Identification of a Oculo-Motor System Human based on Volterra Kernels

Vitaliy D. Pavlenko, Dmytro V. Salata, and Hryhori P. Chaikovskyi

Abstract—A new method of constructing nonparametric dynamic model of the human oculomotor system on the basis of experimental data "input-output" is developed, considering nonlinear and inertial properties of the rectus muscles of the eye. A technology for tracking eye movement is based on the videos. It is possible to determine the dynamic characteristics of the oculo-motor system functions as a transition of the first and second order - integral transforms Volterra kernels.

Keywords—oculo-motor system, modeling, nonparametric dynamic models, Volterra kernels, multidimensional transient characteristics, eye-tracking technology.

I. INTRODUCTION

The innovative technology of Eye tracking which is rapidly being developed nowadays—is the process of determining the point where the eye looks or eye movements relatively to the head [1]–[3]. This high-tech innovation is being developed further and is effectively used in the construction of a mathematical model of a process of tracking eye movement to detect anomalies in data tracking to quantify the motor symptoms of Parkinson's disease [4]–[5]. The use of nonlinear dynamic model of Wiener and Volterra-Laguerre [6] and their identification is based on a random effects test [7], which requires the application of methods of correlation analysis and generation of a large amount of experimental data (long-term experimental studies).

In order to build a model of Volterra [8] Oculo-Motor System (OMS) a person is encouraged to use the test deterministic effects, for example, step signals (the most appropriate for the study of the dynamics of OMS) [9], which simplify the computational algorithm of identification and significantly reduce the time of processing of experimental data. There is a method and computer algorithms identifying deterministic nonlinear dynamical systems in the form of Volterra models using multi-test signals [10].

II. THE PURPOSE AND RESEARCH PROBLEMS

The purpose of the work is to develop a method for constructing nonparametric dynamic model of oculo-motor system, considering its inertial and nonlinear properties, which is based on experimental studies of "input-output" and computational tools and information technology software for experimental data processing. To achieve this goal, the following tasks were set:

- Development of methods for constructing nonlinear dynamic model of OMS as Volterra kernels, which characterises both nonlinear and inertial properties of the nature objects;
- Development of information technology for obtaining experimental data for identification of OMS based on pupil’s movement tracking using video registration;
- Development of computational methods for identification of multidimensional dynamic (transient) characteristics of OMS using test inputs as Heaviside functions of different amplitudes;
- Verification of constructed OMS model.

III. THE VOLTERA MODEL AND IDENTIFICATIONS

Mathematical (informational) model of investigated object is created on the basis of the results of measurements of its input and output variables, and the solution to the problem is connected with the identification of the experimental data and its processing by means of noise measurements.

To describe the objects of unknown structure, it is advisable to use the most universal nonlinear nonparametric dynamical models - Volterra model [8]. The nonlinear and dynamic properties of investigated object are uniquely shown by a sequence of invariants according to the type of input signal of multidimensional weight functions - Volterra kernels.

Continuous nonlinear dynamical system connection between the input \( x(t) \) and output \( y(t) \) signals for zero initial conditions can be represented by a series of Volterra:
\[ y(t) = \sum_{i=1}^{\nu} \int_{0}^{t} w_i(\tau) x(t-\tau) d\tau + \]
\[ + \sum_{i=1}^{\nu} \int_{0}^{t} w_i(\tau, \tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1 d\tau_2 + \]
\[ + \sum_{i=1}^{\nu} \sum_{j=1}^{\mu} \int_{0}^{t} \int_{0}^{t} w_{ij}(\tau, \tau_1, \tau_2, \tau_3) x(t-\tau_1) x(t-\tau_2) x(t-\tau_3) \times (t-\tau_4) x(t-\tau_5) d\tau_1 d\tau_2 d\tau_3 d\tau_4 d\tau_5 + \ldots, \]

where \( w_i(\tau, \ldots, \tau) = \) Volterra kernel of \( n \)-th order, function is symmetric relatively to real variables \( \tau_1, \ldots, \tau_n \); \( y(t) = \) the \( n \)-th partial component of the response system (\( n \)-dimensional convolution integral); \( t = \) current time.

