Dispersion relation of triple-chambered cochlea

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Abstract— We investigated the dispersion relations of the triple-chambered cochlea separated by the basilar membrane and the Reissner's membrane. The slow waves can propagate on the basilar membrane (basilar membrane mode), and also on the Reissner's membrane (Reisnner's membrane mode). Most of the previous studies have focused on the mechanics of vibration of the basilar membrane and there are only a few studies on the wave propagation on Reissner's membrane. In this study, we focused particularly on the Reissner's membrane mode among the many propagation modes and evanescent modes that the structure supports. We analyzed the dispersion of the Reissner's membrane mode and the basilar membrane mode, and studied their dependency on the position along the cochlea. We also studied how the distance between the Reissner's membrane and the basilar membrane affects the dispersion relations.

Keywords—Reissner's membrane, basilar membrane, cochlea, mode analysis, dispersion relation, slow wave, wave propagation

I. INTRODUCTION

THE cochlea is a fluid-filled duct that has three regions: the scala vestibuli, scala media, and scala tympani. They are separated by the Reissner's membrane (RM) and the basilar membrane (BM). Many analytical and numerical models have been developed to study wave propagation in the cochlea [1]. Most of the previous studies have treated the cochlea as two fluid chambers and focused on the mechanics of vibration of BM [2-9]. The slow wave that propagates on BM is called the BM mode [10, 11]. On the other hand, there are only a few studies on the wave propagation on RM (RM mode). Fuhrmann et al. have studied the effects of RM on the wave propagation in the cochlea [12]. They assumed that RM had neither mass nor stiffness and treated it as a rigid boundary with no-slip condition. Rwichenbach et al. have shown that the RM mode plays an important role in transmitting the signals of otoacoustic emissions [13]. While BM includes radial fibers that impose a high stiffness, RM exhibits a comparatively low impedance. It has been shown that the RM mode does not evoke a significant displacement of BM, but, in contrast, a

disturbance moving in the BM mode propagates on both membranes [13]. It has also been shown that in the basal region of the cochlea, and for frequencies above 1 kHz, RM sustains waves with wavelengths
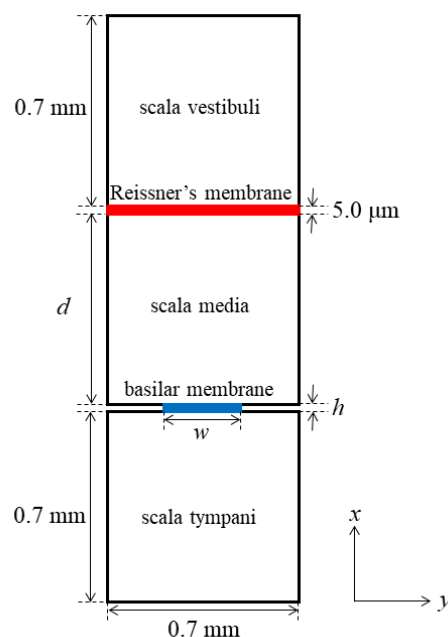

Figure 1. Cross-section of cochlea.

smaller than the height of the scalae and that do not penetrate into the membrane's surrounding fluids [13]. The dispersion relations of the RM mode, however, have not been studied so far. Given the dispersion relations, the phase velocity and group velocity can be obtained. Thereby, it is possible to understand the basic characteristics of the waves, such as fast wave and slow wave, and we can also discuss the possibility of the coupling between two modes.

In a previous study, we treated the cochlea as two fluid chambers and studied the dispersion diagrams of the fast-wave and slow-wave modes and the structural dependence of the angular wavenumber and coupling efficiency of the slow-wave mode [14]. We also investigated the dispersion characteristics of the propagation modes that progress on the tectorial membrane [15]. In this study, we considered the cochlea structure as three regions separated by RM and BM, and investigated the dispersion characteristics of the RM mode as

well as the BM mode. We used COMSOL Multiphysics to analyze the effect of the elastic-wave mode.

II. MODEL

The human cochlea is a fluid-filled structure about 35 mm in length and coiled in a spiral of 2 1/2 to 2 3/4 turns. We modeled it as an uncoiled, triple-chambered fluid-filled duct, which is composed of the scala vestibuli, scala media, and scala tympani.

