Positioning Error Reduction by the Use of Multiple GPS Terminal

Yuki Odaka, Shinya Takano, Yusuke In, Masakazu Higuchi, and Hitomi Murakami

Abstract— Recently, studies of location identification by mobile phone have been attracting a lot of attention, and it is applied to various applications. Now, when mobile phones have become ubiquitous terminals, many mobile phone GPS terminals may be unevenly distributed within local locations. Our study examines the error characteristics of terminals that are located within a short distance. We used GPS loggers in this experiment because we intended to investigate the change effect of satellites. In this paper, we consider that positioning errors in GPS terminals depend on the difference in the number of satellites used for positioning calculation and the satellite numbers. In addition, we perform theoretical analysis of GPS errors and study the error characteristics of GPS loggers and mobile phone GPS.

Keywords— Plural satellite, mobile phone GPS, error reduction, Newton's method

I. Introduction

PRESENTLY in Japan, the mobile phone has become a ubiquitous terminal that is carried by anybody, anywhere, anytime for 24 hours a day. As the background to this, the GPS capability has become a standard feature of mobile phones since 2007. By equipping the mobile phone with location capabilities, the GPS has been applied to navigation, games, etc, and it has become necessary to expand the application area in future [1].

In this kind of environment, people in possession of mobile phones form groups as they move and multiple GPS terminals can be concentrated in small area resulting in their uneven distribution. Additionally, it is expected that in future RFID and other sensors will have GPS capabilities installed and enable them to work as a cluster of sensors.

In this paper, we investigate the error characteristics of closely spaced GPS terminals using multiple GPS-equipped mobile phones [2]-[5]. The result is that we were able to verify that for multiple GPS terminals, there is a huge variation in error. The reason could be that in addition to multipath, there is a variation in the number and numbering of satellites used to calculate the position among the multiple GPS terminals.

On the other hand, not only mobile phone GPS, but clusters of dedicated GPS sensors are also expected in the future. The mobile phone GPS positioning method is different from that of the dedicated GPS. Since the main function of a mobile phone is to make telephone calls, it would be undesirable for the GPS function to consume large amounts of power. In addition, from the cost perspective, using a high-sensitivity GPS antenna is difficult. Therefore, for mobile phone GPS, besides the pseudo-range of the receiver from the satellite, the specific method employed uses the distance of the receiver from the mobile phone base station and compensation data. Also, in order to reduce power consumption, the number of satellites used is less than that used by dedicated GPS units, and depending on the sensitivity of the antenna, the mobile phone GPS cannot compare with the dedicated GPS unit. Because of this difference in the measurement methods used by the mobile phone GPS and the dedicated GPS, it can be thought that a difference in the GPS error characteristics would arise.

In this paper, in a location that is accessible to several GPS satellites, the experimental investigation of satellite fluctuation effect on measurements and theoretical analysis are carried out [6]. Due to the difficulties in making detailed measurements using mobile phone GPS, the satellite fluctuation effect investigation is performed with dedicated GPS units. For the reason that, in future, clusters of dedicated GPS units are expected, this experimental approach was taken. Then, the dedicated GPS error characteristics were examined and compared with those of the mobile phone GPS.

II. Structure of the Experiment

In this experiment, by simultaneously using multiple dedicated GPS units, GPS measurements were taken. Eight terminals of Wintec's WBT-202 GPS devices and a high-end differential GPS (DGPS) device, the Crescent A100 (Hemisphere GPS Inc.), were used. The WBT-202 GPS chip is the u-blox5 by the u-blox company. Using u-blox company's u-center GPS evaluation software, NMEA messages [7] were saved.

At the time of purchase, the WBT-202 is set to Static Hold. This means that position calculations are performed when the terminal is not moving. For this reason, u-center changed this setting so that the Static Hold is disabled. We changed the measurement interval from one second to 1 minute interval, i.e. the measurement mode was changed to cover the range from automobile to stationary measurements. The positioning method was the originally set single point positioning (SPP)

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method. Measurements were taken at one minute intervals for more than 24 hours.

Figure 3 shows the experimental scene. The GPS is connected to the PC where positioning data is stored. Additionally, position measurements by a high-end differential GPS (DGPS) device, the Crescent A100 (Hemisphere GPS Inc.), were performed simultaneously at the same location. The true value was the one obtained by this high-end device.

Fig. 1 Location of experiment

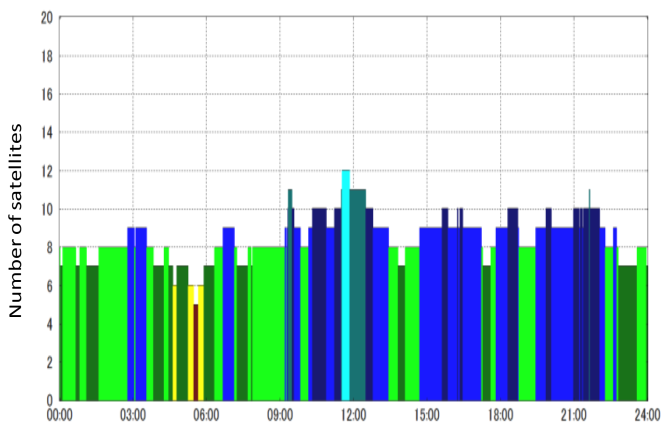

Fig. 2 Number of satellites viewed from the rooftop of Seikei University

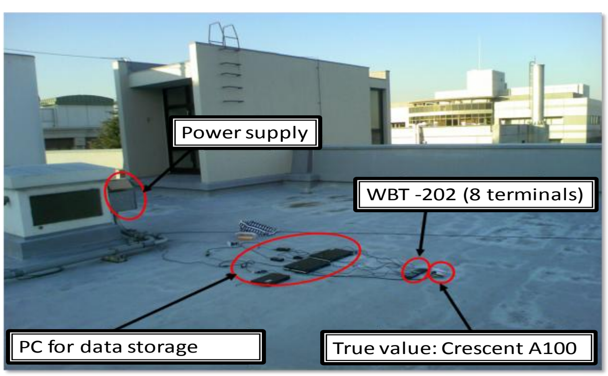

Fig. 3 Experimental scene

III. Experimental Result

The experiments were conducted using eight terminals but position measurements on one terminal stopped during the course of the experiment. Thus, with 7 terminals, over 24 hours of data (over 1400 points of measurement results) was obtained.

Using the same device model, at the same place and at the same time to measure the GPS error, it is expected that the number and type of GPS satellites accessed would be same, but as shown in Figs 4 and 5, a slight difference is observed between the number of accessible satellites and that used to calculate the error. Out of the 1440 measurements, only 189 measurements had all the 7 terminals simultaneously matching the number of accessed satellites and those used in the calculations.