In nonlinear dynamical system multiple-input and multiple-output use multivariate Volterra series, which is as follows:

\[ y_j(t) = \sum_{i_1,i_2,\ldots,i_n=0}^{\nu} \int_{0}^{\infty} \int_{0}^{\infty} \ldots \int_{0}^{\infty} \int_{0}^{\infty} w_{i_1,i_2,\ldots,i_n}(\tau_1, \ldots, \tau_n) \times (t-\tau_1) x_{i_1}(t-\tau_1) x_{i_2}(t-\tau_2) \times \ldots \times (t-\tau_n) x_{i_n}(t-\tau_n) d\tau_1 d\tau_2 \ldots d\tau_n, \]

where \( w_{i_1,i_2,\ldots,i_n}(\tau_1, \ldots, \tau_n) = \) Volterra kernel of \( n \)-th order in \( i_1, i_2, \ldots, i_n \) inputs and \( j \)-th output (\( j=1,2,\ldots,\mu \)), the functions are symmetric relatively to real variables \( \tau_1, \ldots, \tau_n \); \( y_j(t) = \) system response to the \( j \)-th output at the current time \( t \) for zero initial conditions; \( x_{i_1}(t), \ldots, x_{i_n}(t) = \) input signals; \( \nu, \mu = \) quantity of inputs and outputs, respectively.

In the context of the problem stated above - identification of OMS - the model (2) should be used for the mathematical description of the object [8]: two pairs of rectus muscles (input object) provide the eye movement up and down, left and right, and various combinations, shown in Fig. 1; measured responses - the coordinates \( u(t) \) and \( v(t) \) of current position of the pupil relatively to the initial position \( u_0 \) and \( v_0 \) (the outputs of the object). In this case in model (2) \( \nu=2 \) and \( \mu=2 \) are adopted.

![Fig. 1 direct eye muscles](image)

In this paper, to simplify the experiment and data identification, the task was set to determine horizontal pupil’s movement (\( \nu=1 \) and \( \mu=1 \)) on the basis of the model (1).

Problem identification (model constructing) as (1) or (2) means determination of the Volterra kernels based on the experimental data of OMS "input-output". Construction of the model refers to the selection of test actions \( x(t) \) and development of the algorithm, which enables for the measured response \( y(t) \) to define partial components \( y_n(t) \) and determine on this basis the Volterra kernels \( w_n(\tau_1, \ldots, \tau_n), n=1,2,\ldots, [9]. \)

A. Computing Method of Multidimensional Transient Functions for Identification of OMS

Taking into account the specificity of the investigated object, test multistage signals were used for identification. If a test signal \( x(t) \) represents an identity function (Heaviside function) – \( \theta(t) \), it will result in identification of the transition function of the first order and the diagonal section of \( n \)-th order.

To determine the sections subdiagonal transition functions of \( n \)-th order (\( n \geq 2 \)), OMS is tested using the \( n \) step test signal with given amplitude and different intervals between signals. With appropriate processing responses, \( n \)-dimensional transition functions of subdiagonal section are received \( h_n(t-\tau_1, \ldots, t-\tau_n) \), which represent \( n \)-dimensional integral of Volterra kernel of \( n \)-order \( w_n(\tau_1, \ldots, \tau_n) \):

\[ h_n(t-\tau_1, \ldots, t-\tau_n) = \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \ldots \int_{0}^{\infty} w_n(\tau_1, \ldots, \tau_n) x(t-\tau_1) x(t-\tau_2) \times \ldots \times (t-\tau_n) x(t-\tau_n) d\tau_1 d\tau_2 d\tau_3 \ldots d\tau_n. \]

The method for determination of sections of \( n \)-dimensional transition functions is based on the statement, proving of which is similar to that given in [10].

B. The Method for Constructing an Approximate Model of Volterra Nonlinear Dynamical System

The method for constructing an approximate Volterra model of the OMS is developed. The method of identification is based on the approximation \( y(t) \) at an arbitrary deterministic signal \( x(t) \) in the form of integral power of the polynomial Volterra \( N \)-th order (\( N \)-th order approximation model):

\[ \tilde{y}_N(t) = \sum_{n=1}^{N} \hat{y}_n(t) = \]
\[ = \sum_{n=1}^{N} \int_{0}^{t} \int_{0}^{t} \int_{0}^{t} \ldots \int_{0}^{t} w_n(\tau_1, \ldots, \tau_n) x(t-\tau_1) x(t-\tau_2) \times \ldots \times (t-\tau_n) x(t-\tau_n) d\tau_1 d\tau_2 d\tau_3 \ldots d\tau_n. \]

