Control of time delay systems using Matlab toolbox

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***Abstract*—**The paper shows an application of Robust Control Toolbox for Time Delay Systems implemented in the Matlab system. The toolbox is used to solve the problem of uncertain time delay using the D-K iteration and algebraic approach. The algebraic approach represents a new technique for solving problems arising in the robust control. It combines the structured singular value, algebraic theory and algorithm of global optimization solving remaining issues in structured singular value framework. The algorithm of global optimization can be alternated with direct search methods such as Nelder-Mead simplex method giving solutions for problems with one local extreme. As a global optimization method, Differential Migration is used, which proved to be reliable in solving this type of problems. The D-K iteration represents a standard method in the structured singular value theory. The results obtained from the D-K iteration are compared with the algebraic approach.

***Keywords*—**Algebraic approach, robust control, structured singular value, time delay systems, uncertainty.

I. Introduction

TIME delay systems are a constant issue present in control theory. In this paper, the problem of uncertain time delay will be solved using Robust Control Toolbox for Time Delay Systems implemented in the Matlab system. The essential tool is the structured singular value denoted μ (see [11]) giving a measure of robustness. The algebraic approach (see [2] and [3]) and evolutionary algorithm Differential Migration (see [1]) are used treating the problem of multimodality of the cost function and impossibility of deriving controller for performance weights with poles on the imaginary axis. This implies that the final controller provides zero steady-state error which is impossible in the scope of the standard tools using DGKF formulae for obtaining $\mathbf{H}_\infty$ (sub)optimal controllers or other methods such as linear matrix inequality (LMI) approach leading to numerical problems in most of real world cases (see [6], [7] and [8]). Besides this, the algebraic approach overcomes some difficulties connected with the D-K iteration, namely the fact that it does not guarantee convergence to a global or even local minimum (see [13]). Controllers obtained via the algebraic approach can have simpler structure due to the fact that there is no need of scaling matrices absorbance into generalized plant, and hence no need of further simplification causing deterioration of the frequency properties of the resulting controller. Moreover, the controller structure can be chosen in advance, which is not possible in the scope of currently used methods.

Optimization is performed via evolutionary algorithm. Evolutionary algorithms belong to the new branches of engineering (see [9], [10], [14] and [15]) providing solution to the problems which were not solvable using traditional optimization tools.. In this paper, a new evolutionary algorithm – Differential Migration is used having some favourable properties compared to the existing ones. Namely the fact that lower computational time is needed for obtaining a suitable solution.

Pole placement is performed via solving the Diophantine equation in the ring of Hurwitz-stable and proper rational functions ($\mathbf{R}_{\mathrm{PS}}$). The structured singular value assesses the robust stability and performance of the controller.

For comparison reasons, the results obtained from the D-K iteration (see [5]) demonstrate the differences between the standard and proposed method. The overall performance is verified by simulations of step response for different values of time delays with simple feedback loop and two-degree-of-freedom structure (1DOF and 2DOF, see [12]).

The following notation is used: $\|\cdot\|_\infty$ denotes $\mathbf{H}_\infty$ norm, $\bar{\sigma}(\cdot)$ is maximum singular value, $\mathbf{R}$ and $\mathbf{C}^{n\times m}$ are real numbers and complex matrices, respectively, $\mathbf{I}_n$ is the unit matrix of dimension n and $\mathbf{R}_{\mathrm{PS}}$ denotes the ring of Hurwitz-stable and proper rational functions.

II. Preliminaries

Define $\mathbf{\Delta}$ as a set of block diagonal matrices

$$\mathbf{\Delta} \equiv \{\mathrm{diag}[\delta_1 I_{r_1},\ldots,\delta_S I_{r_S},\Delta_1,\ldots,\Delta_F]:\delta_i \in \mathbf{C},\Delta_j \in \mathbf{C}^{m_j\times m_j}\} \quad (1)$$

where S is the number of repeated scalar blocks,
F is the number of full blocks,
$r_1,\ldots, r_S$ and $m_1,\ldots, m_F$ are positive integers defining dimensions of scalar and full blocks.

For consistency among all the dimensions, the following condition must be held

$$\sum_{i=1}^{S} r_i + \sum_{j=1}^{F} m_j = n \quad (2)$$

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Definition 1: For $\mathbf{M} \in \mathbf{C}^{n \times n}$ is $\mu_{\mathbf{\Delta}}(\mathbf{M})$ defined as

$$\mu_{\mathbf{\Delta}}(\mathbf{M}) \equiv \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(\mathbf{I} - \mathbf{M}\Delta) = 0\}} \tag{3}$$

If no such $\Delta \in \mathbf{\Delta}$ exists making $\mathbf{I} - \mathbf{M}\Delta$ singular, then $\mu_{\mathbf{\Delta}}(\mathbf{M}) = 0$.

