Electrophysical Study of Signal Transmission by Unmyelinated and Myelinated Axons of Neurons

Yumi Takizawa and Atsushi Fukasawa

***Abstract*—**Electrophysical study is given for signal transmission characteristics with the axon in neurons. Electrical analysis was done under voltage equilibrium and current continuity conditions. Inductance L and capacity C per unit length are obtained for lossless axon. Attenuation, phase rotation, and phase (signal) velocity are given for axon with loss. It is then presented that higher velocity and wideband signal transmission are realized by reduction of C and L by myelination and large cross section of axons. Loss and phase rotation per unit length are also obtained for axon with serial resistance r and parallel conductance g. Long distance transmission is found available by reduction of serial resistance r and parallel conductance g by voltage dependent Na^+ channels distributed at unmyelinated parts of axons.

***Keywords*—**Electrophysical study of axon, unmyelinated axon, myelinated axon, voltage equilibrium, current continuity, velocity and bandwidth, attenuation, phase rotation.

I. Introduction

GENERATION of electrical signal (action potential) and its transmission (conduction) to post neurons are essential functions of a neuron. The former function was reduced to active operation, which brings oscillation, amplification (energy enhancement) or active switching operation. The latter is reduced to passive operation with a little loss or lossless transmission.

Early models of active transmission along the axon are popular but they are given based on knowledge on physiological experiments. But they were given without consideration in electrical and physical fundamentals (see Chapter V).

This paper present electro-physical modelling of axon transmission, without and with myelins. Electro-physical modelling and equivalent circuit are first given for the axon without myelins.

Schematic diagram of a neuron is shown in Fig. 1. Dendrite and axon work as input and output ports of a cell (neuron). Input and output of axon are at hillock and at axon terminal. Synapses work for connection chemically with previous and post neurons.

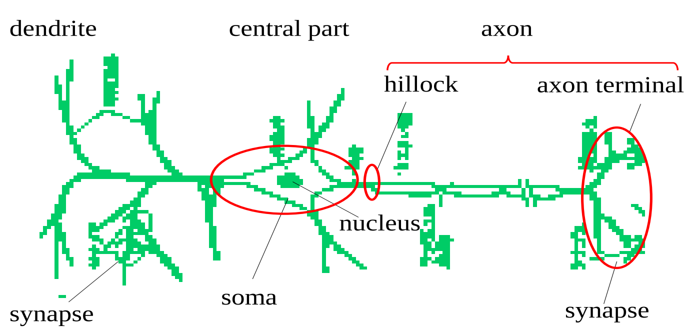

Fig. 1 Schematic diagram of a neuron. Modelling and equivalent circuit are then given for the axon with myelins. It is estimated that myelins provide axons with fast electrical signal transmission capability by myelination.

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Yumi Takizawa is with the Institute of Statistical Mathematics, Tachikawa, Tokyo, 190-8562 Japan (e-mail: takizawa@ ism.ac.jp).

Atsushi Fukasawa was with Chiba University, Inage-ku, Chiba, 263-8522 Japan.

II. ELECTRO-PHYSICAL MODELLING OF UNMYELINATED AXON

A. *Modelling of unmyelinated lossless axon*

Electro-physical modelling of unmyelinated axon is shown in Fig. 2. Cytoplasm and external liquid are separated by the membrane. They work as an inner and an outer conductors and a dielectrics (insulator) to form a coaxial waveguide.

The cylinder coordinate system r, θ, z is used for electrical analysis. Electrical signal transmission is done by TEM (transversal electric and magnetic) mode.

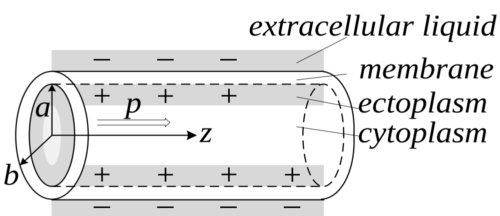

Fig. 2 Electro-physical modelling of axon.

Radial E and circular H vectors exist in cross-sectional (r, θ) plane. Poynting vector p carries signal energy (power) along z axis, where,

$$p = E \times H \qquad (1)$$

where, E, H, p are defined clockwise.

Radius of conductors a and b are shown in Fig.2. In dielectric medium, radial length dr is considered at point r.