Let the input test signals of OMS be fed alternately: \( a_1 x(t), a_2 x(t), \ldots, a_L x(t) \), \( a_1, a_2, \ldots, a_L \) - distinct real numbers satisfying the condition \( |a_j| \leq 1 \) for \( j=1,2,\ldots,L \); then:

\[ \tilde{y}_N[a_1 x(t)] = \sum_{n=1}^{N} \hat{y}_n[a_1 x(t)] = \]
\[ = \sum_{n=1}^{N} a_1^n \int_{0}^{t} \int_{0}^{t} \int_{0}^{t} \ldots \int_{0}^{t} w_n(\tau_1, \ldots, \tau_n) x(t-\tau_1) x(t-\tau_2) \times \ldots \times (t-\tau_n) x(t-\tau_n) d\tau_1 d\tau_2 d\tau_3 \ldots d\tau_n = \]
\[ = \sum_{n=1}^{N} a_1^n \hat{y}_n(t) \]
The partial components in the approximation model \( \hat{y}_n(t) \) are found using the least square method (LSM). This makes it possible to obtain such evaluation, in which the sum of squared deviations of responses identified by the nonlinear dynamical system \( y[a_j x(t)] \) on the model \( \hat{y}_N[a_j x(t)] \) response is minimal, i.e., OMS provides a minimum criterion:

\[
J_N = \sum_{j=1}^{N} [y_j(a_j x(t)) - \hat{y}_N(a_j x(t))]^2 = \sum_{j=1}^{N} \left( y_j(t) - \sum_{i=1}^{K} a_j^i \hat{y}_j(t) \right)^2 \rightarrow \min
\]

where \( y_j(t) = y[a_j x(t)] \). Minimization of the criterion (6) is reduced to solving the system normal equations of Gauss, which in vector-matrix form can be written as:

\[
A^T \hat{A} \hat{y} = A^T y,
\]

where

\[
A = \begin{bmatrix} a_1 & a_1^2 & \cdots & a_1^N \\ a_2 & a_2^2 & \cdots & a_2^N \\ \vdots & \vdots & \ddots & \vdots \\ a_L & a_L^2 & \cdots & a_L^N \end{bmatrix}, y = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_N(t) \end{bmatrix}, \hat{y} = \begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \vdots \\ \hat{y}_N(t) \end{bmatrix}.
\]

From (7) we obtain

\[
\hat{y} = (A^T A)^{-1} A^T y \quad (8)
\]

In (8), matrix operations, we obtain

\[
\begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \vdots \\ \hat{y}_N(t) \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{K} a_j^2 \sum_{j=1}^{K} a_j^3 \cdots \sum_{j=1}^{K} a_j^N \\ \sum_{j=1}^{K} a_j^3 \sum_{j=1}^{K} a_j^4 \cdots \sum_{j=1}^{K} a_j^{N+1} \\ \vdots \\ \sum_{j=1}^{K} a_j^{N-1} \sum_{j=1}^{K} a_j^{N+2} \cdots \sum_{j=1}^{K} a_j^{2N} \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{K} a_j y_j(t) \\ \sum_{j=1}^{K} a_j^2 y_j(t) \\ \vdots \\ \sum_{j=1}^{K} a_j^{N-1} y_j(t) \end{bmatrix} \quad (9)
\]

IV. The results of the research

To implement successfully the technology of experimental determination of the dynamic characteristics of OMS and human diagnostics for technical and medical purposes, it’s necessary to have data sets - the coordinates of the position of the pupil on plane, namely, the values of the horizontal and vertical eye rotation angles relatively to the initial position.

Software tools that automatically identify regions of interest (the pupil) in the sequence of frames of a video recording and coordinate calculations were developed. The most important feature of this software is that it can be easily scaled and it does not require any special hardware. The experiment can be easily carried out using any device with a camera with at least 5 Mpx of resolution and a sample rate of at least 30 frames per second. Calculations can be performed on any device with appropriate adjustments in software code.

Technology stack which was used during development of the software: library algorithms, computer vision, image processing and numerical algorithms from general purpose Open Source - library OpenCV (Open Source Computer Vision Library), which is easily deployed to most of the existing operating systems (Android, Windows, Linux, iOS); Python programming language with appropriate libraries; gradient algorithm.

The experiment was carried out with the help of the proposed system for tracking the behaviour of the pupil based on video recording and performed in the following sequence.

1. A head of the observed person is located in front of the recording device (camera) at the known distance.
2. On fixed intervals, the display shows a graphic test signal in the form of a bright spot (light spot). At the same time the recording of the eye movement from the initial position to a position determined by the light spot (test signal) starts.
3. After passing a series of test signals an experiment is terminated. File with video recording of pupil’s movement is stored in memory of the measuring system.
4. The experiment having been completed, the application that implements intelligent object detection algorithm (pupil) in the captured video is launched. The graph of the pupil’s position changes is built by means of a test signal input (“input-output” experiment).
5. Obtained data is stored in the database and displayed.

