Solving switched capacitors circuits by full graph methods

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***Abstract*—**Circuits with switched capacitors are described by a capacitance matrix and seeking voltage transfers then means calculating the ratio of algebraic supplements of this matrix. As there are also graph methods of circuit analysis in addition to algebraic methods, it is clearly possible in theory to carry out an analysis of the whole switched circuit in two-phase switching exclusively by the graph method as well. For this purpose it is possible to plot a Mason graph of a circuit, use transformation graphs to reduce Mason graphs for all the four phases of switching, and then plot a summary graph from the transformed graphs obtained this way. First we draw nodes and possible branches, obtained by transformation graphs for transfers of EE (even-even) and OO (odd-odd) phases. In the next step, branches obtained by transformation graphs for EO and OE phase are drawn between these nodes, while their resulting transfer is multiplied by $z^{-\frac{1}{2}}$ or $-z^{-\frac{1}{2}}$. This summary graph can then be interpreted by the Mason's relation to provide transparent voltage transfers. Therefore it is not necessary to compose a sum capacitance matrix and to express this consequently in numbers, and so it is possible to reach the final result in a graphical.

***Keywords*—**Switched capacitors, two phases, transformation graph, Mason's formula, voltage transfer, summary graph.

I. Introduction

ANALYSIS of electric circuits is necessary not only for computing of circuit properties but also understanding their principles. The computer methods are a powerful tool for symbolic analysis of circuit parameters [7]. But it is advantageous to have a tool capable to clear and simply symbolic analysis, too. The graphs methods can be considering as this tools. Thanks to its clarity, the graphic method is extremely suitable even for understand of these networks. A clearly arranged set of the transformation graphs derived for different types of the switching circuits can be used for analyzing capacitor switched networks and for understand of course, too. The M-C signal flow graphs are used to the design [8] and analysis [2] continuous time circuits and periodically switched linear circuits, too. The transformation graphs are commonly used for assembly the final matrix considering all phases [6] to solving electronics circuits and matrix is calculated by algebraic minors [10]. It means this method is combination graph and numerical methods, booth. But solving is possible by graphs only in selected circuits, as is described follows.

II. Principle of the methods

A. Summary Graph Construction

Solving circuits with switched capacitors [9] by means of nodal charge equation method system [3], [4], [5], leads generally to an equation system (1)

$$\begin{bmatrix} C_{EE} & -z^{-\frac{1}{2}}.C_{EO} \\ -z^{-\frac{1}{2}}.C_{OE} & C_{OO} \end{bmatrix}\begin{bmatrix} V_E \\ V_O \end{bmatrix}=\begin{bmatrix} Q_E \\ Q_O \end{bmatrix} \quad (1)$$

which can have the following from, for instance (2).

$$\begin{aligned} &+C_{11}.V_{1E}-C_{12}.V_{2E}-z^{-\frac{1}{2}}.C_{11}.V_{1O}+z^{-\frac{1}{2}}.C_{12}.V_{2O}=0\\ &-C_{21}.V_{1E}+C_{22}.V_{2E}+z^{-\frac{1}{2}}.C_{21}.V_{1O}-z^{-\frac{1}{2}}.C_{22}.V_{2O}=0\\ &-z^{-\frac{1}{2}}.C_{11}.V_{1E}+z^{-\frac{1}{2}}.C_{12}.V_{2E}+C_{11}.V_{1O}-C_{12}.V_{2O}=0\\ &+z^{-\frac{1}{2}}.C_{21}.V_{1E}-z^{-\frac{1}{2}}.C_{22}.V_{2E}-C_{21}.V_{1O}+C_{22}.V_{2O}=0 \end{aligned} \quad (2)$$

This system can also be illustrated by a graph, the construction of which can proceed this way: for example the last (fourth) equation can be considered in the following form (3).

$$z^{-\frac{1}{2}}.C_{21}.V_{1E}-z^{-\frac{1}{2}}.C_{22}.V_{2E}+C_{22}.V_{2O}=0 \quad (3)$$

while for the sake of clarity we laid $V_{1O}=0$, so the product $C_{21}.V_{1O}=0$ is thus zero (and will fall out of the equation). This fourth equation will be rewritten so that the fourth variable will be expressed from it.

