proved that commonality exists between a neuron and PCT.

***Keywords*—** Activity of neuron, electro-physical modelling, amplifier and pulse generator, self-systematization of neural group.

I. Introduction

IT is assumed essentially important to clarify principle of operations in neural systems in brain, which are almost unknown still now on.

Divergence has been the main aspect for the study of neural systems depending on the difference in spices, organ, tissues, and so on. Locations of active parts and paths of signals transmissions have been clarified. But the most interested information is the principles of operations and organizations of neural systems to realize sophisticated capabilities of functions against external and internal stimuli.

In this study, a neuron is analyzed by new basis of biology with electro-physical modeling of a neuron. It was found that a neuron operates as an amplifier or a pulse generator to transmit signal information.[1][2]

Because of defect of knowledge on operational principle of a neuron, essential study of systematization of neural group has not been done.

In this paper, a neural group is also analyzed by new basis of telecommunication system knowledge. It was found that a neural group operates as a synchronized system with holding common time inside the system.

In this paper, failures are presented regarding to the membrane model based on electrical and physical requirements.

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II. Dynamics of Electric Charges at a Boundary in Electrolyte

A. Formation of electrical zones and a depletion layer

When electric charges are injected into a zone in electrical medium, charge density at the zone becomes higher and the other zone remains lower. It is assumed that quantity of injected charges is little and velocity of charges is low in the medium. Special phenomena are induced at a boundary between two zones as shown in Fig. 1.

Phase 1

Injected p-charges diffuse to n-zone, and n-charges diffuse to p-zone by the force of gradient of density F_D.

Phase 2

Coulomb’s force FC (force by potential gradient) appears between diffused p- and n-ions. Directions of forces F_D and F_C are opposite. When they are balanced, diffusion is ceased.

Phase 3

A pair of space charges appears at both sides of the boundary. Potential difference appears in the boundary. And electric charges are driven outside the boundary, and two zones and a depletion layer formed at the boundary.

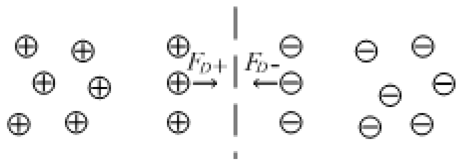
B. Depth and Capacitance of a Depletion Layer

Special phenomena described above is analyzed theoretically by the Maxwell-Hertz equations.

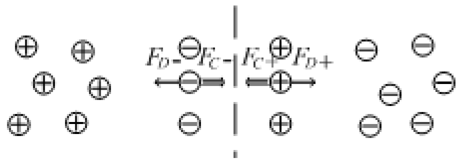
$$\left.\begin{aligned} &\mathrm{rot}\, E + \frac{\partial B}{\partial t} = 0 \\ &\mathrm{rot}\, H - \frac{\partial D}{\partial t} = i \\ &\mathrm{div}\, D = \rho \\ &\mathrm{div}\, B = 0 \end{aligned}\right\} \qquad (1)$$

$$D = \varepsilon_e E, \quad B = \mu_e H$$

Phase 1: Diffusion of charges by gradient of density F_D.



Phase 2: Balance of diffusion F_D and Coulomb's force F_C.



Phase 3: Cease of diffusion and formation of;
(a) p-zone and n-zone, and
(b) space charges and depletion layer with depth *d*.

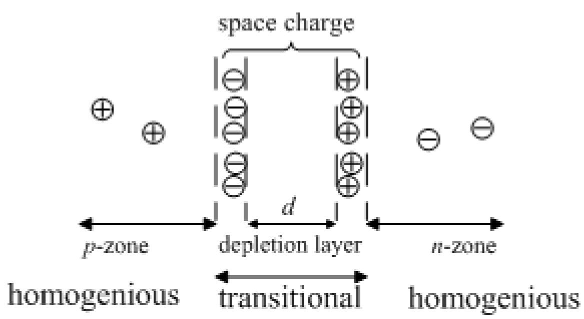


Fig. 1 Formation of zones and a depletion layer at a boundary.

where, *E*, *H* are electric and magnetic field strength, *D*, *B* are electric and magnetic flux density, and ε, μ are permittivity and permeability of medium respectively.