Figures 4 and 5 shows the number of visible satellites and those used from two out of the seven terminals in the experiment.

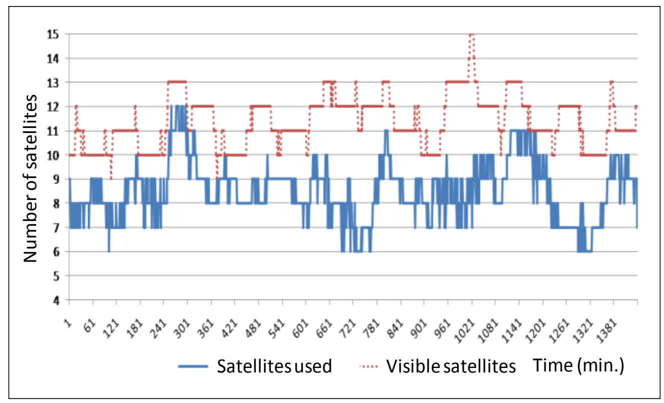

Fig. 4 Terminal A: Satellites visible and used

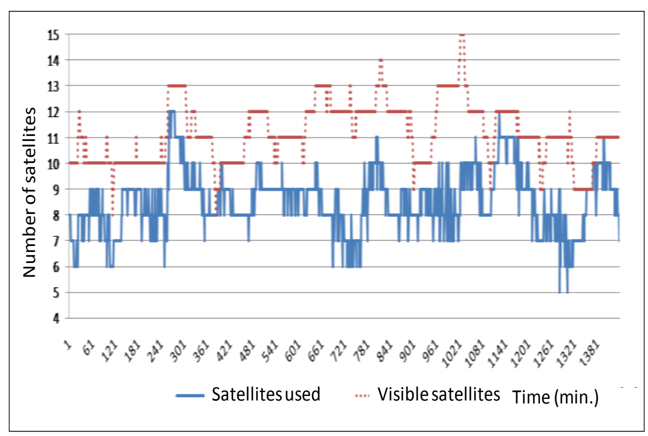

Fig. 5 Terminal B: Satellites visible and used

Table 1 shows the difference in the satellites used by terminals A and B for positioning. In the Table, ○ is for both terminals, ▲ is for terminal A only while ▼ is for B only.

TABLE I. Difference in satellites used by terminals A and B (results truncated)

Satellite No. \ Time (minutes)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	…	58	59	60	61
2	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○		○	○	○	○
4	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○		○	○	○	○
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23	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○		○	○	○	○
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Used No. Ter. A	8	8	9	8	7	8	8	7	7	7	8	8	7	7	7	7	7	8	8	7	7	7	7	8	8	8	8	7	8	8	8	8		9	9	9	9
Used No. Ter. B	8	8	8	8	7	7	7	7	7	7	7	7	7	6	6	6	6	6	6	7	6	6	6	6	8	8	8	8	8	8	8	8		9	8	8	8

Thus, how the observed satellites and the difference in terminal type affect the GPS error of the dedicated GPS devices was examined with respect to the following items:

I. Theoretical analysis of GPS error,

II. Group characteristics dedicated GPS devices and

III. Selection of the true value.

As a method for calculating the error, in *I* the approximate location found in the observed RINEX file [7] data was taken as the true value (ground truth). The error between the ground truth and the observed value was found by WGS-84[9]. For *II* and *III*, calculation was performed by substituting into the Hubeny's formula [10]. The position value for which all the 7 terminals obtained the same value as the Crescent A100 (Hemisphere GPS Inc.) was used as the true value for the position. At a rate of 95% the Crescent A100 can achieve positioning accuracy better than 0.6m.

IV. Theoretical Analysis of GPS Error

As outlined in Chapter 3, for the GPS error calculation, even if say N satellites are accessible (maximum value of N is 15), there were cases in which the actual number of satellites used in the calculation was M (M varies from 5 to 12). It was confirmed experimentally that the value of M varied with time. This means that at the same location and time, the GPS measurement error is different for each terminal. Thus the effect of the change in the number of satellites to GPS error calculation was analyzed.

A. About GPS Position Calculation

At time t, the geometric distance from Receiver i to Satellite j can be expressed as

$$\rho_i^j(t) = \sqrt{(X^j(t)-X_i)^2 + (Y^j(t)-Y_i)^2 + (Z^j(t)-Z_i)^2}, \qquad (1)$$

Wher $\rho_i^j(t)$ is the geometric distance, $X^j(t), Y^j(t), Z^j(t)$ the position of the Satellite j and X_i, Y_i, Z_i is the position of the Receiver i. However, in reality, the error due to the time difference between the receiver and the satellite is added. In this case, at time t, the measured value from Receiver i to Satellite j is given by

$$R_i^j(t) = \rho_i^j(t) + c\Delta\delta_i^j(t). \qquad (2)$$

In (2), $R_t^j(t)$ is the pseudo-range and c is the speed of light. $\Delta\delta_i^j(t)$ is the error obtained by subtracting the Satellite j error $\delta^j(t)$ from the Receiver i error $\delta_i(t)$.

The unknown quantities are the observation coordinates (three components) and clock error making a total of 4 unknowns, so the pseudo-range from a minimum of 4 satellites is used to solve simultaneous equations. Since the pseudo-range contains errors, by using pseudo-range from multiple satellites, the least squares method is used to estimate the observation point.

The position of the satellites is calculated using orbital data that is referred to as ephemeris. Ephemeris consists of three elements, which are: general ephemeris, broadcast ephemeris and precise ephemeris. Normal GPS positioning uses broadcast ephemeris with a precision of 1m. From the broadcast ephemeris, using Keplerian orbital elements in TABLE II, the satellite orbit is calculated.

