Parameter estimation of Wiener model using differential evolution algorithm

Yinggan Tang

Abstract—Wiener model consists of a linear dynamic block followed by a static nonlinear function. This paper presents a novel method for parameter identification of Wiener model using differential evolution algorithm. The linear dynamic block is represented by autoregressive moving average (ARMA) model and the structure of static nonlinear function is assumed to be known in advance. The parameter estimation of Wiener model is converted to a parameter optimization problem. Differential evolution (DE) algorithm is used to search the optimal parameters by minimizing the error between the output of the actual model and that of the identified model. For the convenience of determining the search range of parameters to be estimated, the zeros and poles of linear dynamic block are identified. A new solution representation scheme is proposed for DE algorithm such that it can deal with the case that the linear dynamic block has complex number zeros and poles. The effectiveness of the proposed method is verified through two numerical examples.

Keywords—Wiener model, ARMA model, identification, differential evolution, pole-zeros.

I. INTRODUCTION

The Wiener model consists of a linear dynamic block followed by a nonlinear static function (see in Fig. 1). It is one of the simplest nonlinear systems, which has been widely used to model physical systems that are encountered in engineering, biology, and communication, etc. Fluid flow control [1], pH neutralisation process [2], power amplifier predistortion [3], visual cortex [4], EEG [5] and blind adaption in communication system [6] are successful application examples. Identification of Wiener model is an important issue, whose goal is to estimate both the characteristic of linear dynamic block and the static nonlinear function using observations of input and output. Available approaches in the literature for identifying Wiener model include correlation analysis method [7], two step method [8,9], prediction error method [1], nonparametric approach [10], subspace identification method [2,11], the blind method [12], frequency domain identification method [13], separable least-squares method [14], recursive identification method [15], relay feedback test method [16], etc.

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Fig. 1 Wiener model

Though many works have been done, the identification of Wiener model is still a difficult problem. One reason is that the nonlinearities in practical systems are versatile and it is not easy to find a good representation of the nonlinearity static function for identification purpose. Polynomials [12], cubic spline [17], piecewise linear map [1], neural network [9,18], etc., are often used to characterize the nonlinearities in Wiener model. The other reason is that the intermediate variable is not measurable. To solve this problem, many methods assume that the nonlinear static function is invertible such that the intermediate variable can be recovered from the output data, which can make the identification more easy [2, 10, 12,17]. However, the invertible assumption may be restrictive and limit the applications of these methods. For example, saturations, dead-zones, and hysteresis are not invertible, but they often exist in practice. On the other hand, the inversion of the nonlinearity can lead to severe amplification of possible measurement disturbance [1].

Recently, evolutionary algorithms (EAs) were adopted by several authors to identify block-oriented model. EAs are global optimization technique imposing no restrictions, such as differentiability and continuity, on objective function. In [19], Hatanaka et al. proposed to use genetic algorithm (GA) and evolution strategies (ES) to identify Wiener model, in which, the nonlinear static function is assumed to be invertible and approximated by a piecewise linear function. The parameters of piecewise linear function and interval partitions are computed using GA and ES respectively, while the parameters of linear dynamic block are estimated by the least squares method. Akramizadeh et al. [20] proposed an identification method for Hammerstein model based on GA and LMS algorithm. GA, with AIC criteria to be its fitness function, is responsible for finding the correct structure and parameters of nonlinear static function represented by hyperbolic function. LMS algorithm is used to estimate parameters of linear dynamic block. Identification of Wiener and Hammerstein model using genetic algorithm were considered in [21] under the assumption that the structure of the nonlinear static function is known with unknown parameters. The pole-zeros of linear dynamic block and
parameters of static nonlinear function were estimated simultaneously. The method in [21] implicitly assumed that the form of pole-zeros (i.e., it is real number or complex number) of linear dynamic block is known in advance. However, this information is not known in many practical cases.

Differential evolution (DE), which was developed by Storn and Price in 1995 [22], is a stochastic search algorithm that is originally motivated by the mechanisms of natural selection. Like other EAs, DE is very effective for solving optimization problems with non-smooth objective functions since it does not require derivative information. DE utilizes simple differential operator to create new candidate solutions and one-to-one competition scheme to greedily select new candidate. The one-to-one competition will have a faster convergence speed than other EAs. Due to its simple concept, easy implementation and quick convergence, DE algorithm was successfully applied to many fields mainly for various discrete and continuous optimization problems [22,23,24].