In cytoplasm, the followings are assumed;

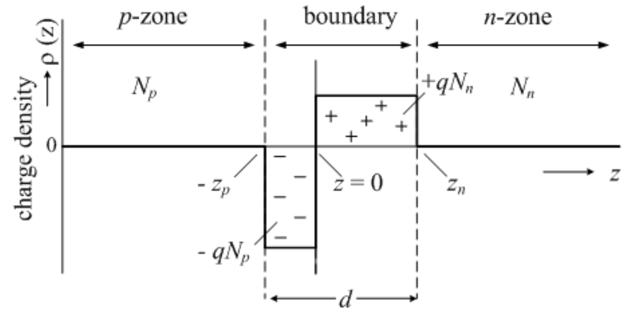
$$B = 0, \quad H = 0 \tag{2}$$

$$\mathrm{rot}\, E = 0 \tag{3}$$

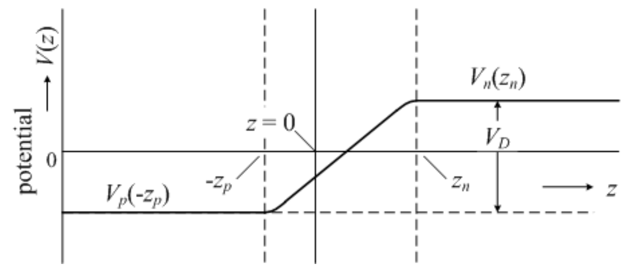
Eq.(1) reduces into Eq.(4).

$$\mathrm{div} D = \rho \tag{4}$$

Potential *V*(*z*) at a boundary is decided by true electric charge density $\rho(z)$ based on the Poison's equation.



(a) Distribution of true electric charge density ρ(*z*).



(b) Potentials *Vp*, *Vn*, and diffusion potential V_D.

Fig. 2 Distribution of true electric charge and diffusion potential of a boundary.

$$\frac{d^2 V(z)}{dz^2} = -\frac{\rho(z)}{\varepsilon_e} \tag{5}$$

where, *z* is the longitudinal axis of a neuron, ε_e is the permittivity of electrolyte solution.

True electric charge is defined as the charge unrestrained to any place. Then polarization charge at the membrane is removed from $\rho(z)$, because the polarization charge is restrained to the membrane in a neuron.

True electric charge density $\rho(z)$ is given by the followings, and is shown in Fig.2 (a).

$$\left.\begin{aligned} \rho(z) &= -q N_p \quad ; \quad -z_p \le z \le 0 \\ \rho(z) &= +q N_n \quad ; \quad 0 \le z \le z_n \end{aligned}\right\} \tag{6}$$

where *Np*, *Nn* are true electric charge densities at *p*- and *n*-side of the boundary. *q* is elementary electric charge.

The electrical diffusion potential *VD* is defined as follows.

$$V_D = V_n(z_n) - V_p(-z_p)$$

$$= \frac{q}{2\varepsilon_e}(N_p z_p{}^2 + N_n z_n{}^2) \qquad (7)$$

The depth of depletion layer is given as,

$$d = z_p + z_n \quad = \left(\frac{2\varepsilon_e (N_p + N_n)}{q N_p N_n} V_D \right)^{\frac{1}{2}} \qquad (8)$$

Now, bias *VB* is assumed applied to a boundary. When *VB* is applied reversely to *n*-zone against *p*-zone, the depth of depletion layer d_B with reverse bias V_B is given as follows.

$$d_B = \left(\frac{2\varepsilon_e (N_p + N_n)}{q N_p N_n} (V_D + V_B) \right)^{\frac{1}{2}} \qquad (9)$$

The positive charge *Q* per unit area at the boundary (*n*-side) is given as follows.

$$Q = qN_n z_n = \left| -qN_p z_p \right| = \left(\frac{2\varepsilon_e q N_p N_n)}{N_p + N_n} (V_D + V_B) \right)^{\frac{1}{2}} \qquad (10)$$

The structure is assumed as an equivalent capacity.

$$c = \left| \frac{dQ}{dV} \right| = \left(\frac{\varepsilon_e}{2} \frac{qN_p N_n}{N_p + N_n} \frac{1}{V_D + V_B} \right)^{\frac{1}{2}} \qquad (11)$$

When *VB* is applied forwardly at the boundary, the capacity is given changing V_B to - V_B .