TABLE II. Keplerian Orbital Elements

Orbital element	Annotation
Ω	Right Ascension of Ascending Node (RAAN)
a	Semimajor axis of the orbital ellipse
i	Inclination
e	Eccentricity of the ellipse
ω	Argument of perigee
T_0	Elapsed time of the perigee

The following are the formulas used for calculating the necessary parameters for computing the position of the satellite time *t*.

$$\begin{aligned}
M &= M_0 + \left[\sqrt{\frac{\mu}{a^3}} + \Delta n\right](t - t_e) \\
l &= l_0 + \Omega'(t - t_e) - \omega_E(t - t_0) \\
\omega &= \omega_0 + C_{uc}\cos(2u) + C_{us}\sin(2u) \\
r &= r_0 + C_{rc}\cos(2u) + C_{rs}\sin(2u) \\
i &= i_0 + C_{ic}\cos(2u) + C_{is}\sin(2u) + i'(t - t_e) \\
&\text{where} \quad u = \omega_0 + v \quad , \quad r_0 = a(1 - e\cos E).
\end{aligned} \qquad (3)$$

In Equation (3), M is the mean anomaly, l the angle of the ascending node from the Greenwich Meridian, ω and i are Keplerian orbital elements shown in Table 2, r_0 is the geocentric distance, E is the eccentricity and r is the corrected geocentric distance. Additionally, t_0 represents the initial time of present GPS and $\omega_E = 7292115.1467 \cdot 10^{-11} rad\ s^{-1}$ is the angular velocity of the earth`s rotation. The other parameters are the broadcast ephemeris values shown in Table 3. Also, from the satellite clock parameter, the satellite clock error at time t can be computed as follows:

$$\delta^s = a_0 + a_1(t - t_c) + a_2(t - t_c)^2 \tag{4}$$

The parameters used in this investigation are shown in TABLE III.

TABLE III. Broadcast ephemeris

Parameter	Meaning
t_e	Broadcast ephemeris standard time
$\sqrt{a}$	Square root of the semi-major axis
e	Eccentricity
M_0	Standard time of the mean anomaly
ω_0	Argument of perigee
i_0	Orbit inclination
l_0	Right ascension of ascending node (RAAN) at the start of this GPS week
Δn	Mean motion difference
i'	Rate of change of orbit inclination
Ω'	Rate of change of ascending node declination
C_{uc}, C_{us}	Correction factor (argument of perigee)
C_{rc}, C_{rs}	Correction factor (geocentric distance)
C_{ic}, C_{is}	Correction factor (orbit inclination)
t_c	Standard time of the satellite clock
a_0	Offset of the satellite clock
a_1	Drift of the satellite clock
a_2	Drift of the satellite clock frequency

Next, we explain the method of computing the position of each terminal. The single point positioning model of Equation (1) consists of a set of nonlinear simultaneous equations. For this reason, in order to calculate the receiver position, X_i, Y_i, Z_i, normally the equations are linearized about an arbitrarily chosen initial value using the Newton's method.

The geometric distance $\rho_i^j(t)$ is initialized to $\rho_{i0}^j(t)$ and the unknown quantity $\Delta\rho_i = [\Delta X_i, \Delta Y_i, \Delta Z_i]$ is resolved. After Taylor expansion, and taking the first term of the result, the linearized equation can be obtained.

$$\rho_i^j(t) \equiv \rho_{i0}^j(t) - \frac{X^j(t) - X_{i0}}{\rho_{i0}^j(t)}\Delta X_i - \frac{Y^j(t) - Y_{i0}}{\rho_{i0}^j(t)}\Delta Y_i - \frac{Z^j(t) - Z_{i0}}{\rho_{i0}^j(t)}\Delta Z_i \tag{5}$$

Substituting the above equation (5) into the single point positioning model, Equation (2), and moving the known quantities to the right-hand side of the equation, we get

$$R_i^j(t) - \rho_{i0}^j(t) + c\delta^j(t) = -\frac{X^j(t) - X_{i0}}{\rho_{i0}^j(t)}\Delta X_i - \frac{Y^j(t) - Y_{i0}}{\rho_{i0}^j(t)}\Delta Y_i - \frac{Z^j(t) - Z_{i0}}{\rho_{i0}^j(t)}\Delta Z_i + c\delta_i(t) \tag{6}$$

Thus the 4 unknown quantities $\Delta X_i, \Delta Y_i, \Delta Z_i, \delta_i(t)$ can be expressed as a linear equation.

For the sake of brevity let

$$l^j = R_i^j(t) - \rho_{i0}^j(t) + c\delta^j(t) \tag{7}$$

$$a_{X_i}^j = \frac{X^j(t) - X_{i0}}{\rho_{i0}^j(t)}, \quad a_{Y_i}^j = \frac{Y^j(t) - Y_{i0}}{\rho_{i0}^j(t)}, \quad a_{Z_i}^j = \frac{Z^j(t) - Z_{i0}}{\rho_{i0}^j(t)} \tag{8}$$

When the number of satellites is N, the linearized equations become

$$\left\{\begin{array}{l} l^1 = a_{X_i}^1 \Delta X_i + a_{Y_i}^1 \Delta Y_i + a_{Z_i}^1 \Delta Z_i + c\delta_i(t) \\ l^2 = a_{X_i}^2 \Delta X_i + a_{Y_i}^2 \Delta Y_i + a_{Z_i}^2 \Delta Z_i + c\delta_i(t) \\ l^3 = a_{X_i}^3 \Delta X_i + a_{Y_i}^3 \Delta Y_i + a_{Z_i}^3 \Delta Z_i + c\delta_i(t) \\ \vdots \\ l^N = a_{X_i}^N \Delta X_i + a_{Y_i}^N \Delta Y_i + a_{Z_i}^N \Delta Z_i + c\delta_i(t) \end{array}\right\} \tag{9}$$

Let's define the following matrices and vectors:

$$\vec{A} = \begin{bmatrix} a_{X_i}^1 & a_{Y_i}^1 & a_{Z_i}^1 & c \\ a_{X_i}^2 & a_{Y_i}^2 & a_{Z_i}^2 & c \\ a_{X_i}^3 & a_{Y_i}^3 & a_{Z_i}^3 & c \\ \vdots & \vdots & \vdots & \vdots \\ a_{X_i}^N & a_{Y_i}^N & a_{Z_i}^N & c \end{bmatrix} \quad \vec{x} = \begin{bmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \\ \delta_i(t) \end{bmatrix} \quad \vec{l} = \begin{bmatrix} l^1 \\ l^2 \\ l^3 \\ \vdots \\ l^N \end{bmatrix} \tag{10}$$

Equation (9) can written as

$$\vec{l} = \vec{A}\vec{x} \tag{11}$$

By solving Equation (11) for $\vec{x}$, the components $\Delta X_i, \Delta Y_i, \Delta Z_i$ obtained are added to the initial value X_{i0}, Y_{i0}, Z_{i0}. Taking this as the new initial value, the matrix and vectors of Equation (10) can be generated and the matrix/vectors of Equation (11) can be calculated [9].

B. Errors Due To Changes In Satellites Used

With the formula from Section 4.1 a GPS positioning program was created, and by varying the number of satellites, theoretical positioning measurements were conducted. The experimental results are shown in Figure 6.