In many practical applications, the structure of nonlinear static function is known while the parameters are unknown. For example, in a control system, the actuators may include nonlinearity such as saturation, dead-zone, backlash and Coulomb friction and so on. Usually, the mathematical model structure is known, however, some or all model parameters may not be known. Observing this fact, we investigate the parameter identification of Wiener model, in which the structure of static nonlinear function is known. The parameters of linear dynamic block and the static nonlinear function are identified simultaneously by minimizing the error between the actual model output and the identified model output using DE algorithm. For the convenience of determining the parameter range, we identify the pole-zeros of linear dynamic block. A new solution representation scheme is proposed. The new solution representation scheme can deal with the case that the pole-zeros of linear dynamic block are complex number.

The rest of this paper is organized as follows. First, a brief review of differential evolution is given in section DE. Section PF addresses the parameter identification of Wiener model using DE. Numerical examples and the conclusion remarks are present in section simulation and section conclusion respectively.

II. DIFFERENTIAL EVOLUTION ALGORITHM

DE is a stochastic population based algorithm. For an optimization problem in \( l \)-dimension search space, each individual, represented by a real-value vector \( \mathbf{X}_i^K = [X_{i1}^K, X_{i2}^K, \ldots, X_{il}^K] \), stands for a candidate solution for the problem. First, a population of \( NP \) individuals, 
\[
\mathbf{X}_i^K, i = 1, 2, \ldots, NP
\]

is randomly generated within user-defined bounds. After initialization, the individuals evolve through mutation, crossover and selection operator to generate new generation guided by their fitness. At each generation \( K \), the mutation and crossover operators are first applied to the individuals, and a new generation arises. Then, selection operator takes place, and the corresponding individuals from both the populations compete to comprise the next generation. The process can be described in details as follows.

For each target individual \( \mathbf{X}_i^K \), a mutant vector is firstly built by adding the weighted difference between a defined number of individuals randomly selected from the previous population to another individual according to
\[
Z_{ij}^K = X_{ij}^K + F(X_{ri}^K + X_{r2}^K)
\]

where \( r1, r2 \in \{1, 2, \ldots, NP\} \) are randomly chosen and mutually different and also different from the running index \( i \), \( F \) is a constant called scaling factor which controls amplification of the differential variation \( (X_{ri}^K + X_{r2}^K) \).

After the mutation operation, the crossover operator is applied to increase the diversity of the population. In the crossover operation, a trial vector is generated as
\[
U_{ij}^K = \begin{cases} 
Z_{ij}^K & \text{if } \text{rand}(j) \leq \text{CR} \text{ or } j = \text{rand}(i) \\
X_{ij}^K & \text{otherwise}
\end{cases}
\]

where \( \text{rand}(j) \) is the \( j \)-th independent random number uniformly distributed in the range of \([0,1]\), \( \text{rand}(i) \) is a randomly chosen index from \([1,2,\ldots,l]\). \( \text{CR} \in (0,1) \) is a constant called crossover parameter that control the diversity of the population.

The selection operation selects, according to the fitness value of initial target individual \( \mathbf{X}_i^K \) and its corresponding trial vector \( U_{ij}^{K+1} \), which will survive to be a member of the next generation \( K+1 \). For a minimum optimization problem, \( U_{ij}^{K+1} \) is compared to \( \mathbf{X}_i^K \) by the following one-to-one based greedy selection criterion,
\[
X_{i}^{K+1} = \begin{cases} 
U_{ij}^{K+1} & \text{if } f(U_{ij}^{K+1}) < f(X_{i}^{K}) \\
X_{i}^{K} & \text{otherwise}
\end{cases}
\]

where \( f \) is the objective function under consideration. \( X_{i}^{K+1} \) is the individual of the new generation. The mutation, crossover and selection are repeated for \( NP \) times to complete one iteration.

The procedure described above is considered as the standard version of DE. However, several variants of DE have been proposed by Storn and Price. These variants are denoted as \( DE/\alpha/\beta/\gamma \). \( DE \) stands for differential evolution, \( \alpha \) represents string denoting the vector to be mutated, \( \text{rand} \) denotes that the vector is a randomly selected vector and \'best' the best vector in current generation. \( \beta \) is the number of difference vectors considered for mutation of \( \alpha \), it can be 1 or 2. \( \gamma \) stands for the type of crossover being used. The crossover type can be exponential or binomial. Hence, the algorithm described above is denoted as \( DE/rand/1/bin \).