III. Electro-Physical Modelling of a Neuron

A. *Depth and Capacity of Depletion Layers in a Neuron*

(1) Whole aspects of a neuron

A neuron is exhibited as a three-port bio-electrical device with dendrite, central part, and axon. Here, the transmission line part is deleted in actual axon.

These ports are assigned as input, ground, and output ports. The ends of dendrite and axon are composed of multiple branches which are connected to previous and post neurons with synapses. Biochemical and electrical couplings are formed by synapses. A bio-electrical modeling is given in Fig. 3. An excitatory synapse is shown in the figure.

(2) Signal *p*-ion injection to a resting neuron

During a neuron is resting, inner potential is kept negative and uniform inside the neuron. When neurotransmitters are released from previous neurons and accepted by the neuron, *p*-charges of Na^+ are injected into the dendrite. Injected *p*-ions play as an excitatory signal into the neuron.

(3) Dynamics of signal *p*-ions at the first depletion layer

The first depletion layer is formed between the dendrite and the central parts.

The depth d_1 and the equivalent input capacity c_d are given as;

$$d_1 = \left\{ \frac{2\varepsilon_e \ (N_d + N_c)}{q \ N_d \ N_c} \ (V_{D1} - V_{B1}) \right\}^{\frac{1}{2}} \qquad (12)$$

$$c_d = \left(\frac{\varepsilon_e}{2} \frac{qN_d N_c}{N_d + N_c} \frac{1}{V_{D1} - V_{B1}} \right)^{\frac{1}{2}} \qquad (13)$$

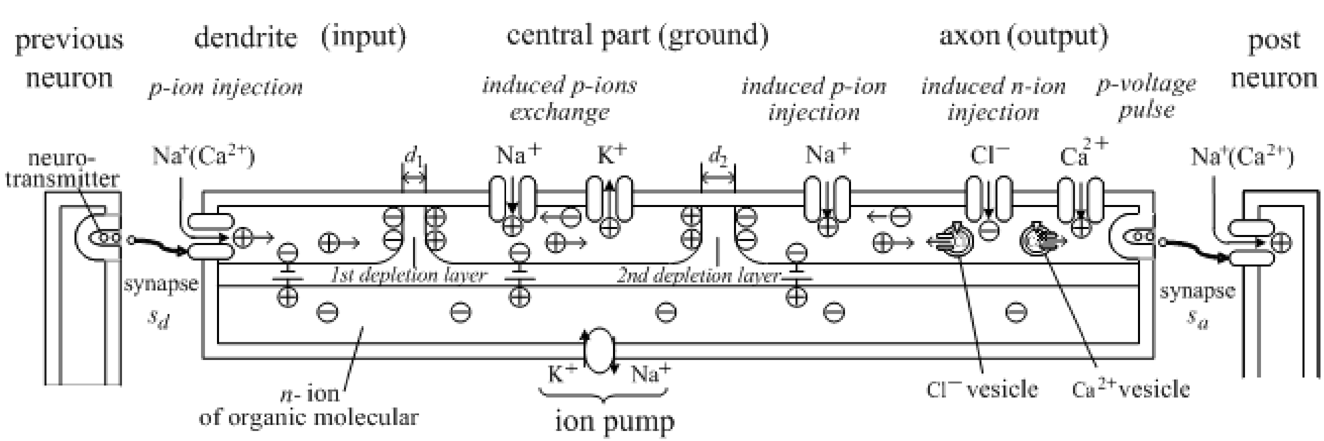


Fig. 3 Electro-physical modeling of an operating neuron. All branches of the dendrite and the axon of a neuron are gathered into each one port. Ion channels of the same kind are gathered in one in an area.

where, *Nd*, *Nc* are true electric charge density at the dendrite and the central part.

V_{D1} V_{B1} are diffusion potential and bias appeared forwardly at the first depletion layer.

