Probabilistic Approach (Simulation-Based Reliability Assessment Method) Applied in Engineering Design and Economics

Karel Frydrýšek, Josef Kašík and Horst Gondek

***Abstract*—**Firstly, this paper reports about the probabilistic solutions of the problems in engineering design (i.e. applications in mining industry - hard rock disintegration process, and applications in biomechanics & traumatology - designing of the external fixators applied in traumatology and orthopaedics).Secondly,this paper reports about the new probabilistic application in economics (evaluation of an investment project). Theory and applications of the Simulation-Based Reliability Assessment (SBRA) Method (i.e.direct probabilistic Monte Carlo approach) are presented in the solutions and evaluations. Applications of the SBRA method in these areas of are new and innovative trends in mechanics and economics.

***Keywords*—**Probability, Simulation-Based Reliability Assessment (SBRA) Method, design, mining, rock mechanics, biomechanics, traumatology, economics, investment, net present value.

I. Introduction

Let us consider the Simulation-Based Reliability Assessment (SBRA) Method, as in [1] to [4], a probabilistic direct Monte Carlo approach, in which all inputs are given by bounded histograms. Bounded histograms include the real variability of the variables.

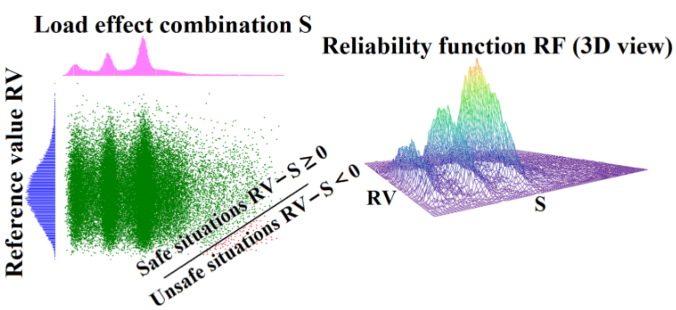

Fig. 1 Reliability function RF (SBRA Method)

Using SBRA method, the probability of failure (i.e. the probability of undesirable situation) is obtained mainly by analyzing the reliability function

$$RF = RV - S, \quad (1)$$

see Fig. 1. Where RV is the reference (allowable) value and S is a variable representing the load effect combination (results of simulations). The probability of failure is the probability that S exceeds RV i.e.

$$P(RF \leq 0). \quad (2)$$

This paper focuses on the SBRA Method applied in the area of engineering design (report about mining - hard rock disintegration process, and report about biomechanics & traumatology - designing of the external fixators applied for treatment of fractures) and in economics (new application - evaluation of an investment project).Another applications of the SBRA Method are mentioned too.

II. Application in the Mining Industry

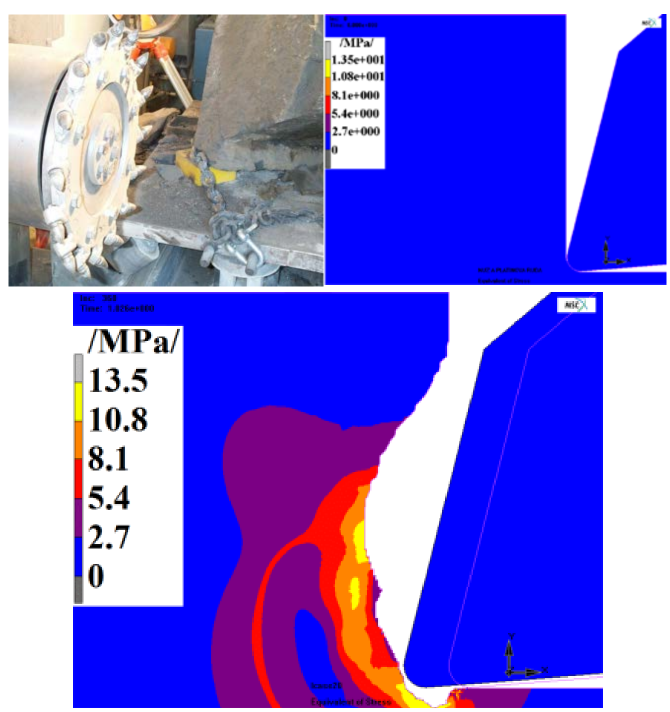

Fig. 2 Example of mechanical interaction between bits and hard rock

Karel Frydrýšek, Department of Mechanics of Materials, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, Czech Republic (corresponding author's phone: +420597 323 495; e-mail: karel.frydrysek@vsb.cz).

Josef Kašík, Department of Business Administration, Faculty of Economics, VŠB – Technical University of Ostrava, Czech Republic (e-mail: josef.kasik@vsb.cz).

Horst Gondek, Department of Production Machines and Design, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, Czech Republic (e-mail: horst.gondek@vsb.cz).

and FE modelling (ore disintegration process)

This chapter is focused on the modelling and designing of excavation tool for the mechanical contact between the bit and the platinum ore and its evaluation (i.e. report about the practical application in the mining technology and rock mechanics), see Fig. 2 and 3.