The satellites and pseudo-range used were obtained from the electronic reference point data at the homepage of the Geospatial Information Authority of Japan [11]. The electronic reference point used consists of 34 locations. From this result, it can be observed that as the number of satellites used in the measurements increases, the measurement error decreases. Specifically, for up to 6 satellites, the precision dramatically improves, but subsequent increase in the number of satellites was found have little effect on the precision.

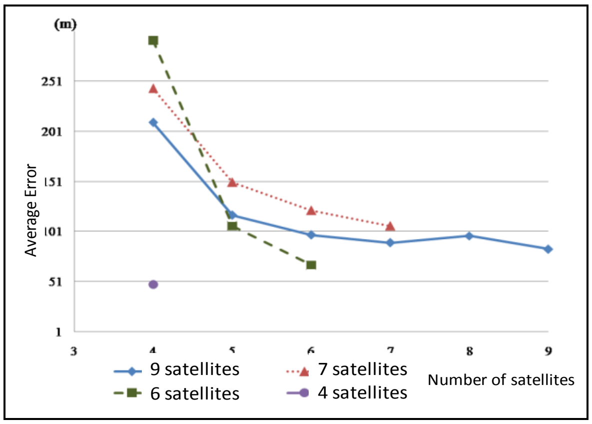

Fig.6 Error characteristics due to change in satellites used (observed satellites)

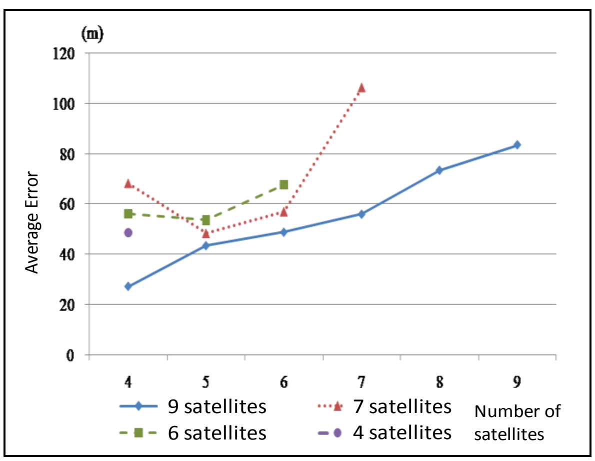

Fig.6 Error characteristics due to change in satellites used (maximum number of satellites)

However, as shown in Figure 7, the possibility that the increase in the number of satellites worsens the pseudo-range precision is high, so there are cases in which the minimum error increases. Thus, even by reducing the number of satellites, adaptively selecting the satellites could lead to a reduction in the error.

V. Group Characteristics of Dedicated GPS Devices

Up to now we have studied the group characteristics of mobile phone GPS [2]-[6]. In this study we made a detailed performance evaluation of the dedicated GPS group characteristics (Handy GPS in Figure 6) and compared the results to those of the mobile phone GPS. As already stated in Chapter 1 of this paper, the mobile phone GPS is a measurement system that is meant to leverage the mobile phone communication. From this difference in measurement systems, it possible to learn the characteristics of the mobile phone GPS.

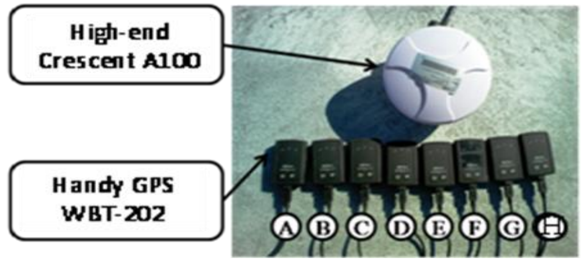

Fig. 6 Dedicated GPS (WBT-202)

A. The Error Characteristic of the Dedicated GPS and the Mobile Phone GPS

First, we investigated the error characteristics of the dedicated GPS and the mobile phone GPS. Mobile phones used in the experiment consist of 8 terminals from 5 different models (TABLE IV). Among the 8 terminals, two sets of 2 terminals and 3 terminals respectively are of the same model. TABLE V shows the mobile phone GPS error characteristics. Although rare, dramatically reducing the effectiveness of the GPS (by increasing the variance of the distance error) led to the detection of large errors. We refer to this error as " out of sync error", which is defined as any error that is about 10 times greater than the average error. TABLE V shows the number of times that the "out of sync error" was detected.

TABLE IV. The composition of the 8 mobile phone terminals of 5 different models used in experiments.

Model	1	2	3	4	5
Maker	A	B	C	C	D
Units	1	1	1	3	2

TABLE V. GPS error characteristics for 5 models, 8 teminals.

Type Name	1-A	2-B	3-C	4-C-1	4-C-2	4-C-3	5-D-1	5-D-2
Average Error (m)	10	12	11	9	9	8	11	11
Maximum Error(m)	58	117	296	191	58	56	238	231
Minimum Error (m)	0	0	0	0	0	0	0	0
Variance (m^2)	59	83	646	72	50	46	180	253
Out of Syn Error (>100m)	0	1	13	1	0	0	2	8
Data points	1608	1348	1475	1590	1448	1367	904	1063

The dedicated GPS error characteristics of this experiment are shown in TABLE VI. All the GPS terminals used were of the same model (WBT-202).

TABLE VI. Experimental results for 8 dedicated GPS terminals.

Terminal	A	B	C	D	E	F	G	H
Average Error (m)	3.56	3.13	3.09	3.05	4.12	3.39	2.98	1.45
Maximum Error(m)	54.89	39.7	38.75	36.76	60.42	44.2	31.02	35.14
Minimum Error (m)	0.03	0.03	0.17	0.12	0.20	0.17	0.03	0.05
Variance (m^2)	19.82	10.75	9.12	5.02	31.13	16.14	7.52	1.60
Out of Syn Error (>30m)	10	6	2	1	17	4	1	1

The average error for the dedicated GPS was 3-4m and 8-12m, which is considerably higher precision than the mobile phone GPS. Additionally, the maximum error did not exceed 100m as is the case with mobile phone GPS. The reason for the poor results from the mobile phone GPS measurements could be because of the number of satellites used and the effect of the correction data used. There were large variations in the maximum error for both the dedicated and mobile phone GPS'.

It can also be observed that the "out of sync error" threshold is different for dedicated GPS and mobile phone GPS, but the number of occurrances of the " out of sync error" vary greatly from terminal to terminal.

B. *The Correlation of GPS Error*

In order to investigate the GPS group characteristics, the distance and direction errors of the dedicated GPS were calculated. The correction of the direction and distance errors obtained by using multiple mobile phone GPS devices of TABLE IV is shown in TABLE VII.