True electric charges $\pm q$ are given on the inner and outer conductors per unit length. Capacitance C is calculated as follows.

$$E = \frac{q}{2\pi\varepsilon\, r} \quad \text{[V/m]} \qquad (2)$$

$$v = \int_a^b E\, dr = \int_a^b \frac{q}{2\pi\varepsilon\, r}\, dr \qquad (3)$$

$$= \frac{q}{2\pi\varepsilon} \ln \frac{b}{a} \quad \text{[V]} \qquad (4)$$

where, E, v are electric field strength in the space and the potential difference (voltage) of inner and outer liquid conductors. Capacity per unit length is given as ;

$$C = \frac{q}{v} = \frac{2\pi\varepsilon}{\ln \frac{b}{a}} \quad \text{[F/m]} \qquad (5)$$

Inductance L is calculated as follows.

$$H = \frac{\mu i}{2\pi\, r} \quad \text{[H/m]} \qquad (6)$$

$$\Phi = \int_a^b H\, dr = \int_a^b \frac{\mu i}{2\pi r}\, dr \quad \text{[Wb]} \qquad (7)$$

where, H, i are magnetic field strength in the space and the current of inner and outer liquid conductors. B and Φ are density and total flux of magnetic field strength in space between inner and outer liquid conductors.

Inductance per unit length is given as;

$$L = \frac{\Phi}{i} = \frac{\mu}{2\pi} \ln \frac{b}{a} \quad \text{[H/m]} \qquad (8)$$

It is pointed that permittivity ε and permeability μ in MKSA system of cytoplasm and outer liquid are as follows;

$$\varepsilon = \varepsilon_0 \varepsilon_r \quad \text{[F/m]} \qquad (9)$$

$$\varepsilon_0 = \frac{4}{3}\pi \times 10^{-12} \quad \text{[F/m]} \qquad (10)$$

and,

$$\mu = \mu_0 \mu_r \quad \text{[H/m]} \qquad (11)$$

$$\mu_0 = 4\pi \times 10^{-7} \quad \text{[H/m]} \qquad (12)$$

$$\mu_r = 1, \qquad (13)$$

and,

$$\varepsilon_0\, \mu_0 = \frac{1}{{c_0}^2} \qquad (14)$$

here, c_0 is the light velocity in vacuum.

Usual voltage of action potential of a neuron is about 100 mV. But current value is so little that the current flows almost the peripheral of inner conductor.

When the distribution is assumed exponential, effective depth δ of penetration is given as;

$$\delta = \sqrt{\frac{2}{\omega\sigma}} \qquad \text{[m]} \quad (15)$$

where, σ is conductance of inner and outer liquid.

B. Modelling of unmyelinated axon with loss

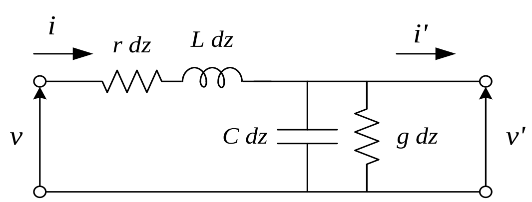

Fig. 3 Equivalent circuit of unmyelinated axon of length *dz* with loss.

The equivalent circuit of the model is shown in Fig.3. The following equations are obtained from conditions of voltage equilibrium and current continuity.

$$v = L\,dz\,\frac{di}{dt} + i\,rdz + v' \qquad \text{[V]} \quad (16)$$

$$i = C\,dz\,\frac{dv'}{dt} + g\,dz\,v' + i' \qquad \text{[A]} \quad (17)$$

and,

$$v - v' = dv \qquad \text{[V]} \quad (18)$$

$$i - i' = di \qquad \text{[A]} \quad (19)$$

From Eqs. (16) ~ (19),

$$\frac{dv}{dz} = L\frac{di}{dt} + i\,r \qquad \text{[V/m]} \quad (20)$$

$$\frac{di}{dz} = C\,\frac{dv'}{dt} + g\,v' \qquad \text{[A/m]} \quad (21)$$

From (20) and (21),

$$\frac{d^2v}{dz^2} = L\frac{d}{dz}\left(\frac{di}{dt}\right) + r\,\frac{di}{dz} \qquad (22)$$