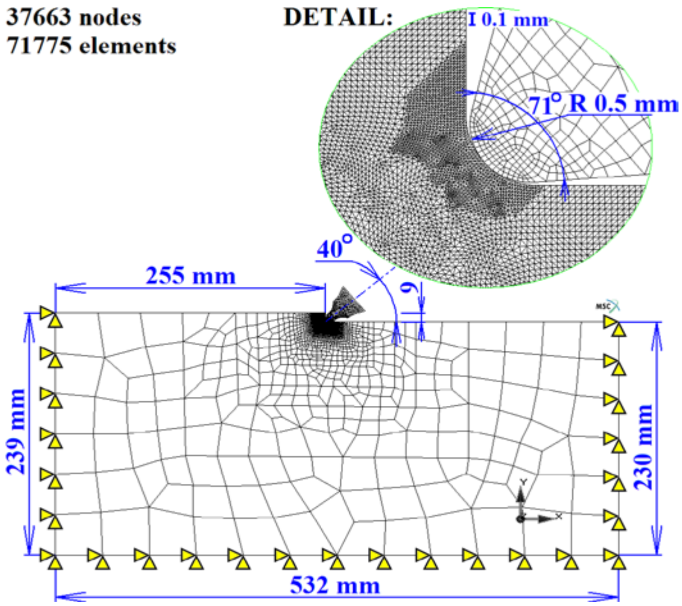

Fig. 3 Ore disintegration process – Finite Element mesh and boundary conditions (solution via FEM)

However, material properties of the ore have a large stochastic variability. Hence, the stochastic approach (i.e. SBRA Method in combination with FEM is applied). The probabilistic inputs, i.e. elastic properties (Modulus of elasticity E and Poisson's ratio μ) and plastic properties (yield stress Rp and fracture stress Rm) are described by bounded histograms, for example see Fig. 4.

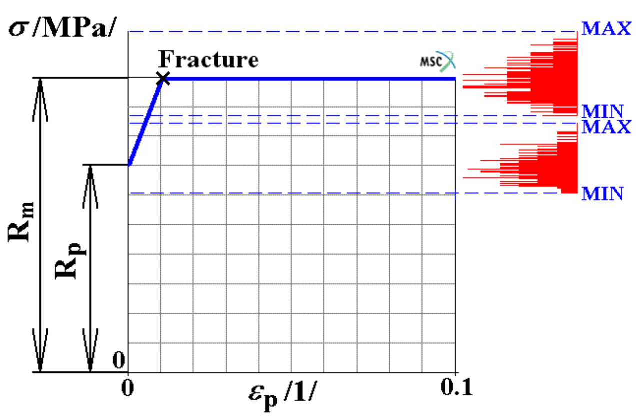

Fig. 4 Material properties of the ore (probabilistic inputs, stress vs. plastic strain)

The results were subsequently statistically evaluated. Because of the material non-linearities, the mechanical contacts with friction, the large number of elements, many iteration steps, and the choice of 500 Monte Carlo simulations, four parallel computers were used to handle the large computational requirements. From the results, the total reaction forces Rv /N/ can be calculated (stochastic result). Reliability function RF, see eq. (1), can be defined by

$$\mathrm{RF} = \mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{ALLOWABLE}}} - \mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{SBRA,\,FEM}}}\ , \quad (3)$$

where $\mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{ALLOWABLE}}}$ /N/ is the allowable reaction force in the cutting bit, which can be acquired from the real capacity of the whole cutter-loader system in the mine and $\mathrm{R}_{\mathrm{V\ MAX}_{\mathrm{SBRA,\,FEM}}} = 5068^{+1098}_{-984}$ N is the maximum total reaction force (acquired from 500 Monte Carlo simulations). If situations when $\mathrm{RF} \le 0$ occur, then the cutter-loader system is overloaded.

Hence, fully probabilistic assessment can be calculated by comparing of probabilities

$$P(\mathrm{RF} \le 0) \le P_{\mathrm{ALLOWABLE}}\ , \quad (4)$$

where $P_{\mathrm{ALLOWABLE}}$ is the acceptable probability of overloading of the cutting-loader system. All the results presented here were applied for optimizing and redesigning of the cutting bit (excavation tool for platinum ore), see Fig. 2a and 5.

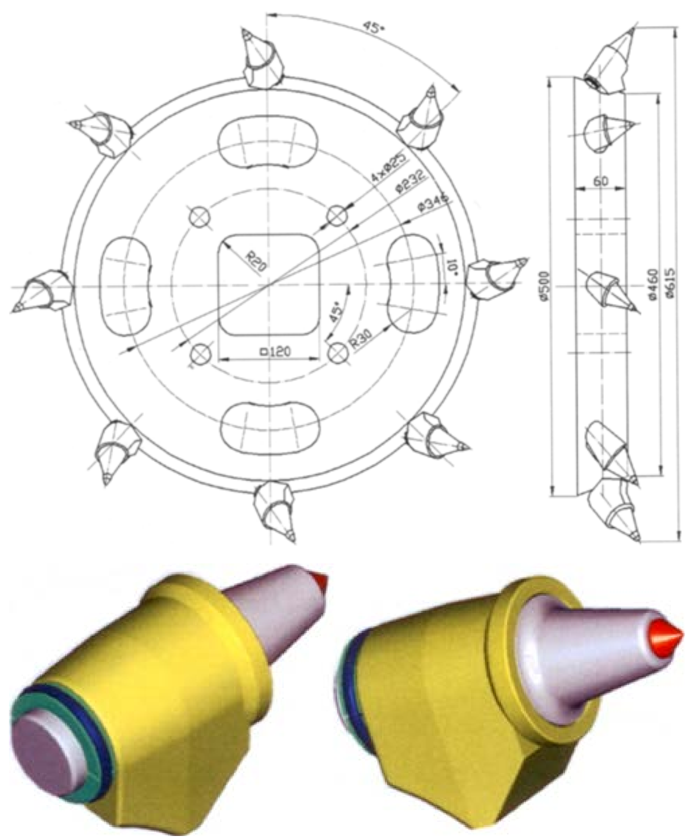

Fig. 5 Final shape of excavation toll and its cutting bit for platinum ore disintegration process

For more information see [1] and [16].