Consider a complex matrix $\mathbf{M}$ partitioned as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \tag{4}$$

and suppose there is a defined block structure Δ_2 which is compatible in size with $\mathbf{M}_{22}$ (for any $\Delta_2 \in \mathbf{\Delta}_2$, $\mathbf{M}_{22}\Delta_2$ is square). For $\Delta_2 \in \mathbf{\Delta}_2$, consider the following loop equations

$$\begin{aligned} e &= \mathbf{M}_{11}d + \mathbf{M}_{12}w \\ z &= \mathbf{M}_{21}d + \mathbf{M}_{22}w \\ w &= \Delta_2 z \end{aligned} \tag{5}$$

If the inverse to $\mathbf{I} - \mathbf{M}_{22}\Delta_2$ exists, then e and d must satisfy $e = \mathbf{F}_L(\mathbf{M}, \Delta_2)d$, where

$$\mathbf{F}_L(\mathbf{M}, \Delta_2) = \mathbf{M}_{11} + \mathbf{M}_{12}\Delta_2(I - \mathbf{M}_{22}\Delta_2)^{-1}\mathbf{M}_{21} \tag{6}$$

is a linear fractional transformation on $\mathbf{M}$ by Δ_2, and in a feedback diagram appears as the loop in Fig. 1.

The subscript L on $\mathbf{F}_L$ pertains to the *lower* loop of $\mathbf{M}$ and is closed by Δ_2. An analogous formula describes $\mathbf{F}_U(\mathbf{M}, \Delta_1)$, which is the resulting matrix obtained by closing the *upper* loop of $\mathbf{M}$ with a matrix $\Delta_1 \in \Delta_1$.

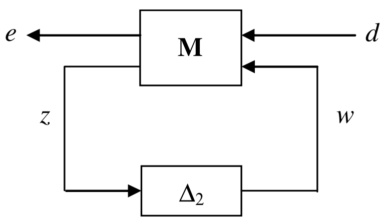

Fig. 1. LFT interconnection

Theorem 1: Let $\beta > 0$. For all $\Delta_2 \in \mathbf{\Delta}_2$ with $\left|\overline{\sigma}(\Delta_2)\right| < \frac{1}{\beta}$, the loop shown in Fig. 1 is well-posed, internally stable, and $\left\|\mathbf{F}_L(\mathbf{M}, \Delta_2)\right\|_\infty \le \beta$ if and only if

$$\sup_{\omega \in \Re} \mu_\Delta[\mathbf{M}(j\omega)] \le \beta \tag{7}$$

Proof: Proof is the same as in [4] and [11] except for the fact that perturbations are complex matrices, which simplifies the proof and complies with the definition of μ.

III. Algebraic μ-Synthesis

The algebraic μ-synthesis can be applied to any control problem that can be transformed to the loop in Fig. 2, where $\mathbf{G}$ denotes the generalized plant, $\mathbf{K}$ is the controller, Δ_{del} is the perturbation matrix, r is the reference and e is the output.

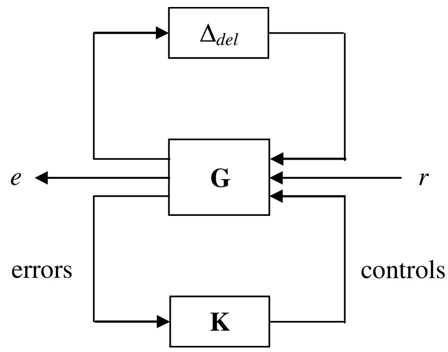

Fig. 2. Closed loop interconnection

For the purposes of the algebraic μ-synthesis, the MIMO system with l inputs and l outputs has to be decoupled into l identical SISO plants. The nominal model is defined in terms of transfer functions:

$$\mathbf{P}_{nom}(s) \equiv \begin{bmatrix} P_{11}(s) & \cdots & P_{1l}(s) \\ \vdots & \ddots & \vdots \\ P_{l1}(s) & \cdots & P_{ll}(s) \end{bmatrix} \tag{8}$$

For decoupling the nominal plant $\mathbf{P}_{nom}$ ($\mathbf{P}_{nom}$ invertible) it is satisfactory to have the controller in the form

$$\mathbf{K}(s) = K(s)\mathbf{I}_l \det[\mathbf{P}_{nom}(s)]\frac{1}{P_{xy}(s)}[\mathbf{P}_{nom}(s)]^{-1} \tag{9}$$

where P_{xy} is an element of $adj[\mathbf{P}_{nom}(s)] = \det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}$ with the highest degree of numerator $\{adj[\mathbf{P}_{nom}(s)]$ denotes adjugate matrix of $\mathbf{P}_{nom}\}$. The choice of the decoupling matrix prevents the controller from cancelling any poles or zeros from the right half-plane so that internal stability of the nominal feedback loop is held. The MIMO problem is reduced to finding a controller $K(s)$, which is tuned via setting the poles of the nominal feedback loop with the plant

$$\begin{aligned} \mathbf{P}_{dec}(s) &= \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)][\mathbf{P}_{nom}(s)]^{-1}\mathbf{P}_{nom}(s) \\ &= \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)]\mathbf{I}_l \end{aligned} \tag{10}$$