Among the same models, the cross-correlation is slightly high, but the average value of the cross-correlation coefficients can be seen to be close to zero. When the cross-correlation of direction errors is low, the measurements obtained from multiple terminals at the point time can be thought to be distributed in various directions centered on the true value. From this observation, by adding the GPS errors, we confirmed that it is possible to improve the error characteristics [3] - [6].

TABLE VII. The correlation coefficients for each combination (mobile phone GPS).

		Indoors		Outdoors	
	Combination	DSEC	DREC	DSEC	DREC
[1]	1-A & 2-B	0.15	0.21	0.16	0.2
[2]	1-A & 3-C	0.16	0.22	0.06	0.13
[3]	1-A & 4-C-1	0.32	0.33	0.1	0.26
[4]	1-A & 4-C-2	0.11	0.13	0.2	0.29
[5]	1-A & 4-C-3	0.11	0.14	0.21	0.3
[6]	1-A & 5-D-1	-0.01	0	0.12	0.29
[7]	1-A & 5-D-2	0.1	0.12	0.09	0.09
[8]	2-B & 3-C	0.09	0.08	0.11	0.22
[9]	2-B & 4-C-1	0.05	0.1	0.26	0.41
[10]	2-B & 4-C-2	0.05	0.1	0.15	0.12
[11]	2-B & 4-C-3	-0.03	0.05	0.26	0.39
[12]	2-B & 5-D-1	-0.01	0.04	0.16	0.33
[13]	2-B & 5-D-2	-0.03	0.02	0.09	0.22
[14]	3-C & 4-C-1	-0.01	0.01	0.05	0.14
[15]	3-C & 4-C-2	0.22	0.23	0.03	0.2
[16]	3-C & 4-C-3	0.01	0.01	0.02	0.08
[17]	3-C & 5-D-1	0.01	0.02	-0.01	0.1
[18]	3-C & 5-D-2	0	0.01	-0.03	0.12
[19]	4-C-1 & 4-C-2	0.35	0.36	0.39	0.46
[20]	4-C-1 & 4-C-3	0.4	0.42	0.28	0.4
[21]	4-C-1 & 5-D-1	-0.01	0	0.14	0.34
[22]	4-C-1 & 5-D-2	-0.03	0	0.09	0.16
[23]	4-C-2 & 4-C-3	0.34	0.35	0.34	0.44
[24]	4-C-2 & 5-D-1	-0.01	0	0.16	0.3
[25]	4-C-2 & 5-D-2	-0.01	0	0.09	0.17
[26]	4-C-3 & 5-D-1	-0.01	0	0.2	0.31
[27]	4-C-3 & 5-D-2	-0.01	0	0.07	0.2
[28]	5-D-1 & 5-D-2	0.09	0.08	0.15	0.07
	Average	0.09	0.11	0.14	0.24

Key: DSEC: Distance Error Correlation, DREC: Direction Error Correlation

On the other hand, the correlation of the distance error and the direction error for the dedicated GPS device is shown in TABLE VIII.

TABLE VIII. The correlation coefficients for each combination (dedicated GPS).

		Outdoors	
	Combination	DSEC	DREC
[1]	A & B	0.5	0.46
[2]	A & C	0.68	0.55
[3]	A & D	0.46	0.31
[4]	A & E	0.52	0.55
[5]	A & F	0.50	0.47
[6]	A & G	0.32	0.46
[7]	B & C	0.64	0.53
[8]	B & D	0.46	0.51
[9]	B & E	0.45	0.51
[10]	B & F	0.50	0.43
[11]	B & G	0.42	0.52
[12]	C& D	0.50	0.40
[13]	C & E	0.37	0.50
[14]	C & F	0.41	0.45
[15]	C & G	0.37	0.45
[16]	D & E	0.35	0.35
[17]	D & F	0.37	0.32
[18]	D & G	0.20	0.44
[19]	E& F	0.67	0.60
[20]	E & G	0.27	0.46
[21]	F & G	0.40	0.51
	Average	0.45	0.47

Key: DSEC: Distance Error Correlation, DREC: Direction Error Correlation

It can be observed that the correlation is higher than that of mobile phone GPS. Even for mobile phone GPS, for terminals of the same model, the cross-correlation is slightly high so, the same could apply to dedicated GPS.

For the dedicated GPS where the correlation of the direction error is high, it is difficult to drastically reduce the error by the method that we propose.

C. *Measurement of True Value by the Dedicated GPS Device*

As explained in Section 5.2, by using the fact that the direction cross-correlation is low, and by adding the GPS distance errors, error characteristics can be improved [3] – [6]. The method used to get the true value estimate can be expressed by equation (12).

$$f^{\wedge}(x) = F(x) + \sum_{i=1}^{N} n(x_i) / N \tag{12}$$

$F(x)$: true position $n(x_i)$: noise at the i th terminal
N : number of terminals for GPS error measurement

From the fact that the cross-correlation of multiple terminals is low, the measurement results are distributed in various directions around the true value. We assume that the mean value of noise is $\sum_{i=1}^{N} n(x_i) / N$ close to 0. In addition, it is expected that the mobile phone holders measure GPS in the same at the same time and same place. However, because the user did not actually understand a true value, latitude and the longitude of one of eight terminals were assumed to be a true value, and the true value estimation was requested by combining 7Cn(n=2~7) with seven remainder. The result is shown in Figure 9. In this Figure, the average value, the maximum estimate and the minimum estimate of the combinations of every terminal, i.e. 7Cn (n=2~7), and the GPS error of the each terminal that would be taken as the base value is shown.

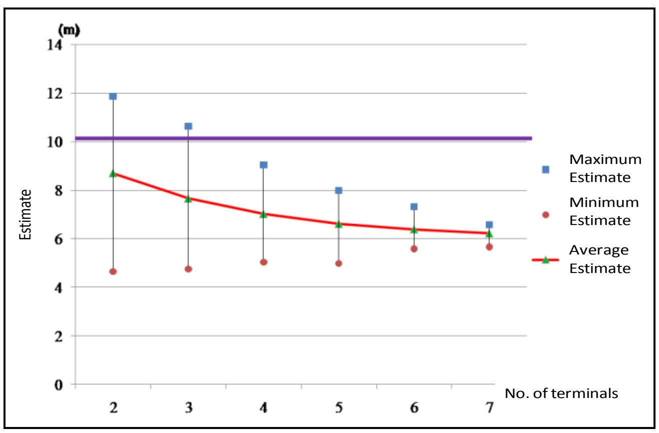

Fig.9 Error estimate for N terminals (mobile phone GPS)

It can be seen from the results that as the number of terminals N increase, the G`PS error is reduced. By using this method of true value estimation, and compared to when the GPS position error measurement are performed independently, using 8 mobile phone terminals can result in a reduction in error to within 50%.