III. Applications in Biomechanics

According to the current studies and research, performed at VŠB – Technical University of Ostrava, Traumatology Centre of the University Hospital of Ostrava and MEDIN a.s., the current design of external fixators (applied in traumatology, surgery and orthopaedics) must be modified, see Fig. 6.

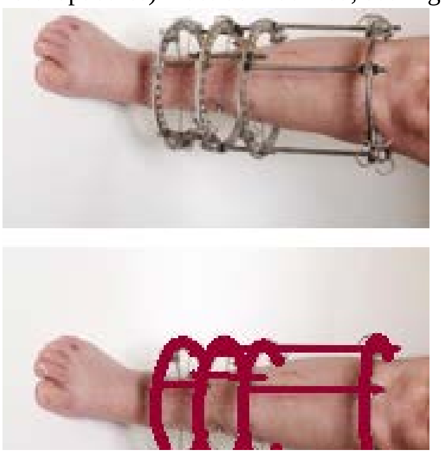

Fig. 6 Design of external fixators a) based on metals - current design, heavier, expensive, etc. b) based on reinforced polymers - new design, lighter, cheap, more friendly etc.)

There are real needs to make a modern design of fixators which satisfy new trends and demands in medicine:

- Applications of new smart materials (the outer parts of fixators must be x-ray invisible, see Fig. 7 - which leads to shortening the operating time and reducing the radiation exposure of patients and surgeons. Antibacterial protection - application of nanotechnologies to prevent or reduce possible infection and weight optimization).
- New design (according to shape, ecological perspective, patient's comfort, reducing the time of the operation, reducing the overall cost, "friendly-looking design").
- Numerical modelling, experiments and measuring of the real loadings (i.e. SBRA application, FEM, strain gauges etc.).

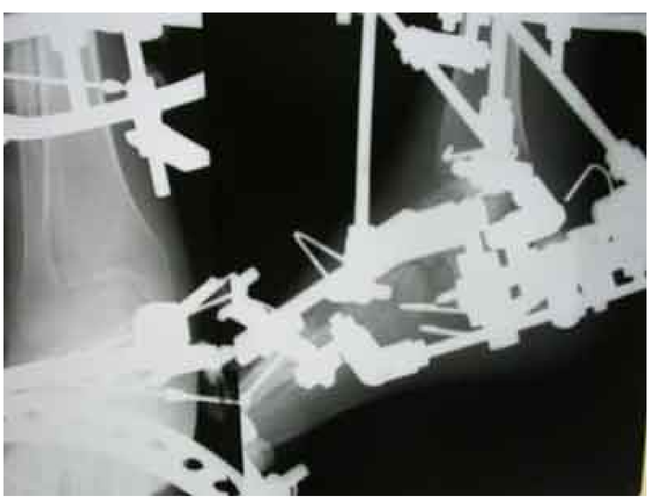

Fig. 7 Problems with high x-ray absorption (it is difficult to see broken limbs because there are too many metal parts)

The new proposed designs are based on SBRA Method application, see Fig. 8.

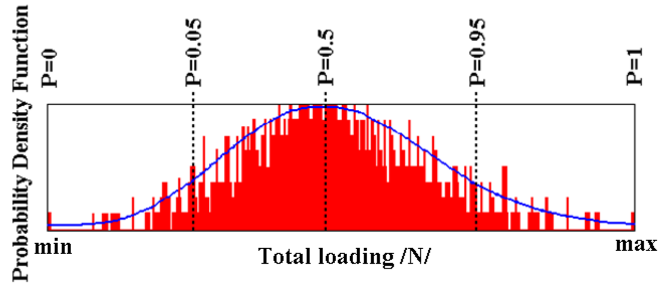

Fig.8 Typical loading spectrum of an external fixator (histogram, overloading is included).

For more information (practical results) see [5] to [9] and [14] and [15].

IV. Application in Economics

The Monte Carlo simulation (i.e. SBRA Method) is a useful tool not only in technical sciences but it can be also effectively used in economics.

One of the most important questions for a business today is how to evaluate and choose the right investment proposal that will increase the business value, which is considered to be the main financial objective of every business [12].

It is not an easy task because it is necessary to decide, whether the expected future benefits (usually expressed in money) are large enough to justify the current expenditure. The managers' decision-makingshould be based on data reflecting the real life situation. They should also take into account the relevant risks related to the investment proposal.

Suppose that you are considering an investment project. The financial evaluation of any investment proposal involves three discrete steps [11]:

1. Estimation of the relevant cash flows,
2. Calculation of a figure of merit for the investment,
3. Comparison of the figure of merit to an acceptance criterion.

The first step, estimating the relevant cash flows, is the most challenging one. It is usually requiring thorough understanding of a business' markets, competitive position, and long-run intentions. The most important problem is the fact that many important costs and benefits are hardly measurable in monetary terms.

The second step, a figure of merit has to be calculated. A common figure of merit is rate of return, which translates the cash inflows and outflows associated with an investment into a single number summarizing its economic worth.

An acceptance criterion (in the third step) is a standard of comparison that helps the managers determine whether a figure of merit is attractive enough to warrant acceptance.