The key parameters in DE are \( NP \) (size of population), \( F \)
is a set of
\[ \gamma(\rho) \]
\[ (14) \]
be
\[ (13) \]
is
\[ (8) \]
are poles and
\( \rho(\gamma) \)
\[ \rho(\gamma) \]
\[ \rho(\gamma) \]
possess the same input-output. Therefore, both systems can't be
distincted from each other on the basis of input-output
the parameters of the system (4). The parameter vector \( \theta \)
may be distributed between the linear dynamic block and the
measurement noise, respectively.
\( q G \)
\[ q G \]
is the transfer function of ARMA model can is
\[ q G \]
\[ q G \]
and system
observation. To avoid this problem, many literatures assumed
that the coefficient of
where
\( y(k) \)
is the output of the identified Wiener model, which is
shown as following,
\[ \hat{y}(k) = f(\hat{x}(k), \hat{\rho}) \],
\[ (11) \]
where
\[ \hat{G}(q^{-1}) = \frac{q^{-d}\hat{B}(q^{-1})}{A(q^{-1})} \]
\[ (12) \]
with
\[ \hat{A}(q^{-1}) = (1 - \hat{p}_1q^{-1})(1 - \hat{p}_2q^{-1})\cdots(1 - \hat{p}_nq^{-1}), \]
and
\[ \hat{B}(q^{-1}) = \hat{b}_0(1 - \hat{z}_1q^{-1})(1 - \hat{z}_2q^{-1})\cdots(1 - \hat{z}_m q^{-1}). \]
\[ (13) \]
\[ (14) \]
The best estimation \( \hat{\theta}^* \) is such that
\[ \hat{\theta}^* = \arg\min_{\theta \in \Gamma} J(\hat{\theta}). \]
\[ (15) \]
where \( \Gamma \) is the search domain admitted for \( \hat{\theta} \). Hence, the
parameter estimation problem is transformed into an
optimization problem. The principle of parameter estimation
can be illustrated with Fig.2.

Fig. 2 The principle of parameter estimation

The optimization problem (15) is multi-dimension. It is
difficult to get the optimal solution of (15) using traditional
gradient based optimization method because there are multiple
local optima in the landscape of \( J \). In this paper, DE algorithm
is used to solve the optimization problem (15).

III. PARAMETER IDENTIFICATION OF WIENER MODEL USING DE

A. Problem formulation
Considering the following Wiener model
\[ \begin{align*}
  x(k) &= G(q^{-1})u(k) \\
  y(k) &= f(x(k); \rho) + e(k)
\end{align*} \]
(4)
where \( q^{-1} \) is the unit delay operator, i.e., \( q^{-1}u(k) = u(k-1) \);
\( u(k) \), \( y(k) \) and \( e(k) \) are the system input, output and
in the landscape of \( J \). In this paper, DE algorithm
is used to solve the optimization problem (15).

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difficult to get the optimal solution of (15) using traditional
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local optima in the landscape of \( J \). In this paper, DE algorithm
is used to solve the optimization problem (15).
B. DE algorithm for parameter identification

Since $p_1$ and $z_j$ maybe complex numbers, DE algorithm can not be used to solve the optimization problem (15) directly for the reason that the individuals in DE are coded using real number. To solve this problem, a new code strategy is proposed. Without loss of generality, suppose $p_1$ and $p_2$ are two different poles of $G(q^{-1})$, i.e., $p_1 \neq p_2$. Let $S = p_1 + p_2$, $T = p_1 p_2$, then

\[(1 - p_1 q^{-1})(1 - p_2 q^{-1}) = 1 + Sq^{-1} + Tq^{-2}\]

\[= (1 - S + \sqrt{S^2 - 4T} q^{-1})(1 - S - \sqrt{S^2 - 4T} q^{-1})\]

It is obviously,

\[p_1 = \frac{S + \sqrt{S^2 - 4T}}{2} = p_{b1} + \sqrt{p_{p1}}\]

\[p_2 = \frac{S - \sqrt{S^2 - 4T}}{2} = p_{b1} - \sqrt{p_{p1}}\]

where $p_{b1} = S / 2$, $p_{p1} = (S^2 - 4T) / 4$, which are real numbers. If $p_{p1} > 0$, then $p_1$ and $p_2$ stand for two real numbers, otherwise, if $p_{p1} < 0$, $p_1$ and $p_2$ stand for a pair of conjugative complex numbers. Finally, if $p_{p1} = 0$, then $p_1$ and $p_2$ are tow identical real numbers. It can be seen from (16) and (17) that, two poles of $G(q^{-1})$ can be represented by two real numbers, no matter whether they are two different real numbers, conjugative complex numbers or tow identical real numbers. Owing to this fact, $p_1$ and $p_2$ can be coded as $[p_{b1}, p_{p1}]$ in DE. For other poles and zeros, similar coding strategy can be adopted. It is shown that this representation is efficient for individual coding when DE is used to solve the optimization problem (15).