In addition, Figure 10 represents the estimated error for the dedicated GPS. It is can be confirmed that as the number of terminals increases, the GPS can be error can be reduced. However, the decrease is even smoother than that of the mobile phone GPS, so for the average estimate, it can be observed that a large increase in N does not result in significant improvement in precision.

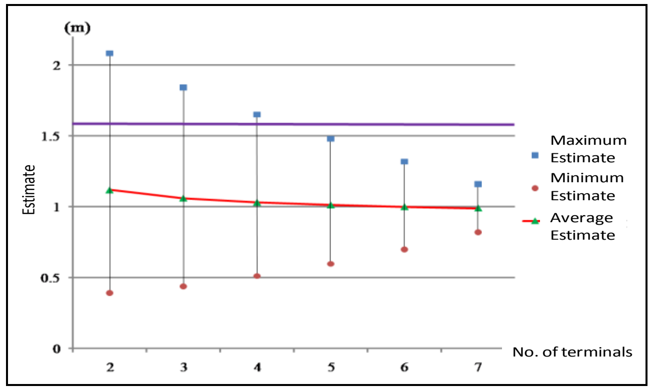

Fig. 10 Error estimate for N terminals (dedicated GPS)

By using this method of true value estimation, and compared to when the GPS position error measurements are performed independently, using 8 terminals of dedicated GPS devices can result in a reduction in error to within 75%. This performance is worse than that of mobile phone GPS, and the reason could be attributed to the high correlation between the distance and the direction errors.

VI. Varying the Error by Selection of the True Value

Up to now, we have made various measurements and analyses of the error but it is also necessary to calculate the true value that would become the standard reference value. All measured values have some error with respect to the true, thus it is necessary to use a correct true value. However, nobody really knows the true value. Although places like the electronic reference point where the exact coordinates are known exist, generally the true value in all other place where position measurements are taken is unknown. We thus investigate if the selection of the true value affects the error. Here, the following three types of true values are used:

I. High-end dedicated GPS' s positioning point
II. Reference coordinates from Google Maps
III. GPS average used in positioning

The high-end dedicated GPS used in *I* was the previously mentioned Crescent A100. The measurement method is DGPS and this method has higher precision than the SPP method used to initialize the WBT-202. The CrescentA100 is equipped with a multipath reduction function and is a high-end device that costs well over 300 thousand Japanese Yen.

The reference coordinates of *II* from Google Maps [12] can be obtained by clicking the point of interest on a map. The coordinates are then displayed. The GPS measurement results are used by Google and others providers so the values from Google Maps can be said to be true coordinates.

Finally, the GPS average value (in this experiment Wintec's WBT-202) used the u-center's evaluation software, called Deviation Map, as a method for obtaining the reference position. Since the state of the satellites, multipath, ionospheric-tropospheric delay, etc vary with time, measurements are taken over a long period of time and the effect of these errors is eliminated by averaging.

A. Shift in Deviation due to the true value

Figures 11 to 13 show the deviation diagrams for the 3 types of true value. For the average value, Fig. 11, the true value becomes the center of the deviation. For the high-end device, Fig. 12, the deviation is centered close to the true value.

However, for the Google Maps, Fig. 13, the deviation is shifted to the west side of the coordinates. It is clear from these results that the true value obtained from the Google Maps is to the east side the true value obtained by the averaging and by the high-end device. Since nobody really knows the true value, it cannot be said that the Google Maps true value is wrong. Neither can it be said that the true value by averaging and the high-end device is correct.

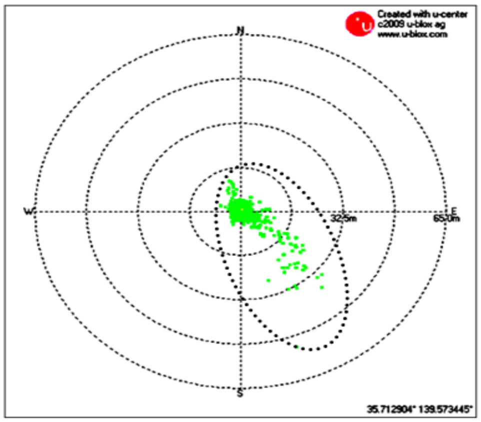

Fig. 11 WBT-202's average value

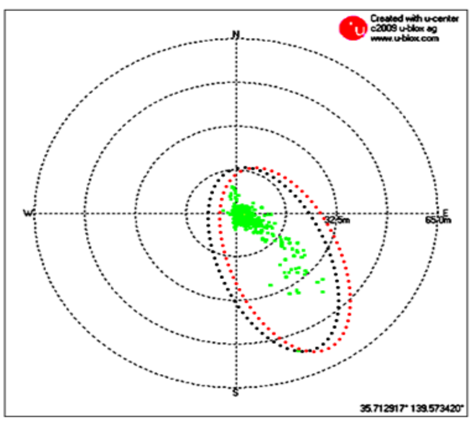

Fig. 12 Crescent A100

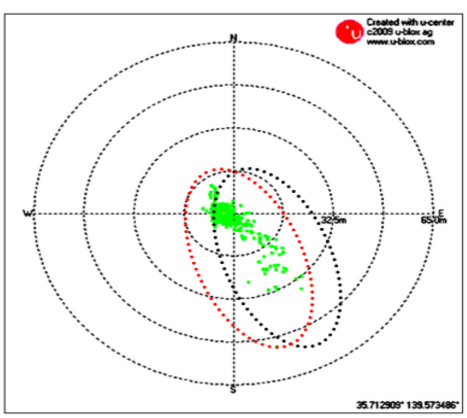

Fig. 13 Google Maps

B. Error characteristics of the high-end GPS device

In this investigation, we used the Crescent A100 GPS, a very high-end GPS device with high level functionality. However, compared to the dedicated GPS (WBT-202) and mobile phone GPS, a different deviation characteristics was obtained for high quality GPS. The deviation map is shown in Figure 14.

Despite the fact that a fixed observation point was used, the position points were observed to be continuous. For the dedicated GPS and mobile phone GPS, the position was independent of time and every time position points were scattered in various directions. Normally the position measurements are as shown in Fig. 15. The data for Figures 14 and 15 consists of 1440 points (one day's data). The distance error for the high quality device is shown in Fig. 16. The distance errors also change continuously and the "sync error" was not detected at all.