For experimental studies of the OMS, Olympus PEN E-PL1 14-42 mm camera with a 4/3 matrix of 12 Mpx is used. The length of a sample is 408,17 ms (50 photo frames). An article [11] describes a method of calibrating a digital camera in order to determine distances by a photographic method. It shows the solution to the problem and its geometrical interpretation. The accuracy of the estimation of the coordinates can reach up to ± 2 pixels. The distance from the test subject to the light disturbance source (in this case, a computer monitor) equals 300 mm.

The study of the oculomotor apparatus of a person involves the analysis of eye movement data. To obtain this data, the test signals to which the eye responds must be reproduced. To generate test signals, an application is developed that runs on the Windows operating system. In the application it is possible to create lists of test signals - coordinates of bright points on the monitor screen or generate signals by random law. The lists of test signals are saved in files when you exit the application. To save the test signal lists, you use the serialization of lists in an XML file. When the application is launched, the XML file is deserialized into the test signal lists.

To create the application, .Net technology and the C# programming language were used. The use of the template WinForms allows you to apply the graphical capabilities of the .Net Framework to play test signals on the monitor screen. An example of displaying test signals on a computer monitor using the developed application is shown in Fig. 2.
Multi-stage approach to eye center detection was used in order to acquire coordinates of the eye centre. The first stage of detection consists of acquiring regions of interests in the form of eye regions by using artificial neural networks, in this case – a Haar cascade classifier [12]. We then applied gradient algorithm of eye center location to the regions of interest, which allowed us to acquire precise location of eye centers.

The main advantage to using Haar cascade classifier is its speed, which in turn makes it easy to process videos. Haar cascade classifier is implemented in OpenCV library [13], which also provides easy and stable tools for training it. The training consists of computing characteristic values based on positive and negative images [12] and saving them in a XML file. The file is then used with an implementation of Viola-Jones method in OpenCV.

The algorithm used is based on [14], with modifications added to it. The optimal centre \( \mathbf{c}^* \) of a circular object in an image with pixel positions \( \mathbf{x}_i, i \in \{1, 2, ..., M\} \), where \( M \) is the size of the image, is given by:

\[
\mathbf{c}^* = \arg \max_{\mathbf{c}} \left\{ \frac{1}{M} \sum_{i=1}^{M} (\mathbf{d}_i^T \mathbf{g}_i)^2 \right\},
\]

(10)

\[
\mathbf{d}_i = \frac{\mathbf{x}_i - \mathbf{c}}{||\mathbf{x}_i - \mathbf{c}||_2}, \quad \forall i: ||\mathbf{g}_i||_2 = 1,
\]

(11)

here, \( \mathbf{d}_i \) – a normalized displacement vector of the possible center \( \mathbf{c} \), \( \mathbf{g}_i \) – a gradient vector at position \( \mathbf{x}_i \). The gradient vector is calculated by means of partial derivatives in this case, but any method can be used without affecting the complexity of the software implementation.

The geometrical solution to the problem essentially comes down to checking, whether the displacement vector and gradient vector have same orientations, and if so, the center in question \( \mathbf{c} \) is the optimal center of the dark object (Fig. 3). The first modification addresses the threshold of dark and light centers. Since the pupil is usually dark compared to sclera and skin, we apply a weight \( w_c \) for each possible centre \( \mathbf{c} \) such that dark centers are more likely than bright centers.

Integrating this into the objective function leads to:

\[
\mathbf{c}^* = \arg \max_{\mathbf{c}} \left\{ \frac{1}{M} \sum_{i=1}^{M} w_{ic} (\mathbf{d}_i^T \mathbf{g}_i)^2 \right\},
\]

(12)

where \( w_{ic} = I'(c_x, c_y) \) is the grey value at \( (c_x, c_y) \) of the smoothed and inverted input image \( I \). The image needs to be smoothed, e.g. by a Gaussian filter, in order to avoid problems that arise due to bright outliers such as reflections of glasses.

Testing of the tracking technology of the pupil’s behavior based on video registration is performed on the basis of the analysis of the OMS work along the horizontal axis. Measured response of the eye \( y_1(t), y_2(t), y_3(t) \) to the input test signals \( a_1\theta(t), a_2\theta(t) \) and \( a_3\theta(t) (L=3) \) for values of the test signal amplitudes \( a_1=0.33, a_2=0.66 \) and \( a_3=1 \) is shown in Fig. 4.

Obtained graphs of OMS first \( \hat{h}_1(t) \) and second \( \hat{h}_2(t,t) \) transient functions are shown in Fig. 5, Fig. 6, respectively. The model response is calculated on the basis of estimates of the transient functions \( \hat{h}_1(t) \) and \( \hat{h}_2(t,t) \):

\[
\hat{y}(t,a) = a\hat{h}_1(t) + a^2\hat{h}_2(t,t).
\]

(13)
Fig. 6 transient functions \( \hat{h}_1(t,t) \).