The depth d_1 becomes narrower as injected *p*-ion at the dendrite and/or the forward-bias V_B increases. Then signal *p*-ions pass over the first depletion layer easily.

(4) *p*-ion injection to the axon

A part of signal *p*-ions reach the axon. Na^+ channels inject *p*-ions inside the neuron.

The potential in this zone is changed into positive, this area forms *p*-zone.

By Na^+ injection, charge allocation at the second depletion layer should be inverted as shown in Fig. 3. This configuration is defined by reverse diode with reverse bias voltage.

(5) Dynamics of signal *p*-ions at the second depletion layer

The second depletion layer is formed between the central part and the axon. The depth d_2 and the equivalent output capacity c_a are given as;

$$d_2 = \left\{ \frac{2\varepsilon_e \ (N_c + N_a)}{q \ N_c \ N_a} \ (V_{D2} + V_{B2}) \right\}^{\frac{1}{2}} \quad (14)$$

$$c_a = \left(\frac{\varepsilon_e}{2} \frac{qN_c N_a}{N_c + N_a} \frac{1}{V_{D2} + V_{B2}} \right)^{\frac{1}{2}} \quad (15)$$

where, *Nc* and *Na* are true electric charge density at the dendrite and the central part. V_{D2} V_{B2} are diffusion potential and bias appeared at the second depletion layer.

The depth d_2 becomes wider than depth d_1.

Signal *p*-ions pass over the second depletion layer by the force of thermal motion of ions.

(6) Dynamics of induced signal *n*-ions

When signal *p*-ions arrive at the axon, *n*-ions are injected into the axon by Cl^- channels.

n-ions move from the right to the left passing over the second and then the first depletion layers. The dynamics of *n*-ions from right to left is forward, and from left to right is reverse.

These *n*-ions play also as the signal together with signal *p*-ions. The *p*- and *n*-ions carry signals to the same direction with the principle of duality.

IV. ELECTRICAL MODELING OF ACTIVITY OF A NEURON

A. *Formulation of Activity in a Neuron*

Electrical modeling of an active neuron is shown in Fig. 4.

i_d is the current of *p*-ions injected in the dendrite, i_a is the current of sum of arrived *p*-ions and *n*-ions injected by Cl-channels at the axon. i_c is the current through resistance R_c of the central part to the outside of a neuron.

α is current multiplication factor and $\alpha \cdot i_d$ is equivalent current source for the axon.

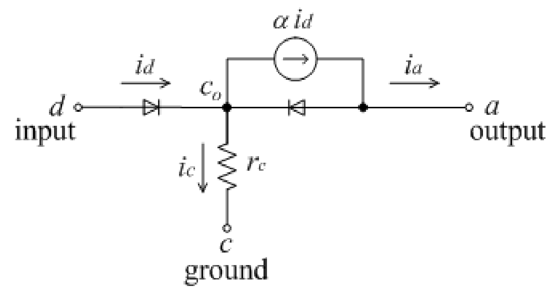


Fig. 4 Electrical modeling of activity of a neuron.

B. *Characteristics as an Amplifier*

Electrical modeling of an operating neuron is shown in Fig.7.

The points of d_0, a_0 are outside of membrane. c_0 is a virtual point taken in the central part. r_d and r_a are resistances of forward diode n_d and reverse diode n_a, r_c is the resistance at the central part to outside of a neuron. R_d and R_a are external resistances of synapses s_d and s_a..

$r_d << R_d$ and $r_a << R_a$. r_c is approximately zero.

The capacities C_d and C_a are caused by the first and second depletion layers respectively.

Input and output synapses s_d and s_a.are shown as forward diodes for excitatory synapses (*p*-ions). These synapses work as backward diodes for inhibitory synapses (*n*-ions).

Voltage amplification gain *G* is given as;

$$G = \frac{v_a}{v_d} = \frac{\dfrac{\alpha R_a}{r_d + r_c}}{1 - \dfrac{\alpha R_a}{r_d + r_c} \cdot \dfrac{r_c}{R_a}} = \frac{K}{1 - K\beta} \quad (16)$$

$$K = \alpha \frac{R_a}{r_d + r_c} \quad (17)$$

$$\beta = \frac{r_c}{R_a} \quad (18)$$

where, v_d and v_a are input and output voltages of a neuron, G, K, β are closed loop gain, open loop gain, and inner feedback ratio of a neuron respectively. Oscillation condition is given by $K\beta \geq 1$.