In practice, we usually use in a decision-making process the net present value NPV concept. According to equation (1), the net present value is the difference between a project's value and its cost, see [10]

$$\mathrm{NPV} = \mathrm{PV} - \mathrm{I}, \tag{5}$$

where PV is the present value of the project cash flows and is usually expressed by equation

$$\mathrm{PV} = \sum_{t=1}^{T} \frac{\mathrm{CF}_t}{(1+r)^t}, \tag{6}$$

where variable CF_t is cash flow in year t, r is the discount rate (also known as the opportunity cost of capital, i.e. the return one could earn on the next best alternative) and T is the total life of the project in years.

The discount rate is frequently used to adjust an investment's cash flows for risk and hence is also known as a risk-adjusted discount rate. The discounted cash flow method considered in this example is relevant whenever a company contemplate an investment project entailing costs or benefits that extend beyond one year.

The parameter I in eq. (5) is the required initial investment. If you want to create value for the owners of a business, you should accept investment projects with positive NPV (the higher the NPV, the better) and eschew negative-NPV projects. Zero-NPV activities are marginal because they neither create nor destroy wealth or business, see [11].

Mathematically expressed according eq. (5), when $\mathrm{NPV} > 0$, then you should accept the investment; when $\mathrm{NPV} < 0$, then you should reject the investment because the costs of the project are greater than the financially expressed benefitand when $\mathrm{NPV} = 0$, then the investment is marginal. Therefore, we should accept only projects with positive net present value.

Although, it is more often used deterministic (old) approach for calculating the net present value of a project, the variables CF_t, i and even I are rather stochastic in reality and NPV has the characteristics analogical to the reliability function defined in eq. (1) or (5). Hence, new and original way, the probabilistic assessment (i.e. a new application of SBRA Method in economics), see eq. (2), can be calculated by comparing of probabilities

$$P(\mathrm{NPV} \le 0) \le P_{\mathrm{ACCEPTABLE}}\ ,, \tag{7}$$

where $P_{\mathrm{ACCEPTABLE}}$ is the acceptable probability of NPV lesser than zero (i.e. acceptable probability of risk, acceptable probability of pecuniary loss). $P_{\mathrm{ACCEPTABLE}}$ is rather subjective in this example because it depends on the investor's attitude to risk, see explanation in Fig. 9.

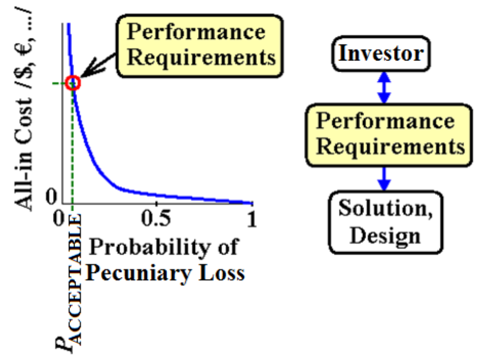

Fig. 9 Definition of the acceptable probability

The probabilistic characteristics for apractical example of investment project (i.e. input data for AntHill software, for example see Fig. 10, 11 and 12) are presented in Table I.

Variable	Type	Parameters	Comment
I /money/, /CZK/	Truncated normal distribution	Min 9.5×10^6 Med 9.999×10^6 Max 1.05×10^7	see Fig. 10
T	Constant	5 years	time
CF_1 /money/, /CZK/	Truncated normal distribution	Min 1.8×10^6 Med 2×10^6 Max 2.2×10^6	1^{st} year, see Fig. 11
CF_2 /money/, /CZK/	Truncated normal distribution	Min 2.125×10^6 Med 2.502×10^6 Max 2.875×10^7	2^{nd} year, see Fig. 12
CF_3 /money/, /CZK/	Truncated normal distribution	Min 2.4×10^6 Med 3×10^6 Max 3.6×10^6	3^{rd} year, see Fig. 13
CF_4 /money/, /CZK/	Truncated normal distribution	Min 2.625×10^6 Med 3.502×10^6 Max 4.375×10^6	4^{th} year, see Fig. 14
CF_5 /money/, /CZK/	Truncated normal distribution	Min 2.8×10^6 Med 3.996×10^6 Max 5.2×10^6	5^{th} year, see Fig. 15
r /1/	Truncated normal distribution	Min 0.09 Med 0.1 Max 0.11	see Fig.16

Table I Input data of an investment project (Min, Med and Max are minimum, median and maximum values)

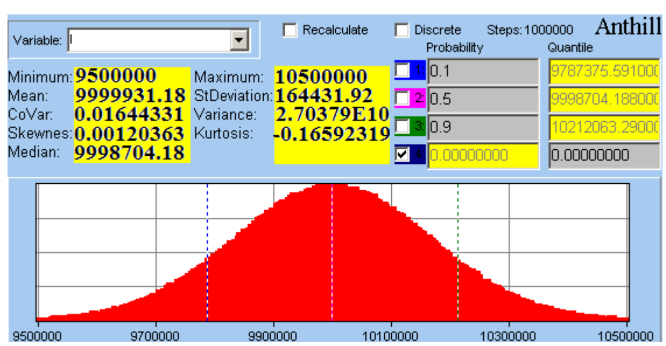

Fig. 10 Histogram of the required initial investment I (106 Monte Carlo simulations, stochastic input)

We can see in Table I and Fig. 10 that it is an investment project for five years with initial investment around 10 million

Czech Crowns /CZK/ (i.e. median value 9998704 CZK). Hence, it can be written in the form $I = 9998704^{+501296}_{-498704}$ CZK.