Comparison of the response of the constructed model \( \tilde{y}(t,a) \) with the response of the identified OMS (with experimental data) \( y(t,a) \) is shown in Fig. 7.

Fig. 7 responses OMS \( y(t) \) and model \( \tilde{y}(t,a_1), \tilde{y}_1(t,a_1), \tilde{y}_2(t,a_1) \).

The verification of adequacy of the mathematical model in relation to the experiment data is calculated by Mean Square Error (MSE) criteria:

\[
\varepsilon = \frac{1}{m} \sum_{i=1}^{m} [y(t_i,a_i) - \tilde{y}(t_i,a_i)]^2
\]

and Percentage-Normalized Mean Square Error (PNMSE) criteria:

\[
\sigma = 100 \sqrt{\frac{\sum_{i=1}^{m} [y(t_i,a_i) - \tilde{y}(t_i,a_i)]^2}{\sum_{i=1}^{m} y^2(t_i,a_i)}}, \%
\]

Provided graphs are practically the same (standard deviation \( \varepsilon=0.0011, \sigma=3.3\% \)) that confirms effectiveness of computational algorithm of identification and adequacy of the constructed model based on experimental data "input-output".

Obtained graphs of OMS first \( h(t) \), second \( h_2(t,t) \) transient functions and third order \( h_3(t,t,t) \) are shown in Fig. 8 respectively.

Fig. 8 transient functions \( h(t), \tilde{h}_2(t,t) \) and \( \tilde{h}_3(t,t,t) \).

The model response is calculated on the basis of estimates of the transient functions \( \tilde{h}_1(t), \tilde{h}_2(t,t) \) and \( \tilde{h}_3(t,t,t) \):

\[
\tilde{y}(t,a) = a\tilde{h}_1(t) + a^2\tilde{h}_2(t,t) + a^3\tilde{h}_3(t,t,t).
\]

Comparison of the response of the constructed model \( \tilde{y}(t,a) \) with the response of the identified OMS (with experimental data) \( y(t,a) \) is shown in Fig. 10.

Fig. 9 responses OMS \( y(t) \) and model \( \tilde{y}(t), \tilde{y}_1(t,a_1), \tilde{y}_2(t,a_1), \tilde{y}_3(t,a_1) \) at an amplitude \( a_1=0.33 \).

Provided graphs are practically the same that confirms effectiveness of computational algorithm of identification and adequacy of the constructed model based on experimental data "input-output".

V. CONCLUSION

Proposed method for constructing nonparametric dynamical models of human OMS considering inertia and nonlinear properties is based on the experimental data "input-output". Technology of tracking the pupil’s behaviour with the help of video recording that allows to determine eye’s dynamic characteristics, has been improved.

Proposed technology of tracking pupil’s behaviour does not need special equipment and can be used in laboratory experimental conditions and applied for widespread use. Important feature of the technology is that it does not require any special hardware and this allows to use it in the modern mobile devices. Verification of the developed model showed its adequacy with the investigation object – virtually identical (within acceptable error) responses of the object and model with the same impact test.

REFERENCES


I used to working at Young scientific researcher, Department of Television, Odessa Electrotechnical Institute of Communications (1974-1975); Senior researcher, Branch Research Laboratory of the Non-Destroyed Testing Products of Electronic, Ministry of Electronic Industry USSR and Odessa Polytechnic Institute (1975-1986); Associate Professor, Department of Computerized Control Systems, Odessa National Polytechnic University (1989-2013); Associate Professor, Department of Maritime Radio Communication, Odessa National Maritime Academy (2001-2013); Professor, Department of Maritime Radio Communication, National University “Odessa Maritime Academy” (2013-2017); Scientific Supervisor, Odessa branch of the minor Academy of Sciences Ukraine (Computer Science), Department of Education and Science, Odessa Regional State Administration (2001-present); Professor, Department of Computerized Control Systems, Odessa National Polytechnic University, Shevchenko avenue, 1, Odessa, Ukraine, 65044 (2013-present).


Professional interests: Modeling and Simulation for Industrial Applications; Non-parametric Identification of Nonlinear Systems; Theory of the Volterra series; Mathematical methods, models and technologies for complex systems research.

Prof. Pavlenko have a Honorary Diploma of the Ministry of Education and Science of Ukraine (2005); Diploma of the Ministry of Education of the Russian Federation (2005); The title of "Honorary Veteran of ONPU" was awarded (2012).