$$C_{22}.V_{2O}=-z^{-\frac{1}{2}}.C_{21}.V_{1E}+z^{-\frac{1}{2}}.C_{22}.V_{2E} \quad (4)$$

The equation can be interpreted so that the addition to the variable V_{2O}, multiplied by the coefficient C_{22} from the variable V_{1E} has the value of C_{21} multiplied by the coefficient $-z^{-\frac{1}{2}}$, and the addition from the variable V_{2E} has the value of C_{22} multiplied by the coefficient $z^{-\frac{1}{2}}$. Therefore the very loop at the node V_{2O} has the transfer C_{22}, the branch from the node

V_{1S} to the node V_{2L} has the transfer $-z^{-\frac{1}{2}}.C_{21}$ and finally, the branch from the node V_{2S} to the node V_{2O} has the transfer $z^{-\frac{1}{2}}.C_{22}$. The above mentioned construction is shown by the graph in Fig.1.

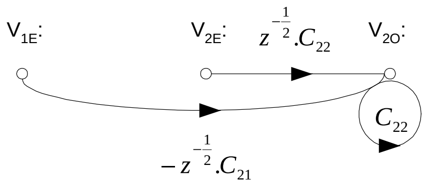

Fig. 1 graph of equation $C_{22}.V_{2O}=-z^{-1/2}.C_{21}.V_{1E}.+z^{-1/2}.C_{22}.V_{2E}$

B. Construction Generally

The above mentioned construction can be used for plotting a summary graph of the SC circuit, which will thus be plotted by first drawing the nodes and possible branches, obtained by transformation graphs for transfers of EE (even-even) and OO (odd-odd) phases, as shown in Fig.2.

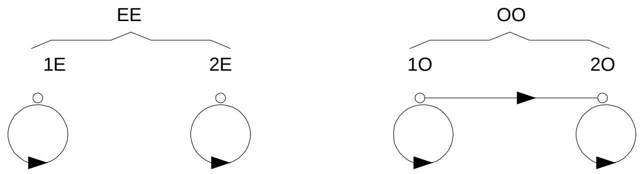

Fig. 2. the first step of the construction: EE and OO transfers

In the next step, branches obtained by transformation graphs for EO and OE phase are drawn between these nodes, while their resulting transfer is multiplied by $-z^{-\frac{1}{2}}$ or $z^{-\frac{1}{2}}$. For this reason e.g. the resulting graph with an EO phase, including its branch between nodes 1 and 2 with the transfer C_1, will be represented by a branch going from the node $1E$ to the node $2O$ in the summary graph and having the transfer $-z^{-\frac{1}{2}}.C_1$. In the same way the resulting graph in the OE phase including its own loop at node 2 with the transfer C_2 will be represented by a branch going from the node $2O$ to the node $2E$ in the summary graph and having the transfer $z^{-\frac{1}{2}}.C_2$, as shown in Fig.3.

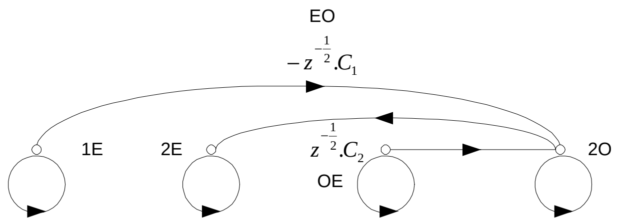

Fig. 3 the second step of the construction: EO and OE transfers

Thereby obtained summary graph is then evaluated by means of the Mason's rule for the T transfer of the graph $T=\frac{\sum p_{(i)}.\Delta_{(i)}}{V-\sum S^{(K)}.V^{(K)}}$ [2]. Therefore it is not necessary to compose a sum capacitance matrix and to express this consequently in numbers, and so it is possible to reach the final result in a graphical way.