Here,

$$\frac{d}{dt}\left(\frac{di}{dz}\right) = C\,\frac{d}{dt}\left(\frac{dv'}{dt}\right) + g\,\frac{dv'}{dt} \qquad (23)$$

From (22) and (23),

$$\frac{d^2v}{dz^2} = LC\,\frac{d^2v'}{dt^2} + (gL + rC)\,\frac{dv'}{dt} + rgv' \quad (24)$$

When v is sinusoidal, it is written as,

$$v = \exp(j\omega t) \qquad \text{[V]} \quad (25)$$

$$\frac{d^2v}{dt^2} = -\omega^2 v \qquad (26)$$

v' is replaced by v if dv/v' is small enough.

$$\frac{d^2v}{dz^2} = \{-\omega^2 LC + j\omega\,(gL + rC) + rg\}\,v \quad (27)$$

$$= \left\{ (j\omega\sqrt{LC})^2 + j\omega\sqrt{LC}\,\left(\frac{r}{\sqrt{\frac{L}{C}}} + \frac{g}{\sqrt{\frac{C}{L}}}\right) + rg \right\} v$$

(28)

$$\frac{d^2v}{dz^2} \approx \left\{ \frac{1}{2}\left(\frac{r}{\sqrt{\frac{L}{C}}} + \frac{g}{\sqrt{\frac{C}{L}}} \right) + j\omega\sqrt{LC} \right\}^2 v \qquad (29)$$

General solution of v on time and space is given as follows;

$$\exp(-j\omega t)\,v = v_1 \exp\{-\,j(\omega t + \gamma z)\} + v_2 \exp\{-\,j(\omega t - \gamma z)\} \qquad (30)$$

It is found that v_1 and v_2 are components of signals transmitting forward and backward directions of z.
Now, γ is transmission coefficient of transmitting wave signals, α and β are attenuation and phase (rotation) constants of γ. Attenuation α is corresponding to transmission loss.

$$\gamma \approx \frac{1}{2}\left(\frac{r}{\sqrt{\frac{L}{C}}} + \frac{g}{\sqrt{\frac{C}{L}}}\right) + j\omega\sqrt{LC} \qquad (31)$$

$$= \alpha + j\beta \qquad (32)$$

Phase velocity c is defined by time-differential of equal phase points along z axis.

$$\frac{d}{dt}(\omega t + \gamma z) = \omega + \gamma\frac{dz}{dt} = 0 \qquad (33)$$

$$c = \frac{dz}{dt} = \left|\frac{\omega}{\gamma}\right| \quad \text{[m/s]} \qquad (34)$$

$$\approx \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \quad \text{[m/s]} \qquad (35)$$

Transmission time T of length l is given as ;

$$T = \frac{l}{c} \quad \text{[s]} \qquad (36)$$

where, r, g are assumed small enough compared to $\sqrt{L/C}$, $\sqrt{C/L}$ respectively. Lossless transmission is realized when r and g are zero approximately.

It is found that velocity c depend on dimensions a and b of the axon. It is also found that long transmission is available by the aid of positive ions injected in the axon.

III. Electro-physical Modelling of Myelinated Axon

A. Electrical scheme of myelinated axon

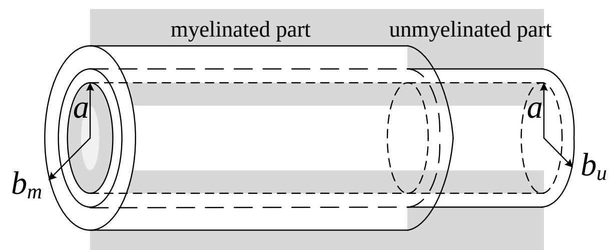

Fig. 4 Electro-physical modelling of axon with myelinated and unmyelinated parts.

Myelinated axon is composed by long myelinated and short unmyelinated parts as shown in Fig.4. A myelinated part is made by a glia cell. The distance between inner and outer conductors is expanded with multiple windings of a glia cell. Unmyelinated part appears between adjacent myelinated parts caused by adjacent glia cells.