The cash flows during the life of the project differ from about 2 million CZK in year 1 (i.e. $CF_1 = 1999920^{+200080}_{-199920}$ CZK), see Fig. 11, to about 4 million CZK in year 5 (i.e. $CF_5 = 3996285^{+1203715}_{-1196285}$ CZK), see Fig. 15. Hence, histograms for CF_1 to CF_5 are presented in Fig. 11 to 15 and Table. I.

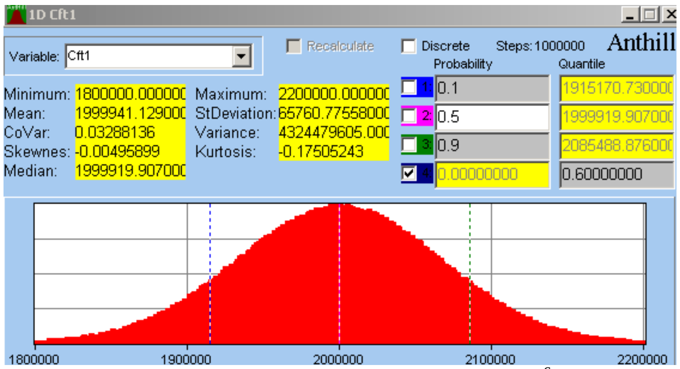

Fig. 11 Histogram of the cash flow in year 1 (CF_1 - 10^6 Monte Carlo simulations, stochastic input)

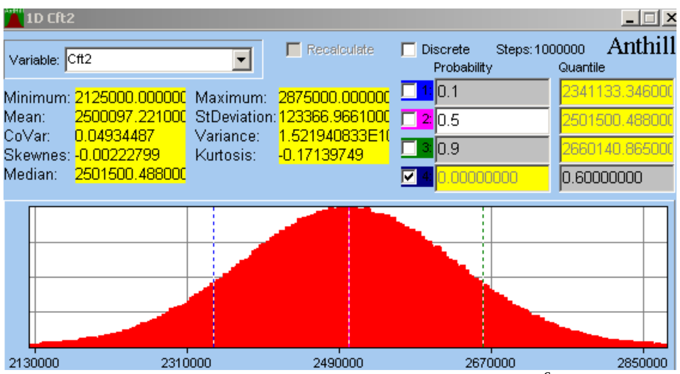

Fig. 12 Histogram of the cash flow in year 2 (CF_2 - 10^6 Monte Carlo simulations, stochastic input)

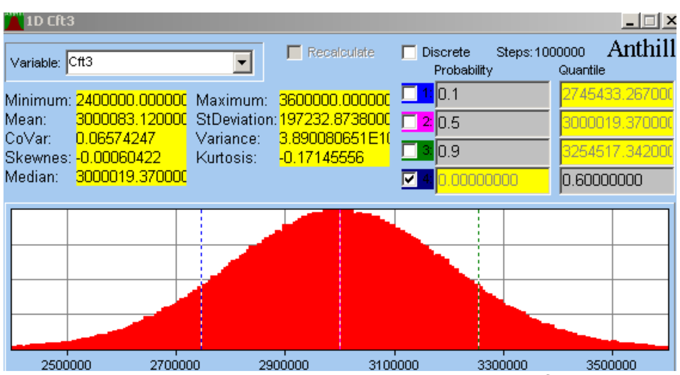

Fig. 13 Histogram of the cash flow in year 3 (CF_3 - 10^6 Monte Carlo simulations, stochastic input)

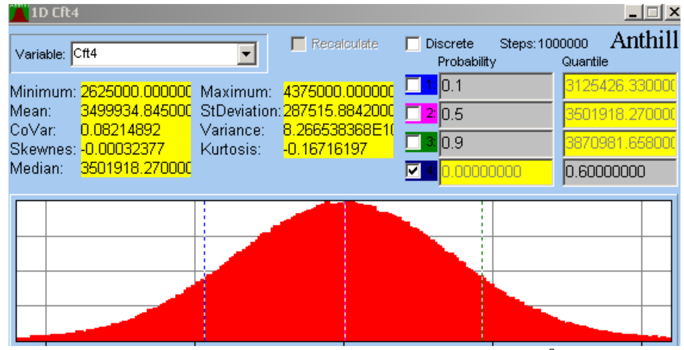

Fig. 14 Histogram of the cash flow in year 4 (CF_4 - 10^6 Monte Carlo simulations, stochastic input)

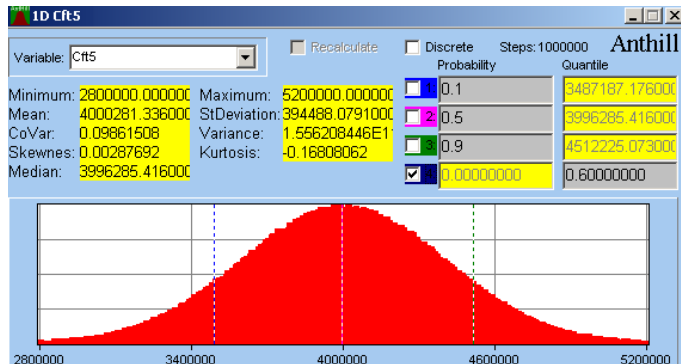

Fig. 15 Histogram of the cash flow in year 5 (CF5 - 10^6 Monte Carlo simulations, stochastic input)

The discount rate (the opportunity cost of capital, i.e parameter r) is within the range 0.09 – 0.11, see Fig. 16 and Tab. I.