C. Evaluation of the Transformation Graph

Evaluation of the transformation graph in OE and EO phases can proceded this way: In an equation system (1), we laid $V_O=0$, so the product is (5).

$$\begin{bmatrix} C_{EE} \\ -z^{-\frac{1}{2}}.C_{OE} \end{bmatrix}[V_E]=\begin{bmatrix} Q_E \\ Q_O \end{bmatrix} \tag{5}$$

Equation $Q_O=-z^{-\frac{1}{2}}.C_{OE}.V_E$ can be interpreted so that the addition to the variable Q_O from the variable V_E. The capacity is given by the relation $C=a^V.\tilde{C}.a^Q.\alpha$ therefore for this situation is $V=V_E$, and $Q=Q_O$, and the transfer of the voltage branch is E and transfer of the charge is O. Therefore the loop from the node C to the node $\tilde{C}$ has the transfer of the voltage ⟶ , and return to the node C has the transfer of the charge ▷ . Described construction is shown in Fig.4 for phase OE and for EO phase, too.

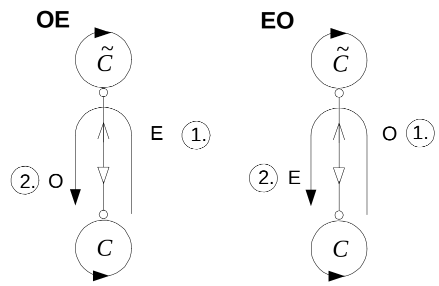

Fig. 4 the construction of the T-graph for OE and EO

III. Example of the Solving

The above described way of a graph evaluation will be illustrated by the following example. A circuit with a switched capacitor has got the schematic wiring diagram shown in Fig.5.

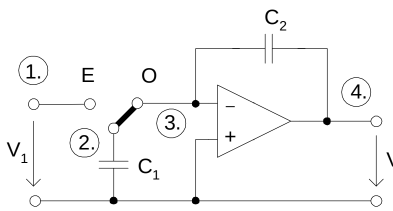

Fig. 5 schematic diagram from the example

The circuit has four nodes; therefore the starting graph of the circuit in Fig.6 has also four nodes. The C_1 capacitor is connected to the second node, the C_2 capacitor then between the third and fourth nodes, which in the simplified starting graph in Fig. 5 is marked by noting C_1 above the second node and C_2 between the third and fourth nodes. In the even-numbered EE phase nodes 1 and 2 will be connected by closing the switch, which is demonstrated in the graph by their transformation – uniting into a single node 1E.=2E. The capacity in this resulting node is given generally by the relation $C = a^V.\tilde{C}.a^Q.\alpha$, where $\tilde{C}$ is the capacity of the original node, a^V, a^Q are then the branches of the transformation graph with the transfers of voltage ⟶ and of charge ▷. Thus the resulting capacity here will be C_1.