The capacity *Cm* at myelin part is reduced to be less values than the capacity *Cu* at unmyelinated part in Fig. 5. On the other hand, inductance *L* is reduced as low as the cross sectional dimension increases.
Na^+ ions are injected into the axon through Na+ channels distributed at each unmyelinated part.

B. Characteristics of Myelinated axon

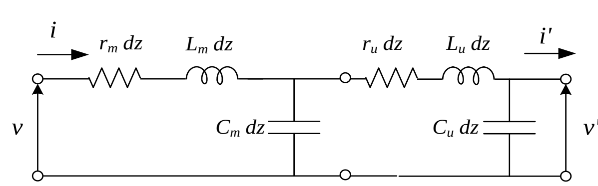

Fig. 5 Equivalent circuit of axon with myelinated and unmyelinated parts. *Lm*, *Cm*, and *Lu*, *Cu* are circuit parameters corresponding to myelinated and unmyelinated parts of an axon.

Myelinated axon is shown in Fig. 5. L_m and C_m are inductances and capacitance of the myelinated part. L_n and C_n are inductances and capacitance of unmyelinated part.

The following relations are pointed as follows;

$$Lm = Lu = L \tag{37}$$

$$Cm << Cu = C \tag{38}$$

and,

$$lm >> lu \tag{39}$$

then,

$$\omega_c \,(myelinated) >> \omega_c \,(\text{un}myelinated) \tag{40}$$

It is found that transmission of wideband signals through long axons is realized by myelination of axon. Then sharp pulses are realized to transmit for long distance by myelination.

It is concluded that larger information per unit time is realized by myelination.

Wideband transmission is realized by the effect of reduction in *L* and *C*.

Long distance transmission is realized by reduction of attenuation by increase of conductance.

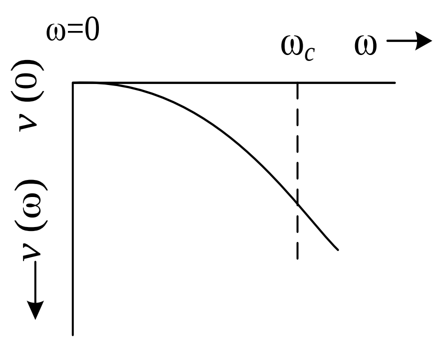

Fig. 6 Frequency characteristics and cut off ω_c.
When transmission coefficient γ is complex number.

C. Cutoff angular frequency

When transmission coefficient γ is complex number;

$$\gamma(\omega) = \alpha(\omega) + j\beta(\omega) \tag{41}$$

$\alpha(\omega)$ and $\beta(\omega)$ are attenuation and phase shift of signal transmission.

Signal transmission characteristics depends on the frequency ω. The frequency characteristics of signal transmission is expressed by Fig 6.

At the end of line $z = l$,

$$v\,(\omega, l) = v(0, l)\exp\{-(\alpha + j\beta)\omega\} \tag{42}$$

ω_c is the cutoff angular frequency at the point, where amplitude decreases equal to $1/e$ of $v(0)$. e is the basis of natural logarithm.

$$|v(\omega_c, l)| = \exp\{-\frac{1}{e}v(0, l)\} \tag{43}$$

$$\omega_c = 1/\sqrt{LC} \tag{44}$$

It is found that cutoff angular frequency ω_c increases depending on product values of *LC*.

D. Long distance transmission

It was found by G. Kato that Na^+ channels distribute at discrete points of unmyelinated part of axon. It shows that ε could be controlled by influx and efflux of positive ions. If positive ions are taken much into axon, effective ε decreases lower, and reduction of transmission time τ of signal.

Many ion channels could exist at large diameter of axon. It is concluded that the signal transmission (phase) velocity becomes higher as increase of size of cross section of axon.

IV. Higher Mode Transmission with the Axon

If wavelength λg of transmitting wave signals is longer enough compared a half of spacing $b - a$, only Transversal Electric and Magnetic (TEM) mode is transmitted. TEM mode is the dominant mode of coaxial line.

If wavelength λg is shorter than a half of spacing $b - a$, higher mode is excited from the dominant mode.

$$b - a > \frac{1}{2}\lambda_g \tag{45}$$

$$\lambda_g = \frac{1}{\sqrt{\varepsilon}}\lambda_0 \tag{46}$$

where, λ_0, λ_g are the wave length of signal in free space and in liquid.