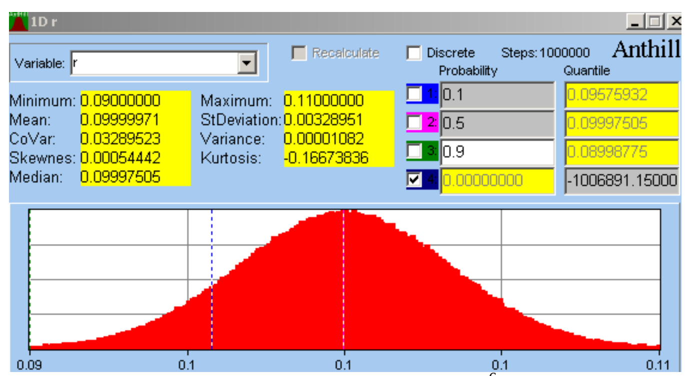

Fig. 16 Histogram of the discount rate r (10^6 Monte Carlo simulations, stochastic input)

After calculations of 10^6 simulations of Monte Carlo method (i.e. application of AntHill software, see Fig. 17), the functions PV, see Fig. 18, and NPV, see Fig. 19, can be calculated. The output data from AntHill software are presented in Table II.

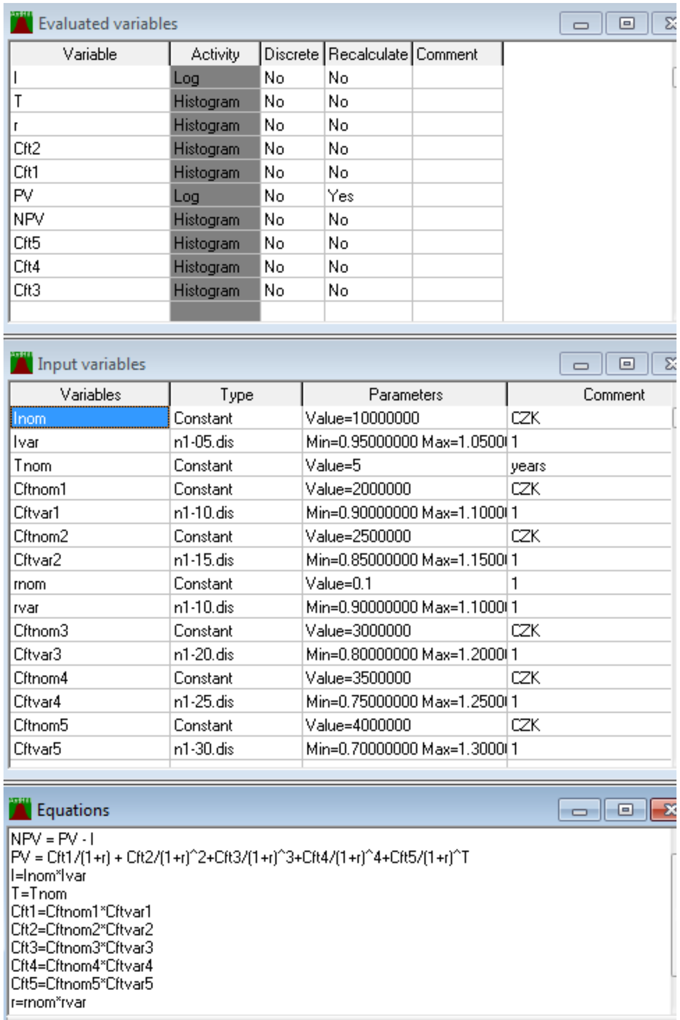

Fig. 17 Programming (AntHill software, application in economics)

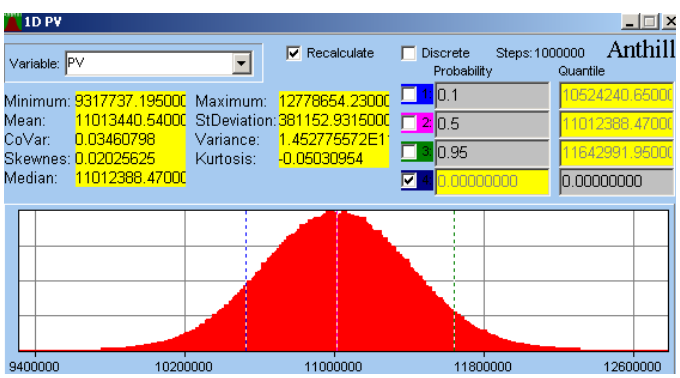

Fig. 18 Histogram of present value PV (10^6 Monte Carlo simulations, stochastic output)

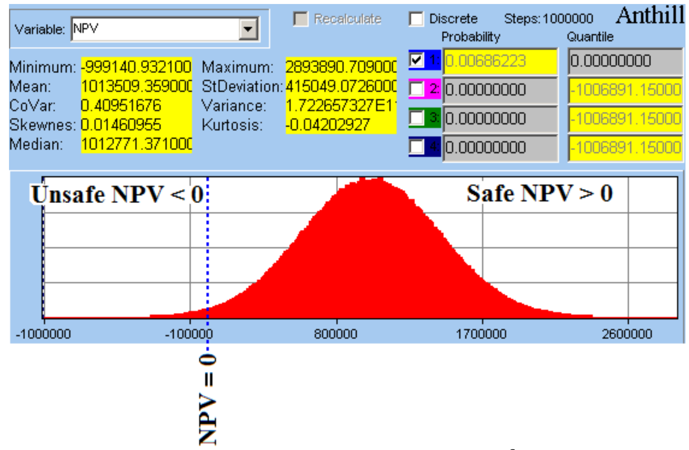

Fig. 19 Histogram of net present value NPV (10^6 Monte Carlo simulations, stochastic output)

Variable	Parameters	Comment
PV /money/	Min 9.318×10^6 Med 11.012×10^6 Max 12.778×10^6	see Fig. 18
NPV /money/	Min -9.991×10^5 Med 1.013×10^6 Max 2.894×10^6	see Fig. 19 and 20

Table II Output data of an investment project (Min, Med and Max are minimum, median and maximum values)

We can also calculate via Anthill software the probability $P(\text{NPV}\leq 0) = 0.0069$ (i.e. the risk of nonpositiveNPV is 0.69 %). This is clearly presented in Fig. 19 and Fig. 20 using geometric probability. In this example, the result is very acceptable even for risk-averse investor.