The process of parameter identification of Wiener model using DE is summarized as follows.

Step 1: Generate $L$ input-output pairs $(u_k, y_k)$ using random input signal according to the actual model.

Step 2: Set the value of $NP$, $F$ and CR.

Step 3: Initialize $NP$ individuals. Each individual is a $l$-dimension real vector representing the parameter of Wiener model. The vector is represented using the method described above.

Step 4: For each individual, generate the estimated output $\hat{y}_k$ and calculate its fitness according to (10).

Step 5: For each individual, perform mutation operation to obtain mutation vector $Z_j$.

Step 6: For each individual, perform crossover operation to obtain crossover trial vector $U_j$.

Step 7: Calculate the fitness of each crossover trial vector $U_j$, as in Step 4.

Step 8: Perform selection operation according to (3).

Step 9: Repeat Step 4-Step 8 until a termination condition is reached.

Step 10: Output the final solution with minimum fitness.

IV. SIMULATION STUDY

In this section, two examples are given to illustrate the effectiveness of the proposed method. All the algorithms in this paper are programmed using MATLAB 7.0 language and executed on Pentium-IV 2.66GHz with 256MB random memory. The identification results of DE algorithm are compared to those of GA and particle swarm optimization (PSO). The implementation code of GA is taken from the GAT toolbox at http://www.isc.ncsu.edu/mirage/GAToolBox/gat/. PSO is implemented according to [25]. To make a fair comparison, the same computation effort, i.e., the maximum generation number, population size and search range, is adopted in GA, PSO and DE. The other parameters of GA, PSO and DE are set as follows:

GA: The chromosome is code using real number, the selection method is "normGeomSelect" with parameter option 0.08, the crossover method is "heuristicXover" with pa-ramerter option [2 3] and the mutation method is "multiNonUnifMutation" with pa-ramerter option [6 genMax 3].

PSO: $c_1 = c_2 = 2$, the inertia weight varies in a linearly decreasing manner suggested in [25] and $w_{max} = 0.9$, $w_{min} = 0.4$.

DE: mutation factor $F=0.5$, crossover rate $CR=0.6$ and the mutation strategy generating new trial vector is DE/rand-to-best/exp.

In experiments, the input signal $u(k)$ is random number uniformly distributed in [-2,2]. Measurement noise $e(k)$ with SNR=20dB is added. Each algorithm is executed 20 times and the mean value is used as the final parameter identification result. To evaluate the performance of the proposed method, the following error index is used,

$$\delta = \frac{\|\hat{\theta} - \theta\|}{\|\theta\|} \times 100\%$$

where $\hat{\theta}$ denotes the estimation of $\theta$, $\|\|$ represents 2-norm of a vector.

Example 1 Considering a Wiener model with linear dynamic block as

$$G(q^{-1}) = \frac{q^{-1}B(q^{-1})}{A(q^{-1})} = \frac{q^{-1} + 0.5q^{-2}}{1 - 0.2q^{-1} + 0.35q^{-2}}$$

the nonlinear static function...
The nonlinear static function is discontinuous as shown in Fig.3.

The pole-zero form of (19) is written as

\[
G(q^{-1}) = \frac{q^{-1}(1-z_1q^{-1})}{(1-p_1q^{-1})(1-p_2q^{-1})},
\]

with \( z_1 = -0.5 \), \( p_1 = 0.1 + 0.583i \), \( p_2 = 0.1 - 0.583i \). \( p_1 \) and \( p_2 \) can be expressed as

\[
p_1 = 0.1 + \sqrt{-0.34}, \quad p_2 = 0.1 - \sqrt{-0.34}.
\]

The true parameter value describing system (19) and (20) is

\[
\theta = [-0.5, 0.1, -0.34, 1.0, 0.5, 0.4, 0.3] .
\]

The identified results are listed in Table 1. It can be seen from Table 1 that the identified parameters values using DE algorithm are almost the same as the true values. Meanwhile, the \( \delta \) of DE is the smallest compared to those of GA and PSO. The evolution process of fitness of each algorithm is shown in Fig.4. The true and estimation nonlinearity are shown in Fig.5. The estimated nonlinearity is identical to the real nonlinearity.

After the model parameters are identified, a sine testing signal \( u(k) = \sin(\pi k/25) \) is used to verify their performance, where the evaluation index is the mean square error (MSE). Fig.6 depicts the output error between the actual model and the identified models using GA, PSO and DE. It can be seen that the output error and MSE of DE is the smallest, which verifies that the identification of DE is more accurate.