Define

$$P_{dec} \equiv \frac{1}{P_{xy}(s)}\det[\mathbf{P}_{nom}(s)] \tag{11}$$

Transfer function P_{dec} can be approximated by a system P_{dec}^*

with lower order than P_{dec}

$$P^*_{dec}(s) = \frac{b(s)}{a(s)} \tag{12}$$

which can be rewritten in terms of its coefficients and transformed to the elements of $\mathbf{R}_{\mathrm{PS}}$

$$P^*_{dec}(s) = \frac{\dfrac{b_0 + b_1 s + \ldots + b_n s^n}{(\alpha_1+1)(\alpha_2+1)\cdot\ldots\cdot(\alpha_n+1)}}{\dfrac{s^n + a_0 + a_1 s + \ldots + a_{n-1}s^{n-1}}{(\alpha_1+1)(\alpha_2+1)\cdot\ldots\cdot(\alpha_n+1)}} = \frac{B}{A},\ A, B \in \mathbf{R}_{\mathrm{PS}} \tag{13}$$

The controller $K = N_K/D_K$ is obtained by solving the Diophantine equation

$$AD_K + BN_K = 1 \tag{14}$$

with A, B, D_K, $N_K \in \mathbf{R}_{\mathrm{PS}}$. Equation (14) is often called the Bezout identity. All feedback controllers N_K/D_K are given by

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT}, \qquad N_{K_0}, D_{K_0} \in \mathbf{R}_{\mathrm{PS}} \tag{15}$$

where N_{K_0}, $D_{K_0} \in \mathbf{R}_{\mathrm{PS}}$ are particular solutions of (14) and T is an arbitrary element of $\mathbf{R}_{\mathrm{PS}}$.

The controller K satisfying equation (14) guarantees the BIBO (bounded input bounded output) stability of the feedback loop in Fig. 3. This is a crucial point for the theorems regarding the structured singular value. If the BIBO stability is held, then the nominal model is internally stable and theorems related to robust stability and performance can be used. The BIBO stability also guarantees stability of $\mathbf{F}_L(\mathbf{G},\mathbf{K})$ making possible usage of performance weights with integration property implying non-existence of state space solutions using DGKF formulae (see [6]) due to zero eigenvalues of appropriate Hamiltonian matrices. Such methodology results in zero steady-state error in the feedback loop with the controller obtained as a solution to equation (14). This technique is neither possible in the scope of the standard μ-synthesis using DGKF formulae, nor using LMI approach (see [7]) leading to numerical problems in most of real-world applications.

The aim of synthesis is to design a controller which satisfies the condition:

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_\Delta[\mathbf{F}_L(\mathbf{G},\mathbf{K})(\omega,\alpha_1,\ldots,\alpha_{n+n_1+n_2},t_1,\ldots,t_{n_2})] \le 1,\ \omega \in (-\infty,+\infty) \tag{16}$$

where $n + n_1 + n_2$ is the order of the nominal feedback system, n_1 is the order of particular solution K_0, t_i are arbitrary parameters in $T = \dfrac{t_0 + t_1 s + \ldots + t_{n_2} s^{n_2}}{(\alpha_{n_1+1}+s)\cdot\ldots\cdot(\alpha_{n_1+n_2}+s)}$ and μ_Δ denotes the structured singular value of LFT on generalized plant $\mathbf{G}$ and controller $\mathbf{K}$ with

$$\Delta \equiv \begin{bmatrix} \Delta_{del} & 0 \\ 0 & \Delta_F \end{bmatrix} \tag{17}$$

where Δ_{del} denotes the perturbation matrix and Δ_F is a full-block matrix corresponding with the robust performance condition.

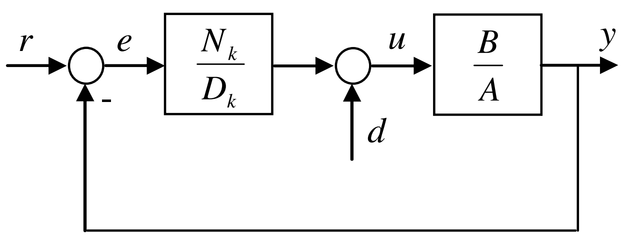

Fig. 3. Nominal feedback loop

Tuning parameters are positive and constrained to the real axis since parameters of the transfer function have to be real and due to the fact that non-real poles cause oscillations of the nominal feedback loop.

A crucial problem of the cost function in (16) is the fact that many local extremes are present. Hence, local optimization does not yield a suitable or even stabilizing solution. This can be overcome via evolutionary optimization, which solves the task very efficiently.

IV. Problem Formulation

The problem to solve is general 1st order system with uncertain time delays:

$$\mathbf{P} \equiv \left\{ \frac{b_3 s^3 + b_2 s^2 + b_1 s^1 + b_0}{a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0} e^{-\tau s} : 0 \le \tau \le T_0 \right\} \tag{18}$$

The control objective is to find a controller that guarantees the robust stability and performance for every plant from the set P. The time delay is treated by multiplicative uncertainty

$$\{P(1 + W_2\Delta) : \|\Delta\|_\infty \le 1\} \tag{19}$$

Let the nominal plant be

$$P(s) = \frac{b}{a} = \frac{b_3 s^3 + b_2 s^2 + b_1 s^1 + b_0}{a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0} \tag{20}$$

then for the weighting function W_2 the following inequality must be held

$$\left| \frac{P'(j\omega)}{P(j\omega)} - 1 \right| < \left| W_2(j\omega) \right|, \ \forall \omega \in \overline{\Re}_+, \ \forall P' \in \mathbf{P} \tag{21}$$

which is equivalent with

$$\left| e^{-\tau j\omega} - 1 \right| < \left| W_2(j\omega) \right|, \ \forall \omega \in \overline{\Re}_+, \ \tau \in [0; T_0] \tag{22}$$

The weight W_2 is defined as an envelope curve of $\left| e^{-\tau j\omega} - 1 \right|$. For $\tau = 10$, W_2 can have the Bode plot depicted in Fig. 4.