Figure 1 shows the analysis model. To simplify the problem, we assumed that RM and BM are located parallel to each other. Here, we also assumed that the cochlea duct was enclosed by rigid boundaries. The sizes of the three chambers; namely, the scala vestibuli, scala media, and scala tympani, vary along the cochlea duct. It has been shown in particular that the width of RM is slightly dependent on the location along the cochlea and has a length ranging from around 0.6 to 0.8 mm [16]. We assumed here that the width of RM is constant at 0.7 mm. The size of the chambers is uniform along the waveguide, and the heights and widths of the scala vestibuli and the scala tympani are equal at 0.7 mm, which is the same length as the width of RM. We used the width w, height h, and Young's modulus E of BM as parameters [17].

$$w = 0.1 \text{ [mm]} + \frac{0.4 \text{ [mm]}}{35 \text{ [mm]}} \times z, \quad (1)$$

$$h = 7.5 \text{ [μm]} - \frac{5 \text{[μm]}}{35 \text{ [mm]}} \times z, \quad (2)$$

$$E = 50 \text{ [MPa]} - \frac{47 \text{ [MPa]}}{35 \text{ [mm]}} \times z, \quad (3)$$

where z is the length along the cochlea duct. Here, the structural parameters of RM were assumed to be independent from the location along the cochlea. The length d between RM and BM was used as a parameter, and we studied the influence of d on the dispersion relations of the cochlea. The density and bulk modulus of the ideal fluid were respectively 1.034×10^3 kg/m^3 [18] and 2.2×10^9 Pa [1]; the density and Poisson's ratio of RM and BM were respectively 1.2×10^3 kg/m^3 [18] and 0.49 [18]; and Young's modulus of RM was 15 MPa [19]. We used COMSOL Multiphysics with 662 nodes and 8838 elements. In the analysis, the acoustic and elastic wave equations were solved by using the finite element method.

III. MODE ANALYSIS

Figure 2 shows the dispersion relation of the RM mode and the BM mode when $z = 10$, 20, and 30 mm, where the horizontal axis is the acoustic-wave frequency f and the vertical axis is the angular wavenumber k. Here, the length d between RM and BM is 0.7 mm and the three chambers have the same square shape. As shown in this figure, the dispersion curves of the RM mode are almost the same regardless of z. On the other hand, those of the BM mode largely vary depending on z. When $z = 10$ and 20 mm, the RM mode has bigger angular wavenumbers compared with the BM mode over the whole frequency range, while they are smaller than those of the BM mode over the whole range when $z = 30$ mm.

Figure 3 shows the x-direction displacement of the

membrane of (a) RM mode and (b) BM mode when $f = 1000$ Hz, and (c) RM mode and (d) BM mode when $f = 10000$ Hz, where $z = 10$ mm. In these figures, the solid and dashed lines show the displacement of RM and BM, respectively. The vertical axis is normalized by the maximum value. As shown in Fig. 3(a), the RM mode vibrates only RM, while as in 3(b), the
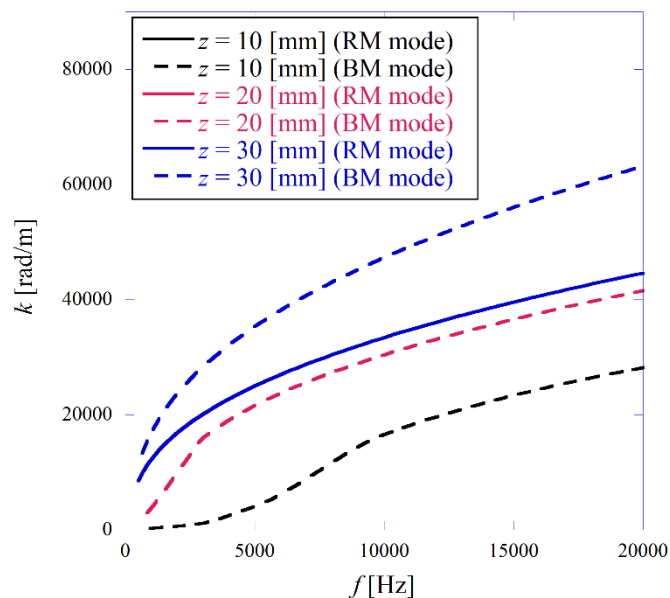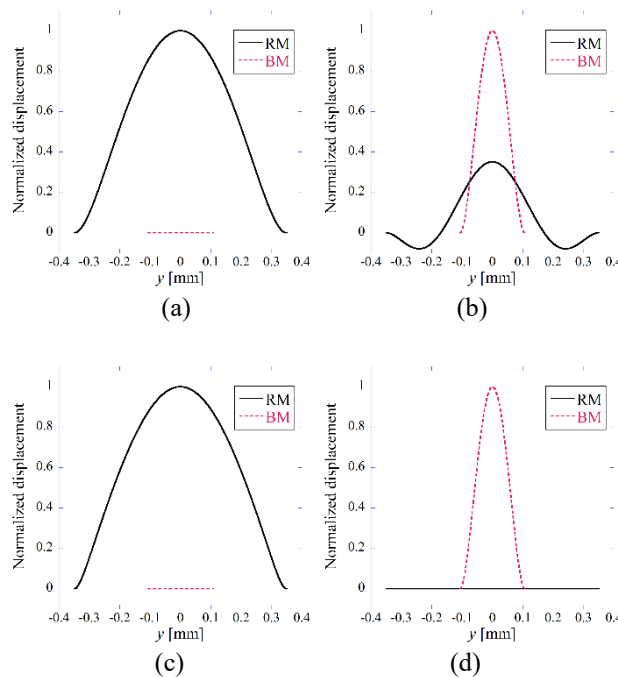