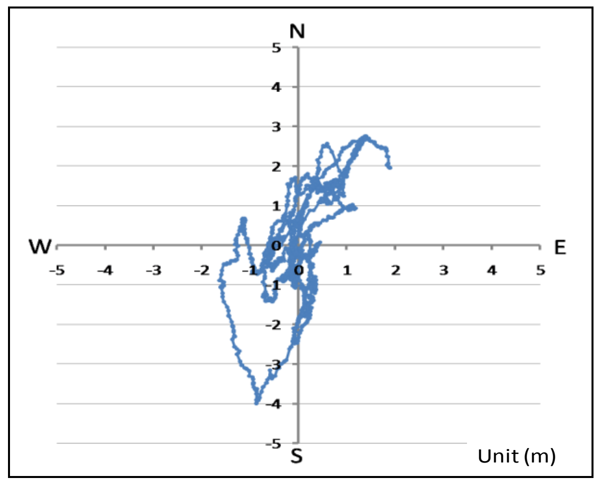

Fig. 14 Deviation map for the high-end device (Crescent A100)

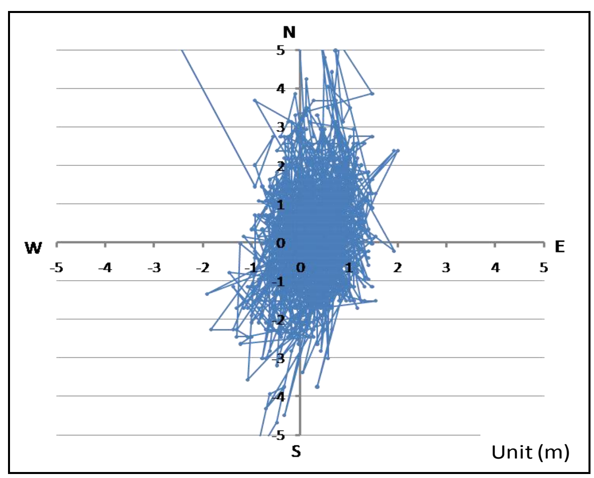

Fig. 15 Deviation map for the dedicated device (WBT-202)

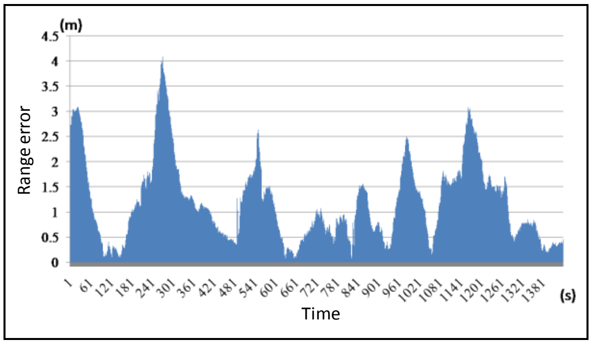

Fig. 16 Time characteristics of the high-end device's range error

Figure 17 shows the positioning results of a high-end device at certain range and the range difference measured after one minute. For the high-end device, the range difference was mostly 10cm. On the other hand, Figure 18 shows the positioning results of a dedicated GPS device at certain range and the range difference measured after one minute. For the dedicated GPS device, there is a shift in the positioning result of about 1m every-time the positioning points are taken with no regard to the past position.

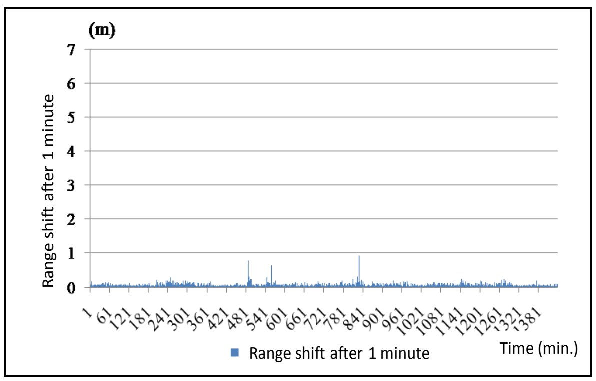

Fig. 17 The shift of positioning results with time (high-end device)

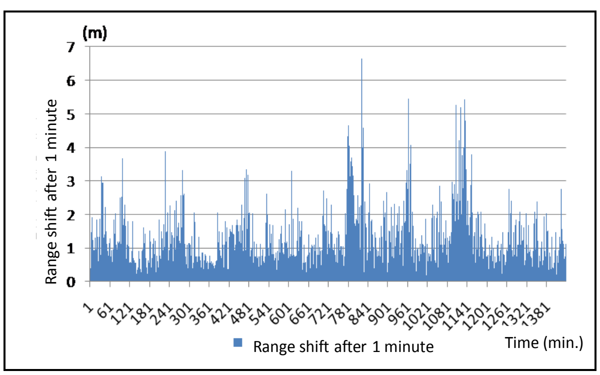

Fig. 18 The shift of positioning results with time (dedicated GPS device)

This is because the positioning interval is 1 second and short; the receiver applies a filter to the previous positioning result which assumes that there will be motion. By such kind of feature, for a fixed observation point, there are cases in which the correct position cannot be obtained when positioning time is short. For the high-end device, in order to obtain the true value, making the positioning time long and using a lot of positioning results to get the true value would be better.

VII. Conclusion

The experimental results can be summarized as follows.

① Even if position measurements are taken at the same time and place, using similar models of GPS, the number and type of satellites accessed is different.

② By increasing the number of satellites used, the positioning error decreases and by adaptively selecting the satellites, the position error can be reduced.

③ The cross-correlation between the range error and the direction error of the same type of terminal is high. The dedicated GPS terminal has a higher correlation than the mobile phone GPS.

④ The method that we propose for true value estimation is inferior to that used in mobile phones but it has been shown to be effective for dedicated GPS. It has also been confirmed that the method is more effective when the cross-correlation is low.

⑤ In calculating the GPS error, deciding on the true value is very important. In this experiment, we believe that using the high-end GPS device to measure the position over a long period of time is the most appropriate method for deciding the true value.

The mobile phone GPS is growing as social infrastructure. It is necessary to further expand it's effectiveness in future. Maybe one of the directions of expansion could be to use mobile GPS terminals as a group. In this kind of environment, in order to use GPS terminals as a group, we foresee that it is very important to know the GPS group characteristics. We examined the mobile phone GPS and dedicated GPS devices' group characteristics, and theoretically analyzed the error related to the number of satellites employed using the error of multiple terminals.

Appendix

As stated in Chapter 4, in calculating the GPS position, the orbit is treated as a Kepler orbit. Figure 19 shows the Kepler orbit.