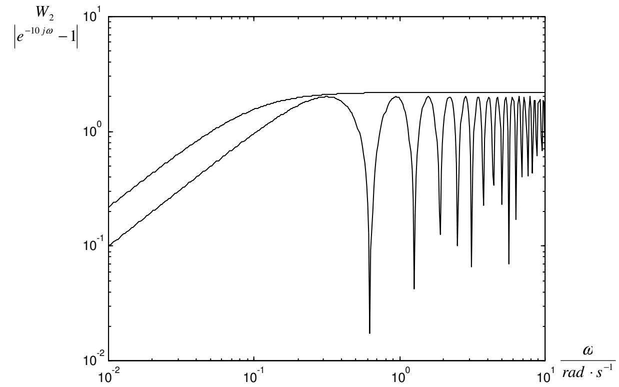

Fig. 4 Bode plot of W_2 and $\left| e^{-10j\omega} - 1 \right|$

V. Problem Solution

A. Structured Singular Value Framework

The problem defined in previous section can be solved using interconnection in Fig. 5. Here, $\mathbf{G}$ denotes the generalized plant partitioned to

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & G_{22} \end{bmatrix} \tag{23}$$

where the block structure of $\mathbf{G}$ corresponds with the input and output variables in Fig. 5:

$$\begin{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix} \\ v \end{bmatrix} = \mathbf{G} \cdot \begin{bmatrix} \begin{bmatrix} w \\ d \end{bmatrix} \\ u \end{bmatrix} \tag{24}$$

The design objective is to find a stabilizing controller K such that

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_{\tilde{\Delta}}[\mathbf{F}_l(\mathbf{G}, K)] \tag{25}$$

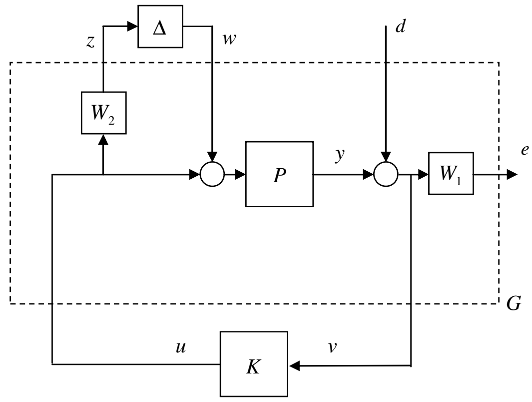

Fig. 5 Closed-loop interconnection for μ-synthesis

is minimal, where

$$\mathbf{M} = \mathbf{F}_l(\mathbf{G}, K) = \mathbf{G}_{11} + \mathbf{G}_{12} K (1 - G_{22} K)^{-1} \mathbf{G}_{21} \tag{26}$$

is the lower linear fractional transformation on generalized plant $\mathbf{G}$ and controller K (see Fig. 5).

B. Algebraic Approach

The controller $K = \frac{N}{M}$ is obtained by solving the Diophantine equation

$$AM + BN = 1 \tag{27}$$

The Diophantine equation (27) is known as Bezout identity and all feedback controllers N/M are given as

$$\frac{N}{M} = \frac{N_0 - AT}{M_0 + BT} \tag{28}$$

where $N_0, M_0 \in \mathbf{R}_{PS}$ represent a particular solution of (27), and T is an arbitrary element of $\mathbf{R}_{PS}$ such that the denominator of (28) is non-zero.

By the analysis of the polynomial degrees of a and b, the transfer functions A, B, M and N were chosen so that the number of closed loop poles is minimal and the asymptotic tracking is achieved:

$$A = \frac{a}{\prod_{i=1}^{n}(s + \alpha_i)}, \ B = \frac{b}{\prod_{i=1}^{n}(s + \alpha_i)} \tag{29}$$

$$M = \frac{sm}{\prod_{i=n+1}^{2n}(s + \alpha_i)}, \ N = \frac{n}{\prod_{i=n+1}^{2n}(s + \alpha_i)} \tag{30}$$

where n is the actual degree of polynomial a obtained by omitting zero parameters a_i.

The resulting controller has the general PID structure:

$$K(s)=\frac{n_n s^n+\ldots+n_1 s+n_0}{s(s^{n-1}+s^{n-2}m_{n-2}+\ldots+m_0)} \tag{31}$$

VI. User Interface

The main window of the toolbox consists of three parts (see Fig. 7):

- System Definition
- Controller Design
- Simulation and Verification

A. System Definition

System definition has the button for displaying the dialog for entering parameters of the control plant. Here, the parameters of transfer function and the maximum value of time delay can be entered (Fig. 8).