Figure 2. Dispersion diagram.

Figure 3. Displacement of membranes: (a) RM mode ($f = 1000$ Hz), (b) BM mode ($f = 1000$ Hz), (c) RM mode ($f = 10000$ Hz), (d) BM mode ($f = 10000$ Hz).

BM mode vibrates both BM and RM when $f = 1000$ Hz. On the other hand, Fig. 3(c) and 3(d) show that each mode vibrates only its membrane when $f = 10000$ Hz.

Figure 4 shows the angular wavenumber k of the RM mode

and the BM mode versus z, when f = 1000, 3000, and 10000 Hz. This figure shows that the wavenumber of RM mode is constant from the base to the apex of the cochlea regardless of the acoustic frequency. On the other hand, the wavenumbers of the BM mode start to increase rapidly at a certain point along z. For example, when f = 10000 Hz, the wavenumber has a small value
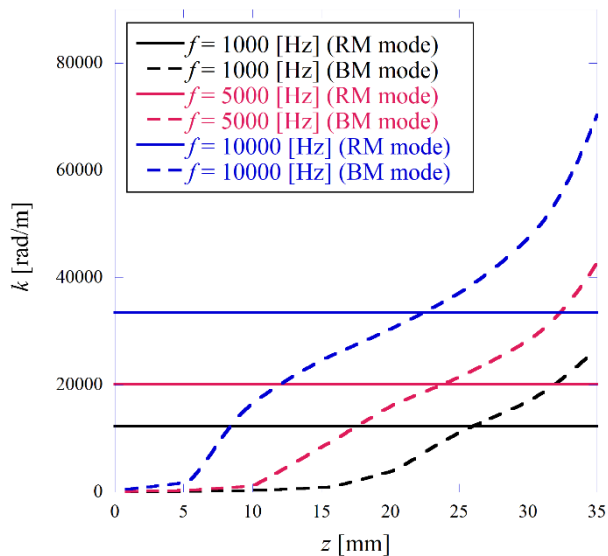
unchanged at 0.7 mm. Fig. 5 shows the angular wavenumber k of the RM mode and the BM mode versus d when f = 1000 Hz. It shows that d hardly affects the angular wavenumbers of both modes when d is more than 0.2 mm. The differences of the angular wavenumbers between the two modes increase when d is smaller than 0.2 mm. The variation tendency, however, is not dependent on the mode type. For example, the angular

Figure 4. Angular wavenumber versus z.

(a)
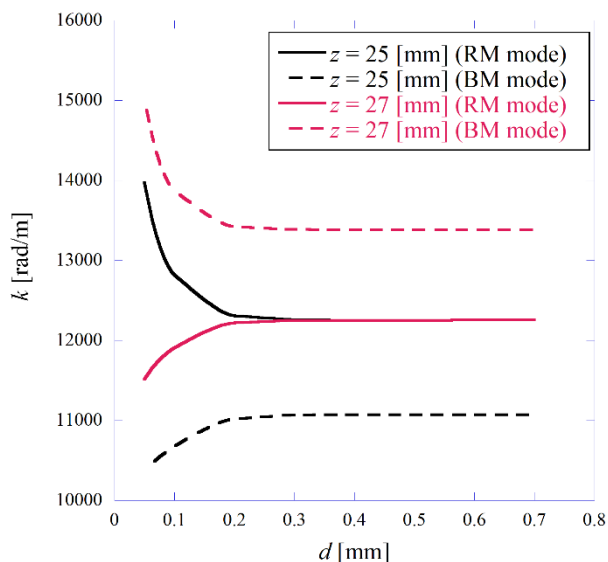
Figure 5. Angular wavenumber versus d when f = 1000 Hz.
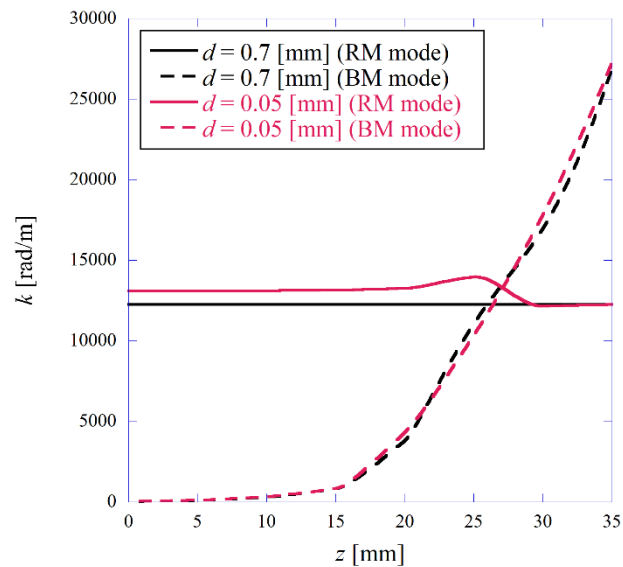
(b)