**Table 1** Parameter identification results of Example 1 using GA, PSO and DE

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>( z_1 )</th>
<th>( p_{n1} )</th>
<th>( p_{n1} )</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( D )</th>
<th>( b )</th>
<th>( \delta ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>-0.5061</td>
<td>0.0985</td>
<td>-0.3379</td>
<td>0.9956</td>
<td>0.5098</td>
<td>0.3533</td>
<td>0.2545</td>
<td>4.85</td>
</tr>
<tr>
<td>PSO</td>
<td>-0.4990</td>
<td>0.1001</td>
<td>-0.3413</td>
<td>0.9983</td>
<td>0.5125</td>
<td>0.3492</td>
<td>0.2531</td>
<td>5.41</td>
</tr>
<tr>
<td>DE</td>
<td>-0.4999</td>
<td>0.1001</td>
<td>-0.3399</td>
<td>1.0005</td>
<td>0.4990</td>
<td>0.3983</td>
<td>0.2998</td>
<td>0.21</td>
</tr>
<tr>
<td>True value</td>
<td>-0.5000</td>
<td>0.1000</td>
<td>-0.3400</td>
<td>1.0000</td>
<td>0.5000</td>
<td>0.4000</td>
<td>0.3000</td>
<td>-</td>
</tr>
</tbody>
</table>
Example 2 Considering a Wiener model with linear dynamic block as

\[ x(k) = 1.4138x(k-1) - 0.6065x(k-2) + 0.1044u(k-1) + 0.0883u(k-2) \]  

the nonlinear function is

\[ y(k) = f[x(k)] = \frac{0.3163x(k)}{\sqrt{0.1 + 0.9x^2(k)}} \]  

The above model is the fluid-flow control model [8], where \( u(k) \) is the pneumatic control signal, \( x(k) \) is the stem position, and the resulting flow through the valve is given by the nonlinear function \( f[x(k)] \) of the stem position.

The linear dynamic block has one zero \( z_1 = -0.8458 \), two poles \( p_1 = 0.7069 + 0.3268i, \ p_2 = 0.7069 - 0.3268i \). Using the code...
scheme, \( p_1 \) and \( p_2 \) can be expressed as:

\[ p_1 = 0.7069 + \sqrt{-0.1068} \quad p_2 = 0.7069 - \sqrt{-0.1068}. \]

The true parameter value describing model (22) and (23) is:

\[ \overline{\theta} = [z_1, P_{r_1}, P_{r_2}, a, b, c] = [-0.8458, 0.7069, -0.3268, 0.3163, 0.1, 0.9]. \]

Table 2 lists the identification results of each parameter of model (22) and (23). It can be seen from Table 2 that the identified parameter value of DE algorithm is more close to the true parameter value. Meanwhile, the \( \delta \) of DE is the smallest compared to those of GA and PSO. The evolution process of fitness of each algorithm is shown in Fig.7. The true and estimation nonlinearity are shown in Fig.8. The estimated nonlinearity is identical to the real nonlinearity.

After the model parameters are identified, a sine testing signal \( u(k) = \sin(\pi k / 25) \) is used to verify their performance, where the evaluation index is the mean square error (MSE). Fig.9 depicts the output error between the actual model and the identified models using GA, PSO and DE. It can be seen that the output error and MSE of DE is the smallest, which verifies that the identification of DE is more accurate.

### V. Conclusion

A new approach to the parameter identification of Wiener model has been presented. Under the assumption that the structure of the nonlinear function is known, the parameters of the linear dynamic block and the nonlinear function are estimated by minimizing an objective function. DE algorithm is used to optimize the objective function. The advantage of the proposed method is that the parameters of the linear dynamic block and the nonlinear function can be identified simultaneously without any information about the intermediate signals. Numerical results show that the accurate and consistent estimation results can be obtained using the proposed method.

### Acknowledgment

This work is supported by NSFC under Grant 61273260, Natural Scientific Research Foundation of the Higher Education Institutions of Hebei Province under Grant 2010165.
Fig. 9. Test results of Example 2. (a) input signal (b) output of actual model (c) output error of GA (MSE = 5.1829e-005) (d) output error of PSO (MSE = 7.7812e-006) (e) output error of DE (MSE = 1.0310e-006)

REFERENCES


Yinggan Tang was born in Nanchang, Jiangxi Province, China, in 1975. He received the BSc degree in Measurement Technology & Instrument in 1999, the MS and Ph.D degree in Control Engineering and Control Theory in 2001 and 2006, respectively, all from Yanshan University, Qinhuangdao, Hebei Province, China. Since 2007, he has been an associated professor with the automation department of Institution of Electrical Engineering, Yanshan University. His research interests include image processing, system identification and intelligent optimization algorithm.