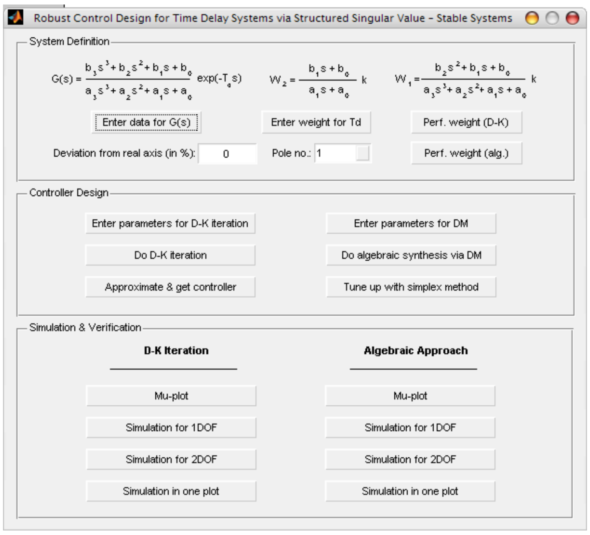

Fig. 7 The main window

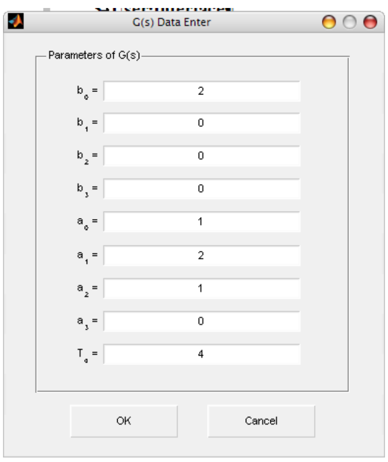

Fig. 8 Dialog for entering parameters of the control plant

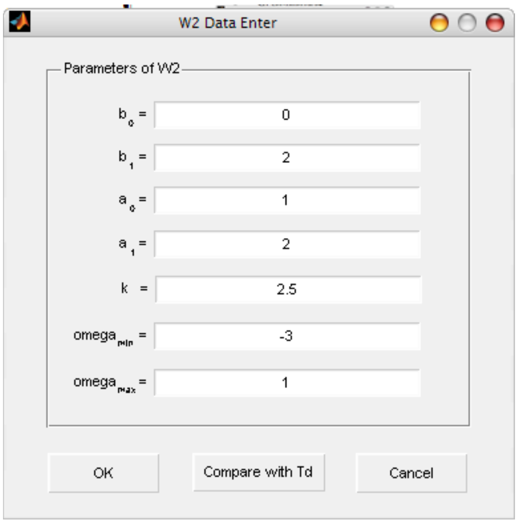

Fig. 9 Dialog for entering the parameters of the weight W_2

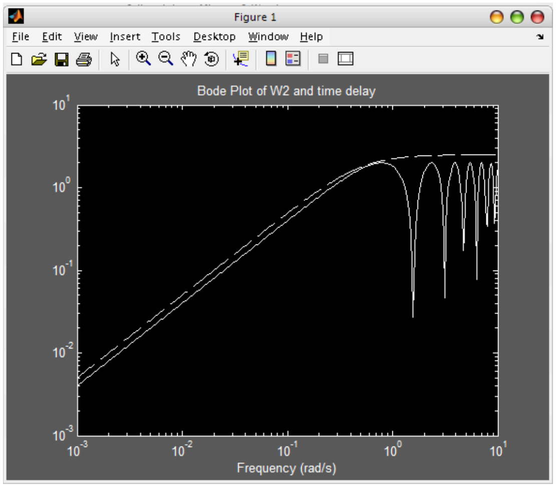

Fig. 6 Bode plot of the weight W_2 compared to the left side of (22)

Another button displays the dialog for entering the parameters of the weight W_2 treating uncertain time delay (Fig. 9). In the dialog, there is a button for showing the Bode plot of the weight W_2 compared to the left side of (22) (see Fig. 6).

In the last part of system definition, buttons showing dialogs for entering parameters of the performance weight W_1 are placed. There are separate weights for the D-K iteration and algebraic approach. Each dialog has a button for showing the Bode plot of the particular weight.

B. Controller Design

The controller design part is divided into two sections – D-K iteration and algebraic approach. In the first row, there are the buttons for entering parameters for both the D-K iteration and algebraic approach.

In the second row, there are buttons for performing the design of controllers. The design is interactive and uses the command line window of the Matlab system for communication with user.

The D-K iteration asks the user for the starting mu-iteration. Then, the first gamma for the suboptimal controller is searched using the bisection method. Then, the user is prompted for the change of frequency range and bounds or tolerances. Then, the current step of the D-K iteration is finished and the μ-plot is displayed. In the next step, the μ-plot is approximated using scaling matrices D and D^{-1}. To this effect, the user is prompted for his choice. Command apf can be used for auto-prefit, which automatically finds the parameters for this step. After exiting this part using e command parameters for gamma search can be set. Then, the user is again prompted for change of the frequency range and bounds or tolerances. Finally, the μ-plot is calculated and displayed. These steps are repeated until the user terminates the whole process. Then, the resulting controller is obtained and displayed in the Matlab window.