Figure 6. Angular wavenumber versus z when (a) f = 1000 Hz and (b) f = 10000 Hz.
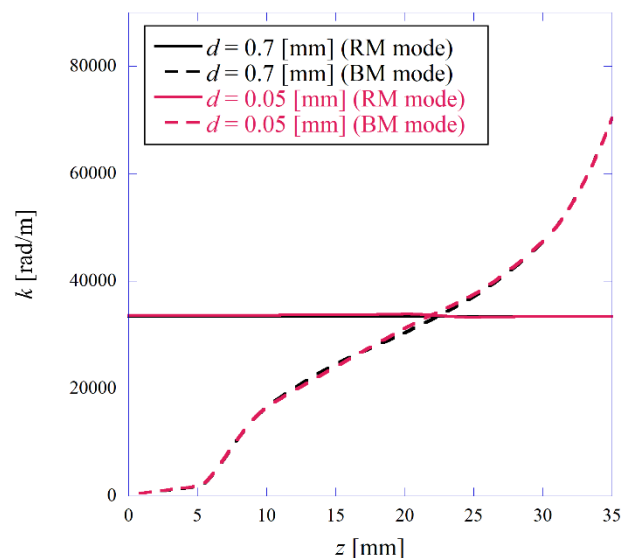
until z is less than about 5 mm, and it starts to increase at that point. This is due to the variation along z of the width, height, and Young's modulus of BM. We assumed that the Young's modulus of BM varies according to equation (3). Therefore, it is equal to that of RM when z = 26 mm. We can see that the angular numbers of the two modes are equal when the Young's modulus of BM is slightly larger than that of RM.

Next, we investigated the influence of the height of the scala media d on the angular wavenumber, wherein the heights of the other two chambers and the width of the three chambers are

wavenumber of the RM mode increases as d decreases when z = 25 mm, but it decreases when z = 27 mm.

Figures 6(a) and 6(b) show the angular wavenumber k of the RM mode and the BM mode versus z, when the height of the scala media d is 0.05 or 0.7 mm. Here, f = (a) 1000 and (b) 10000 Hz, respectively. As shown in Fig. 6(a), when f = 1000 Hz, the angular wavenumber k of the RM mode increases as d is reduced from 0.7 to 0.05 mm. The angular wavenumber

difference is about 1000 rad/m from $z = 0$ to 25 mm, and the difference becomes very small when z is more than 30 mm. On the other hand, d has little influence on the angular wavenumber of the BM mode. From Fig. 6(b), we can see that both modes have little dependence on d over the whole range of z when $f = 10000$ Hz.

IV. CONCLUSION

We investigated the cochlea, which has three regions separated by RM and BM, by using modal analysis. The dispersion diagrams of the RM mode and the BM mode showed that the wavenumber of the RM mode is constant from the base to the apex of the cochlea regardless of the acoustic frequency, while the wavenumber of the BM mode largely changes depending on the position along the cochlea. The analytical results of the displacement of the membranes demonstrated that when $f = 1000$ Hz, the RM mode vibrates only RM, while the BM mode vibrates both BM and RM, and that when $f = 10000$ Hz, each mode vibrates only its membrane. We also studied how the distance between BM and RM affects the dispersion relations of the RM mode and the BM mode. The results showed that when $f = 1000$ Hz, the angular wavenumber of the RM mode increases as the distance is reduced, whereas when $f = 10000$ Hz, both modes have little dependence on the distance over the whole range of the length along the cochlea duct. It was shown that the distance hardly affects the angular wavenumbers of both modes when it is more than 0.2 mm.

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