In case that the axon has little Cl channels, $\alpha < 1$, $K\beta << 1$. Therefore a neuron operates as an amplifier with threshold for input signal with positive inner feedback.

C. Characteristics as a pulse generator

The neuron operates as an oscillator to generate pulses when the product of open loop gain K and feedback ration β exceeds 1.

This oscillator is composed by self injection without input trigger.

$$T_1 = C_d \frac{r_c R_a}{r_c + R_a} \quad (19)$$

$$T_2 = C_a R_a \quad (20),$$

where, $R_d + r_d >> r_c$, $r_a = \infty$

are assumed for simplified analysis.

The period of oscillation T is given as the total time length as following;

$$T = T_1 + T_2 = C_d \frac{r_c R_a}{r_c + R_a} + C_a R_a \quad (21).$$

D. Timing of output pulses

An oscillator operates in free running condition without external input. Timing of output pulse is adjusted in pull-in condition when external input i_d is added.

Output pulses v_a under free-running and pulled-in conditions are shown with dotted and solid lines in Fig. 6 respectively.

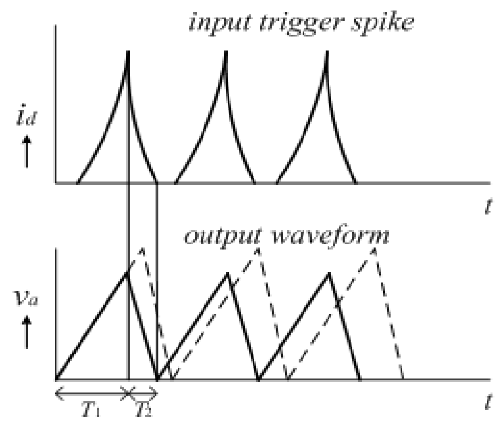


Fig. 6 Astable pulse generator by external injection.
Dotted line is an original waveform. Solid line is the waveform synchronized to input trigger pulse.

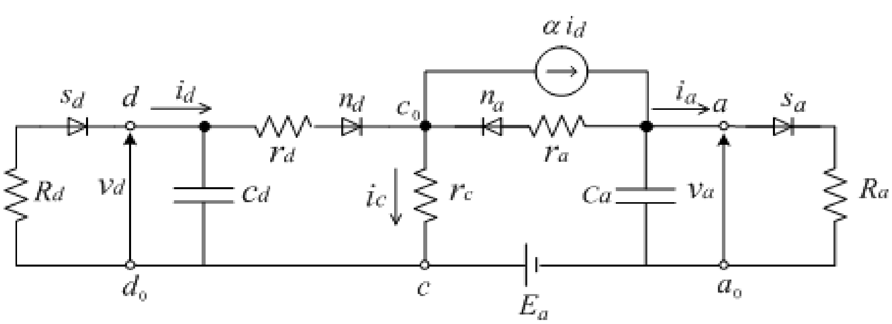


Fig. 5 Electrical modeling of an active neuron.

V. SELF-SYSTEMATIZATION BY MUTUAL PULSE INJECTION AMONG NEURONS

A. Formation of a Neural Group

Actual formation of a neural group is given in Fig. 7. A small circle represents a neuron. Input and output signals of a neuron are at a branch of the dendrite and at a branch of the axon. A set of pair neurons is shown in Fig. 7 (a). Connection between two neurons is performed by arrows with dual directions. A system of four neurons is shown in Fig.7 (b).

B. System Synchronization

The timing of output pulse of an oscillator is adjusted by the other. When two oscillators are connected with each other, the timing is set at a certain timing between two. As number of oscillators increases, the variation of timings among neurons is reduced and system synchronization is established.