The algebraic approach launches the evolutionary search, which performs the predefined number of migration loops defined in the parameters dialog for the algebraic approach. The search can be interrupted by pressing Ctrl+C. The controller can be obtained by pressing Approximate and get controller button.

Besides the evolutionary search, Nelder-Mead simplex method can be used for the tune up of the controller by pressing the button Tune up with simplex method.

C. Simulation and Verification

Simulation and verification part has two columns of buttons each for the particular design method, i.e. D-K iteration and algebraic approach.

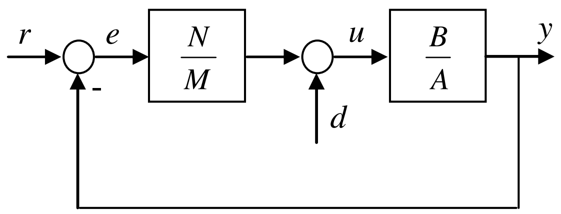

Fig. 10 Simple feedback loop

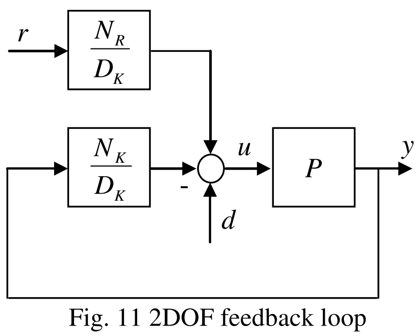

Fig. 11 2DOF feedback loop

In the first row, buttons for displaying the μ-plots are present. If the Mu-plot button in the algebraic approach is pressed then a comparison of both approaches can be viewed in terms of the μ-plots for both the D-K iteration and algebraic approach in one figure.

Under the μ-plot buttons, buttons for performing simulation in Matlab Simulink are placed. The simulation can be performed for both simple feedback loop and two-degree-of-freedom (2DOF) feedback loop (see Fig. 10 and 11)..

Finally, buttons for showing the simulation in one plot are at the bottom of the main window.

VII. Example of Time Delay System Control

The plant family is defined as follows:

$$\mathbf{P} \equiv \left\{ \frac{e^{-\tau s}}{s^2 + 2s + 1} : 0 \le \tau \le 4 \right\} \tag{32}$$

The control objective is to find a controller that will guarantee the robust stability and performance for every plant from the set $\mathbf{P}$. The time delay is treated by multiplicative uncertainty

$$\{P(1 + W_2 \Delta) : \|\Delta\|_\infty \le 1\} \tag{33}$$

Let the nominal plant be

$$P(s) = \frac{b}{a} = \frac{1}{s^2 + 2s + 1} \tag{34}$$

Then, for the weighting function W_2 the following inequality must be held

$$\left| \frac{P'(j\omega)}{P(j\omega)} - 1 \right| \le |W_2(j\omega)|, \ \forall \omega \in \overline{\Re}_+, \ \forall P' \in \mathbf{P} \tag{35}$$

which is equivalent with

$$\left| e^{-\tau j\omega} - 1 \right| \le |W_2(j\omega)|, \ \forall \omega \in \overline{\Re}_+, \ \tau \in [0;4] \tag{36}$$

The weight W_2 can be defined as envelope curve of $\left| e^{-\tau j\omega} - 1 \right|$ for $\tau = 4$ (see Fig. 6):

$$W_2(s) = 2.5 \frac{2s}{2s + 1} \tag{37}$$

The performance condition is of the form:

$$\|W_1 S\|_\infty < 1 \tag{38}$$

where S is the sensitivity function and weight W_1 is designed so that the asymptotic tracking is achieved:

$$W_1(s) = \frac{0.004}{10s^3 + 100s^2 + s + 10^{-5}} \cdot 10 \tag{39}$$

Multiplicative uncertainty (33) and performance condition exactly fit in the LFT framework. The closed-loop interconnection for the μ-synthesis is shown in Fig. 5.

The generalized plant $\mathbf{G}$ can be partitioned to

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & G_{22} \end{bmatrix} \tag{40}$$

where block structure of $\mathbf{G}$ correspond with the input and output variables in Fig. 5:

$$\begin{bmatrix} \begin{bmatrix} z \\ e \end{bmatrix} \\ u \end{bmatrix} = \mathbf{G} \cdot \begin{bmatrix} \begin{bmatrix} w \\ d \end{bmatrix} \\ v \end{bmatrix} \tag{41}$$

The design objective is to find a stabilizing controller K such that

$$\left\| \mu_{\tilde{\Delta}} [\mathbf{F}_l(\mathbf{G}, K)] \right\|_\infty \tag{42}$$

is minimal, where

$$\mathbf{M} = \mathbf{F}_l(\mathbf{G}, K) = \mathbf{G}_{11} + \mathbf{G}_{12} K (1 - G_{22} K)^{-1} \mathbf{G}_{21} \tag{43}$$

is the lower linear fractional transformation on generalized plant $\mathbf{G}$ and controller K (see Fig. 5).