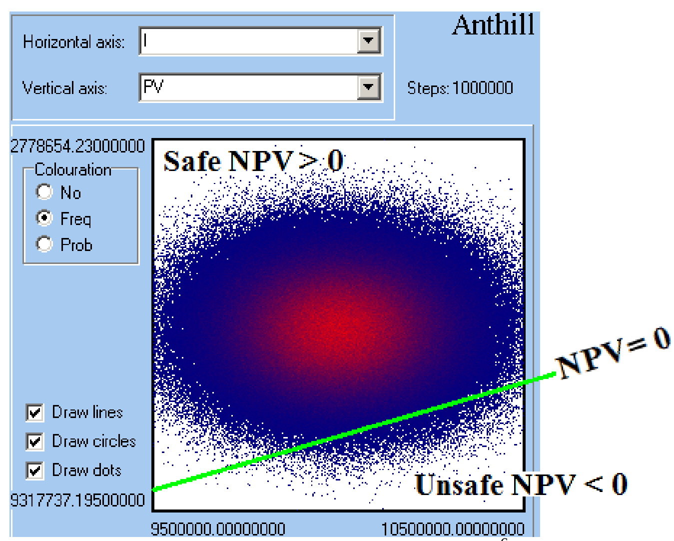

Fig. 20 2D histogram of reliability function (10^6 Monte Carlo simulations, stochastic output)

V. Conclusion

Report about the application of the Simulation-Based Reliability Assessment (SBRA) Method in the area of rock mechanics (hard rock disintegration process - design of excavation tool), biomechanics & traumatology (design of external fixators) was presented.

New application in economics (investment project evaluation), which leads to practical applications, waspresented.

Application of the SBRA method is a modern and innovative trend in engineering and it can be also used in economics. The development of SBRA Method is evident (see also [1], [2], [6], [13] and [16]). Nature is stochastic and therefore SBRA Method is a good tool to perform real reliability assessments, for example see its new application in geomechanics presented in Fig. 21 and 22 and references [2] and [13].

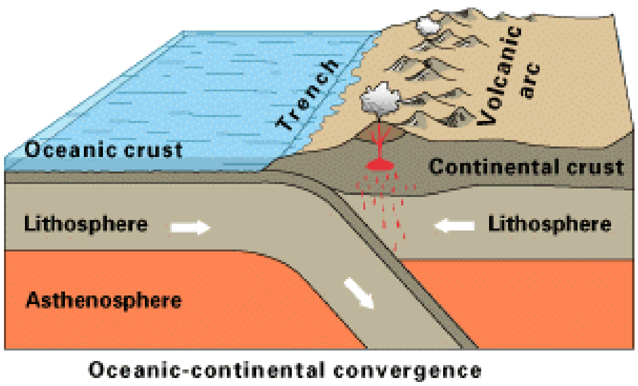

Fig. 21 Application of SBRA Method in mechanics of continental plates, see [2] and [13] (introduction)

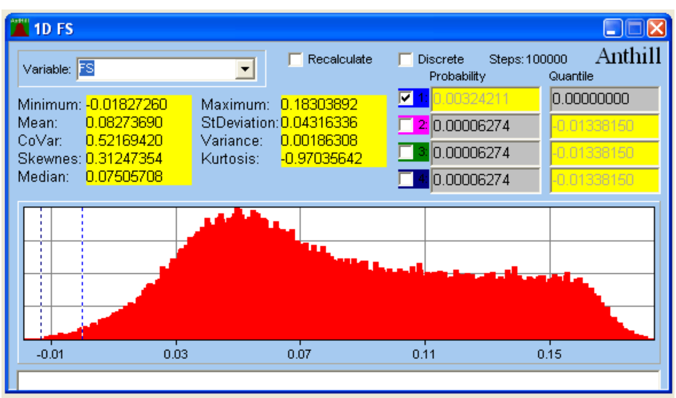

Fig. 22Application of SBRA Method in mechanics of continental plates, see [2] and [13] (reliability function – stochastic output)

This article was supported by the Czech grant projects FR-TI3/818, TA03010804 and by the European Social Fund withinthe project CZ.1.07/2.3.00/20.0296.

References

[1] Frydrýšek, K.: Probabilistic Approach Applied in the Design of Machine Parts and Structures, In*: Recent Researches in Engineering and Automatic Control*, ISBN 978-1-61804-057-2, North Atlantic University Union, WSEAS Press, Puerto De La Cruz, Tenerife, Spain, 2011, pp.30-37.

[2] Frydrýšek, K., Wandrol, I., Kalenda, P.: Report about the Probabilistic Approaches Applied in Mechanics of Continental Plates, In*: Mathematical Models and Methods in Modern Science*, vol. 3, ISBN 978-1-61804-106-7, ISSN 2227-4588, WSEAS Press, Porto, Portugal, 2012, pp.146-149.