Typical higher mode *Ez* is excited inside the line. Longitudinal electric vector *Ez* occurs along *z* axis. However if higher mode is brought from dominant TEM mode, not only signal power varies but also higher mode works as noise against signal, it causes data error and reduction of reliability of communication systems.

The electro-salutatory transmission along an axon with myelin was presented by I. Tasaki (1939)[7]. He proposed that rapid transmission with salutatory effect, which means leaping transmission of current between myelins outside the axon. This provides unreasonable result of signal transmission not inside but outside the axon.

The electro-salutatory transmission implies signal transmission by *Ez* component provides not only low reliable transmission but also severe crosstalk among parallel axons in a neural system.

V. Validation of the Proposed Modelling

A. *Early modelling of axon*

Lossless and fast signal transmission were observed in myelinated and thick axons by G. Kato, 1923[1]. An early model of axon transmission was given by G. Kato and I. Tasaki, 1934[2], 1939[3]. Generation and transmission of action potentials are combined in this model. Active transmission model with higher electro-magnetic mode (*Ez* mode) was given by H.S. Gasser, 1936[4]. But any scheme was not given about activity.

Afterward variety of data on velocities were measured about various axons by A. Siegel and H. Sapru, 2014[5].

Here, it should be pointed that early neuron models do not meet electro-physical requirement.

B. *Initial model by the authors* [2008]

At an initial state of the study in neurons, it was pointed that novel models of neuron and axon must be given separately. It was thought that the neuron itself should operate as an active device, and the axon with some length of nerve fiber should operate as a passive device.

By an initial paper of the study of neurons, an axon was assigned by repetition of serial inductance *L* and parallel capacitance *c* as shown in Fig. 2, p.323 in the paper, 2008[].

Activity was shown by potential switch for the states between resting and active potentials as shown in Fig.1 and 2 of the same paper.

Novel modelling and characteristic analysis are given in this paper based on the above predictions.

VI. Conclusion

The fundamental characteristics of axons are first analyzed in this paper.

It is clarified that signal transmission (phase) velocity in axon depends on inductance *L* and capacitance *C* of axon which are defined by myelination and cross-sectional dimension of axon.

It is clarified that wideband signal and long distance transmission are realized by myelination, and large cross-sectional dimension (thick) of axons.

It is also presented that increase of conduction of the axon is brought by injection of positive ions through ion channels distributed at unmyelinated part of axon.

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Yumi Takizawa received the B.S. degree in Physics from Shinshu University, Japan, in 1984, and the Ph.D. degree from the University of Tokyo in 1994. She was a research leader of signal processing for communication system Lab., OKI Electric Industry Co. Ltd., Tokyo since 1984. She joined the Institute of Statistical Mathematics, Japan as an associate professor in 1995.

She has been engaged nonstationary signal processing methods and its application to wide areas of industrial systems. She also has been engaged in digital mobile communication system of Wideband CDMA. Then she has been engaged in neuroethology of weakly electric fish as a visiting researcher at University of Virginia, 1997-1998 and 1999-2000. She continued research of biological signal processing including behavior of neural systems.

She received the Prize on Telecommunication System Technology from the Foundation of Telecommunication Association, Japan in 2004. She is an associate Professor. .

Atsushi Fukasawa received the B.S. degree in Electrical Engineering from Chiba Unversity in 1962. He received the Master of Arts degree in Electrical communication from Waseda University, Tokyo, in 1967 and the Ph.D. degree from Waseda University in 1983.

He joined Graduate University of Science and Technology, Chiba University as a professor in 1997. He joined the department of Electrical and Electronics Engineering, faculty of Engineering, Chiba University in 1998, and the department of Urban Environmental and System Engineering, Chiba University in 1999. He joined Transdisciplinary Research Integration Center, Research Organization of Information and Systems as a project professor in 2004.

He received the Prize of the Agency of Science and Technology, Japan in 1967, and received the Ohm Science and Technology Prize from Ohmsha publishing in 1994 respectively. He received also the Prize on Telecommunication System Technology from the Foundation of Telecommunication Association, Japan in 2004.

And he is also received WSEAS/NAUN the Best Paper Award of NEUROLOGY, 2012.

He is a senior member of the IEEE.