C. Synchronous Signal Processing

This formation enables system synchronization and synchronized signal processing simultaneously. Signal processing for multiple inputs and multiple outputs are available for dynamic processing including correlation, comparison, and detecting variations. This formation will be required for complex, reliable, and fast operation and signal processing [2]

VI. ENERGY DIAGRAM OF ELECTRICAL CHARGES IN ACTIVE NEURON

Energy of *p*- and *n*-ions in a neuron are illustrated in Fig. 8. The energy of *p*- and *n*-ions are assumed with a small difference to Fermi level as shown in the figure.

Cl channels at the axon inject *n*-ions to left at the second depletion layer passing over a slope shown in the figure.

The dynamics of *p*- and *n*-ions is well informed by tracing the curve to right (*p*-ion) and to left (*n*-ion). The three port configuration is kept in spite with a slope at the axon (ref. [13,14]).

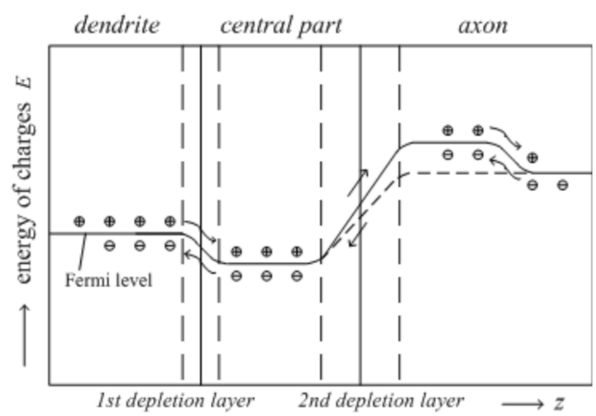


Fig. 8 Energy diagram of negative and positive ions with Cl^{-} channels at axon.

(a) A set of two neurons. (b) A system by four neurons.

Fig. 7 Synchronization and signal processing by mutual injection.

VII. Point Contact Transistor (PCT)

Semiconductor transistors present electrical activity with solid composition together with triode electron tubes. PCT is shown in Fig. 9. A *n*-type semiconductor substrate is used and two metal electrodes, emitter *E* (input) and collector *C* (output) are attached with signal gap.

The lower surface is covered by metal as base *B* (ground). The space between *E* and *C* 50 μm or less to get 20 dB amplitude gain, which is too narrow to put base between them.

The surface of substrate was heated by large current flow from *C* to *B* ports to improve the characteristics of activity. This processing is seemed to compensate the disadvantage of distant base port.

Analysis of a neuron presented in this paper is efficiently applied also to the PCT.

VIII. Conclusion

Activity of a neuron was presented for validation of the modelling referring unicellular organism with common behavior of excitatory cells.

Modelling was first given with electro-physical structure composed of three zones and two depletion layers formed in ectoplasm membrane for activity of a neuron.

An electrical modelling was then given for amplification and pulse generation. Positive pulse generation was shown by self and mutual injection. Stability of phase and period is realized by mutual injection to yield stable timing (clock) inside systems.

Validation of proposed modelling was given by comparison of activities by neuron and Point Contact Transistor (PCT) as one of active devices of solid semiconductor.

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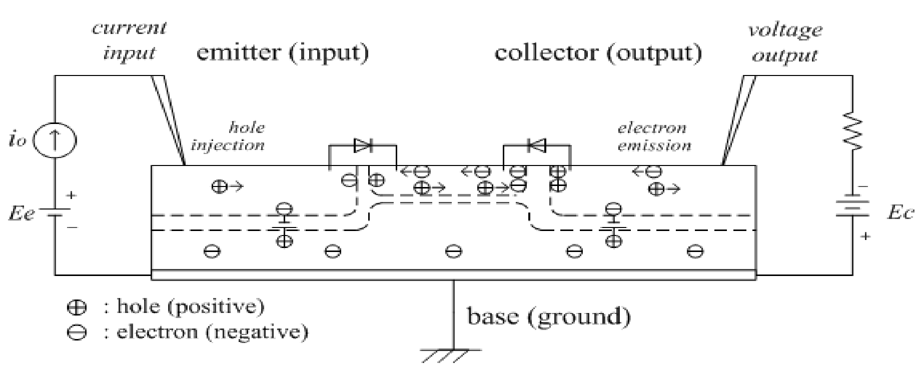

Fig. 9 Electro-physical modeling of Point Contact Transistor (PCT).

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