The transfer matrix $\mathbf{M}$ can be partitioned to

$$\mathbf{M} = \begin{bmatrix} M_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \tag{44}$$

where the relationship between inputs and outputs is the following

$$\begin{bmatrix} z \\ e \end{bmatrix} = \begin{bmatrix} z \\ e \end{bmatrix} = \mathbf{M} \cdot \begin{bmatrix} w \\ d \end{bmatrix} \tag{45}$$

Then the transfer function from d to e is the upper linear fractional transformation on $\mathbf{M}$ and Δ

$$e = \mathbf{F}_u(\mathbf{M}, \Delta) d = \mathbf{M}_{22} d + \mathbf{M}_{21} \Delta (1 - M_{11} \Delta)^{-1} \mathbf{M}_{12} d \tag{46}$$

For performance and stability the following corollary of Theorem 1 holds.

Corollary 1: Closed loop in Fig. 5 is stable for all $\Delta \in \mathbf{C}$, $|\bar{\sigma}(\Delta)| < 1$, the performance condition (38) holds, and $\|F_u(\mathbf{M}, \Delta)\|_\infty \le 1$ if and only if

$$\mu_{\tilde{\Delta}}(\mathbf{M}) \le 1 \tag{47}$$

for all frequencies.

The term $\mu_{\tilde{\Delta}}$ in (47) corresponds with the perturbation set in the form

$$\tilde{\mathbf{\Delta}} \equiv \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_P \end{bmatrix}, \Delta, \Delta_P \in \mathbf{C} \tag{48}$$

and takes into account performance condition (38).

A. Algebraic Approach

The controller $K = \frac{N}{M}$ is obtained by solving the Diophantine equation (27). By the analysis of the polynomial degrees of a and b, the transfer functions A, B, M and N were chosen so that the number of closed loop poles is minimal and the asymptotic tracking is achieved:

$$A = \frac{a}{\prod_{i=1}^{2} (s + \alpha_i)}, \quad B = \frac{b}{\prod_{i=1}^{2} (s + \alpha_i)} \tag{49}$$

$$M = \frac{sm}{\prod_{i=3}^{4} (s + \alpha_i)}, \quad N = \frac{n}{\prod_{i=3}^{4} (s + \alpha_i)} \tag{50}$$

and degrees of polynomials m, n are:

$$\partial m = 1, \partial n = 2 \tag{51}$$

The resulting controller has the PID structure:

$$K(s) = \frac{n_2 s^2 + n_1 s + n_0}{s(s + m_0)} \tag{52}$$

By the optimization of the poles α_i via the Differential Migration and subsequent tuning by the Nelder-Mead simplex method, resulting poles were obtained:

$$\alpha_1 = 1.70, \alpha_2 = 0.097, \alpha_3 = 0.63, \alpha_4 = 33.70 \tag{53}$$

yielding the controller

$$K_A(s) = \frac{6.916 s^2 + 4.825 s + 1.752}{s^2 + 34.12 s} \tag{54}$$

B. Comparison Study

As a reference, *D-K* iteration is used, which is a common method for μ-synthesis. In order to satisfy state-space formulae assumptions for $\mathbf{H}_\infty$ suboptimal controller the weight W_1 has to be modified so that it does not have integrating behaviour:

$$W_1(s) = \frac{0.004}{10s^3 + 100s^2 + s + 10^{-5}} \cdot 100 \tag{55}$$

The controller obtained from the *D-K* iteration was approximated by 3rd order transfer function:

$$K_{D-K}(s) = \frac{5.295s^3 + 2.924s^2 + 0.8805s + 0.03513}{s^3 + 9.305s^2 + 0.1146s + 1.163 \cdot 10^{-6}} \tag{56}$$

The μ-plot in Fig. 14 shows that both controllers have the supremum of μ below one and the robust stability and performance condition is satisfied.

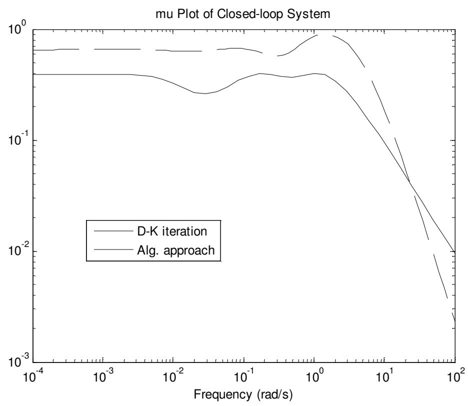

Fig. 14 Mu-plot for the controllers obtained by the D-K iteration and algebraic approach

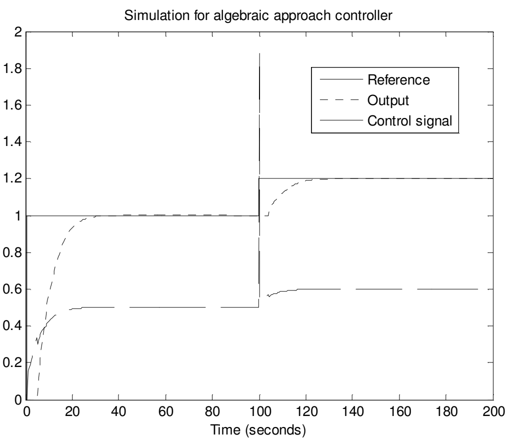

Fig. 13 Simulation for simple feedback loop with $T_0 = 4$ s – algebraic approach