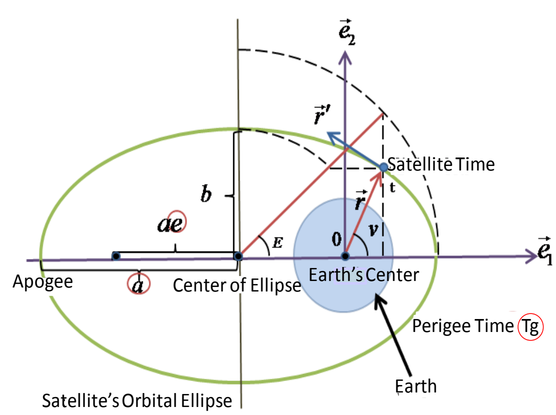

Fig. 19 The Kepler orbit

The angle v is called the true anomaly. It is the closest point of the GPS satellite from the center of the earth along the perigee and represents the angular position of the present satellite. This value determines the position of the satellite. Here, we explain the derivation of the satellite position using the broadcast ephemeris data.

To find the true anomaly at time t, first, the mean anomaly M and eccentric anomaly E are determined from the broadcast ephemeris data. The mean anomaly is obtained by Equation (13).

$$M = M_0 + \left[\sqrt{\frac{\mu}{a^3}} + \Delta n \right] (t - t_e) \tag{13}$$

Here, M_0 is the mean anomaly of the standard time, μ is the product of the gravitational constant and the mass of the earth, and is a constant value μ =986004.418×10^8 m³/s². a is the semimajor axis of the orbital ellipse, Δn is the mean motion difference and t_ε is the standard time of the broadcast ephemeris. These values are contained in the ephemeris data so if the ephemeris data is available, they can be determined. The eccentric anomaly can be obtained from Equation 14.

$$E = M + e \sin E \tag{14}$$

e is the eccentricity, a value that is obtained from the broadcast ephemeris data. Then, from the eccentric anomaly E, the true anomaly v can be obtained by using Equation (15).

$$v = 2 \arctan \left[\sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \right] \tag{15}$$

From this value, the position of the satellite on the Kepler's orbit $\vec{r}$ is given by Equation (16).

$$\vec{r} = r \begin{bmatrix} \cos v \\ \sin v \end{bmatrix} \tag{16}$$

The GPS coordinates are not calculated in the elliptical orbit coordinate system, but in the terrestrial reference system with the earth's center as the origin. For this reason, it is necessary to perform coordinate transformation. Figure 20 shows the position of the satellite with terrestrial reference system.

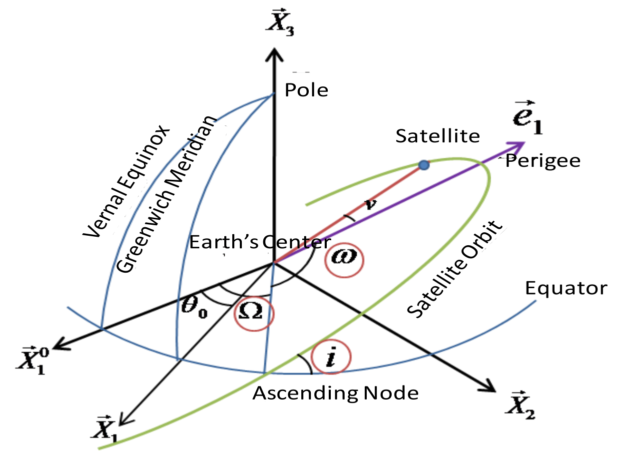

Fig. 20 Kepler's orbit on the terrestrial reference system

Here, l is the angle of the ascending node from the Greenwich Meridian, ω is the argument of perigee, i the inclination, Ω the right ascension of ascending node (RAAN) and θ_0 the angle of the vernal equinox to the Greenwich Meridian. On the Kepler's orbit, the position of the satellite $\vec{r}$ can be transformed to the terrestrial reference system by applying the rotation matrix $\vec{R}$. Let's refer to the coordinates after rotation as $\vec{\rho}$. The transformation is done by Equation (17).

$$\vec{\rho} = \vec{R}\vec{r} \tag{17}$$

Since after transformation, the coordinates consists of three components, orthogonalizing the Kepler orbit system's $\vec{e}_1, \vec{e}_2$ introduces the $\vec{e}_3$ axis and the $\vec{e}_3$ axis component is made equal to zero.

Rotation matrix for the transformation from Kepler orbit coordinate system to the terrestrial reference system

$$\vec{R} = \vec{R}_3\{-l\}\vec{R}_1\{-i\}\vec{R}_3\{-\omega\}$$

$$= \begin{bmatrix} \cos l \cos \omega - \sin l \sin \omega \cos i & -\cos l \sin \omega - \sin \Omega \sin \omega \cos i & \sin l \sin i \\ \sin l \cos \omega + \cos l \sin \omega \cos i & -\sin \Omega \sin \omega + \cos l \cos \omega \cos i & -\cos l \sin i \\ \sin \omega \sin i & \cos \omega \sin i & \cos i \end{bmatrix}$$

Fig. 21 Rotation matrix

The rotation matrix $\vec{R}$ of Fig. 20 can be expressed as product of three rotation matrices as shown in Fig. 21. Here, l is given by $l = \Omega - \omega$. The angle obtained is derived from Equation (3) of Chapter 4. In this way, the coordinates of the satellite can be calculated from the broadcast ephemeris data.

As explained in Chapter 4, there 4 unknowns in GPS calculations. For this reason, it is desirable to be able to track a minimum of 4 GPS satellites. However, in general, more than 4 satellites are tracked for position calculation. When there are more equations than unknowns, the least-squares method is used for approximation.

As in Chapter 4, the difference in position coordinate can be expressed in matrix form as follows:

$$\vec{l} = \vec{A}\vec{x} \tag{18}$$

n is the number of satellites used, and since there are 4 unknowns, the design matrix has n rows and 4 columns. When n is greater than 4, it is generally not possible to obtain a unique solution. To obtain a unique solution, a noise vector is added to Equation (17). The new equation is given in (19).

$$\vec{l} + \vec{n} = \vec{A}\vec{x} \tag{19}$$

This equation is solved by the least-squares principle,

$$\vec{n}^T \vec{P}\vec{n} = \min \tag{20}$$

and a single solution is obtained. Applying the least-squares principl (20) to the observation equation (19), Equation (21) is obtained.

$$\vec{A}^T \vec{P}\vec{A}\vec{x} = \vec{A}\vec{P}\vec{l} \tag{21}$$

Acknowledgment

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