[3] Kala, Z., Sensitivity Analysis of Steel Plane Frames with Initial Imperfections, in *Engineering Structures*, vol.33, no.8, 2011, pp. 2342-2349.

[4] Marek, P., Brozzetti, J., Guštar, M., Tikalsky, P.: *Probabilistic Assessment of Structures Using Monte Carlo Simulation Background, Exercises and Software, (2nd extended edition)*, ISBN 80-86246-19-1, ITAM CAS, Prague, Czech Republic, 2003.

[5] Frydrýšek, K., Pleva, L, Janečka, M., Klučka, R., Sivera, M., Jořenek, J.: Report about the New External Fixator for Treatment of Pelvis and Acetabulum Fractures, In*: Latest Advances in Biology, Environment and Ecology*, ISBN 978-1-61804-097-8, North Atlantic University Union, WSEAS Press, Iasi, Romania, 2012, pp.144-147.

[6] Frydrýšek, K., Pleva, L, Učeň, O, Kubín, T., Rojíček, J., Šír, M., Madeja, R.: Report about the New External Fixator for Treatment of Complicated Limb Fractures, In*: Latest Advances in Biology, Environment and Ecology*, ISBN 978-1-61804-097-8, North Atlantic University Union, WSEAS Press, Iasi, Romania, 2012, pp.148-151.

[7] Frydrýšek, K., Koštial, P., Barabaszova, K., Kukutschová, J.: New Ways for Designing External Fixators Applied in Treatment of Open and Unstable Fractures, j. *World Academy of Science, Engineering and Technology*, ISSN 2010-376X (print version) ISSN 2010-3778 (electronic version), vol. 7, issue 76, 2011, pp. 639–644.

[8] http://aplmech.vsb.cz/zevni-fixace/

[9] Seligson, D., Mauffrey, C., Roberts, C.S., *External Fixation in Orthopedic Traumatology*, ISBN 978-1-4471-2197-8, Springer-Verlag London Limited, GB, 2012, pp.1-228.

[10] Brealey, R. A., Myers, S. C., Marcus, A. J., *Fundamentals of Corporate Finance*, McGraw-Hill, 2012.

[11] Higgins, R. C., *Analysis for Financial Management*, 10th ed. McGraw-Hill, 2012.

[12] Ross, S., *Modern Financial Management*. 8th ed. McGraw-Hill, 2012.

[13] Frydrýšek, K., Wandrol, I., Kalenda, P., Application of SBRA Method in Mechanics of Continental Plates , *International Journal of Mechanics*, ISSN 1998-4448, vol. 6, issue 4, North Atlantic University Union NAUN, 2012, pp. 230-237.

[14] Frydrýšek, K., Pleva, L., Jořenek, J., Ječmínek, V., Klučka, R., Sivera, M., New External Fixators for Treatment of Pelvis and Acetabulum Fractures, *International Journal of Biology and Biomedical Engineering*, Issue 2, Volume 7, 2013, pp. 36-42.

[15] Frydrýšek, K., Pleva, L., Učeň, O., Kubín, T., Šír, M., Madeja, R., Žilka, L., New External Fixators for Treatment of Complicated Periprosthetic Fractures, *International Journal of Biology and Biomedical Engineering*, Issue 2, Volume 7, 2013, pp. 43-49.

[16] Frydrýšek, K., Monte Carlo Approach Applied in the Design of Machine Parts and Structures, *International Journal of Mechanics* , ISSN: 1998-4448, vol. 6, issue 4, North Atlantic University Union NAUN, 2012, pp. 222-229.

Assoc. Prof. M.Sc. Karel FRYDRÝŠEK, Ph.D., ING-PAED IGIP (Department of Mechanics of Materials, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, Ostrava, Czech Republic) - born in June 27th 1972, married, one daughter.

Study: M.Sc. - 1995, Ph.D. - 1999, Assoc. Prof. - 2009 in the branch of "Applied Mechanics" all at the Faculty of Mechanical Engineering VŠB – Technical University of Ostrava. He also studied pedagogy in the branch of "Academic Pedagogy for Teachers-Engineers According to the European Standards IGIP" at the Centre for Study of Academic Teaching in the Prague -2003.

Scientific-research activities and cooperation with industry: Theory and practice of FEM and other numerical methods, strength and elasticity, plasticity, material tests, fatigue, thermal stresses, creep, comparing of experiments and calculations, stress-strain analyses in bodies, proposition of testing machines and its parts, rock mechanics, geomechanics, mechanics of composites and structures on elastic foundation. He has a rich cooperation with industry (automotive industry, railway industry, civil engineering, mining, metallurgy, forming, casting, heat technology, steel structures, pipe systems, biomechanics etc.). In the last years, he is focused on probabilistic reliability assessment (SBRA Method applications) and biomechanics (problems of design of external & internal fixators for treatment of open and unstable fractures in traumatology and orthopaedics).

M.Sc. Josef KAŠÍK, Ph.D. (Department of Business Administration, Faculty of Economics, VŠB – Technical University of Ostrava, Ostrava, Czech Republic) – born on January 21st 1974, married, one son and one daughter.
Study: M.Sc. – 1998 in Finance, Ph.D. –2005 in Business Administration.
Scientific-research activitiesand cooperation with industry: economics, business administration, business diagnostics, business valuation, business process modelling and statistics.

Prof. M.Sc. Horst GONDEK, DrSc. (Department of Production Machines and Design, Faculty of Mechanical Engineering, VŠB – Technical University of Ostrava, Ostrava, Czech Republic).
Scientific-research activitiesand cooperation with industry: mining, geomechanics, geology, machines, engineering design.