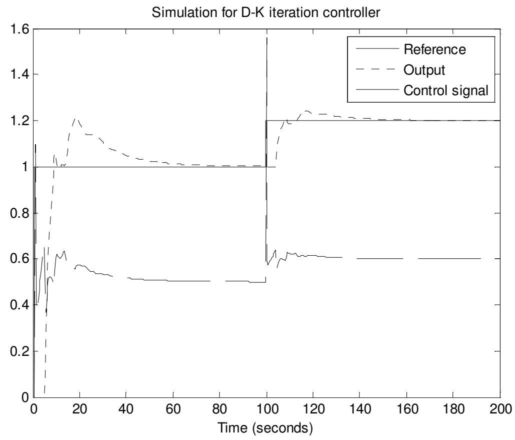

Fig. 12 Simulation for simple feedback loop with $T_0 = 4$ s – D-K iteration

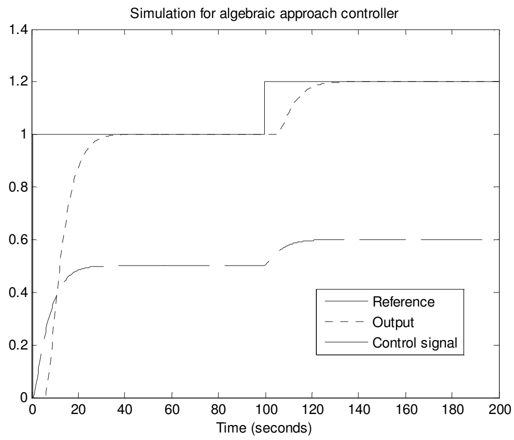

Fig. 16 Simulation for 2DOF feedback loop with $T_0 = 4$ s – algebraic approach

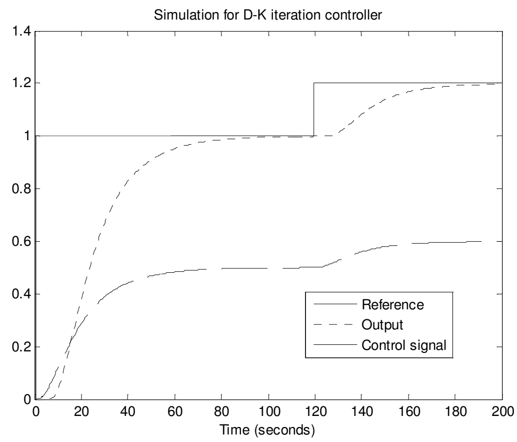

Fig. 15 Simulation for 2DOF feedback loop with $T_0 = 4$ s – D-K iteration

Simulations for the full and half time delay in Fig. 12, 13, 15, 16, 18 and 17 show that the algebraic approach has no steady state error which is not true for the D-K iteration. The controllers are stable for both full and half time delay and the algebraic approach gives faster set point tracking.

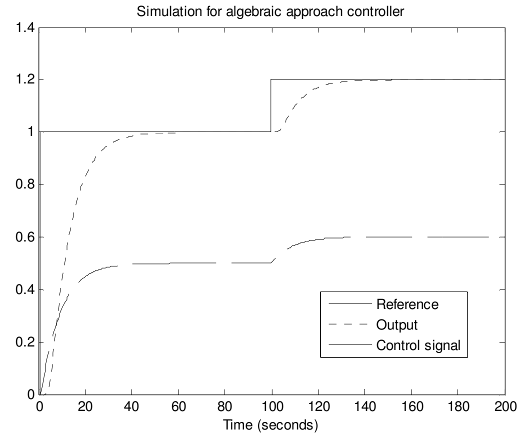

Fig. 18 Simulation for 2DOF feedback loop with $T_0 = 2$ s – algebraic approach

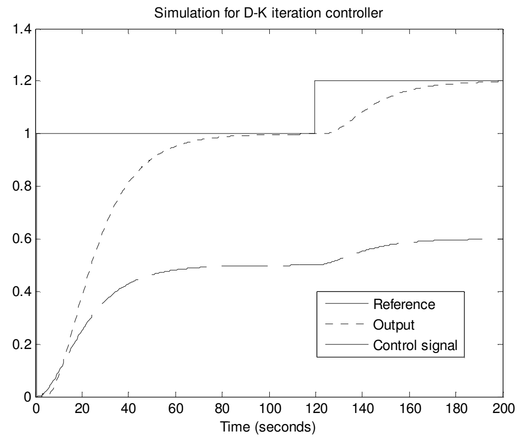

Fig. 17 Simulation for 2DOF feedback loop with $T_0 = 2$ s – D-K iteration

VIII. Download

The Robust Control Toolbox for Time Delay Systems toolbox can be downloaded from:
http://web.fai.utb.cz/?id=0_5_2_8_1&lang=cs&type=0

IX. Conclusion

The paper showed usage of the Robust Control Toolbox for Time Delay Systems for the Matlab system. An outline of the algebraic approach was given with application to time delay system. An example of control using the presented Matlab toolbox showed the benefits of the algebraic approach in comparison with the standard method for robust control design using structured singular value.

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