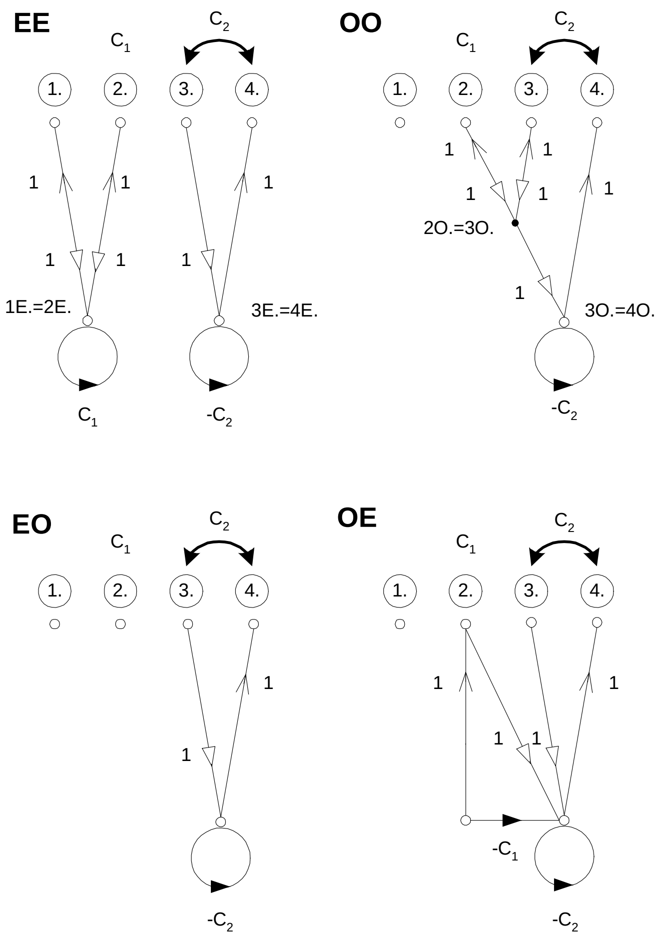

Fig.6 the transformations graphs for EE, OO, EO and OE phases

The operational amplifier is connected to the third node by its inverting input and into the fourth node by its output, and consequently the branch with the charge transfer of the transformation graph goes from the node 3, the branch with the voltage transfer of the transformation graph enters the node 4. Following this transformation graph, the capacity C_2 connected between nodes 3 and 4 then transforms into the resulting capacity of the amount $-C_2$, as the capacitor C_2 is connected to the node 3 by one of its ends, therefore the inherent look at this node has the transfer C_2 and is transformed according to the equation $C = a^V.\tilde{C}.a^Q.\alpha$. The branch between the nodes 3 and 4 with the transfer C_2 is transformed to the inherent loop with the transfer $-C_2$, because in the relation $C = a^V.\tilde{C}.a^Q.\alpha$ is now $\alpha = -1$, as the branch of the original graph converts to the inherent loop in the resulting transformed graph. In the odd phase OO by closing the switch the nodes 2 and 3 will be connected, which will demonstrate in the graph by their transformation – uniting into a single node 2O.=3O., and this resulting node is at the same time the input node of the operational amplifier.

In the remaining phases EO and OE, we start, according to the equation $C = a^V.\tilde{C}.a^Q.\alpha$, along the branch with the voltage transfer a^V from the resulting node to the original node and we enter back to the resulting node along the branch with the charge transfer a^Q. The transformation graphs for all the four cases are in Fig.6.

The summary graph obtained from the partial transformed graphs from the Fig.6 by the above mentioned procedure is then shown in Fig.7. First the results of the transformed graphs for EE and OO phases are plotted (in case of this example only) as nodes.

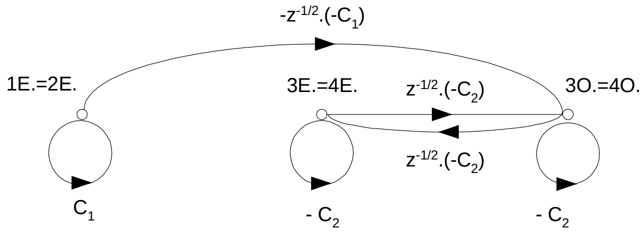

Fig.7 the summary graph of the SC circuit from Fig.5

In the next step, the results of the transformed graph for the EO and OE phases multiplied by $-z^{-\frac{1}{2}}$ or $z^{-\frac{1}{2}}$ are then drawn between these nodes as branches, i.e. the branch with the transfer $-z^{-\frac{1}{2}}.(-C_1)$ between the nodes 1E.=2E. , and 3O.=4O., and the branches with the transfers $z^{-\frac{1}{2}}(-C_2)$ between the nodes 3E.=4E., and 3O.=4O.

By evaluating this summary graph, which is done by substitution into the Mason's formula $T = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)}.V^{(K)}}$,

we get the following final results this way .

From the graph it is obvious that the entry node is: 1E or the first node in the even phase, therefore there will only be transfers from the even phase of the first node. It is further evident from the graph that the exit (i.e. fourth) node exists here both in the even phase as: 4E(4E.=3E.) and in the odd phase as: 4O(4O.=3O.). It is thus possible to express in numbers the two following transfers: $\frac{V_{4E}}{V_{1E}}$ and: $\frac{V_{4O}}{V_{1E}}$, for which it holds that:

$$\frac{V_{4E}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)}.V^{(K)}} =$$

$$= \frac{-z^{-\frac{1}{2}}.(-C_1).z^{-\frac{1}{2}}.(-C_2)}{(-C_2).(-C_2) - z^{-\frac{1}{2}}.(-C_2).z^{-\frac{1}{2}}.(-C_2)} = \frac{z^{-1}.C_1}{z^{-\frac{1}{2}}.C_2 - C_2} \quad (6)$$

and for the second one

$$\frac{V_{4O}}{V_{1E}} = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)}.V^{(K)}} =$$

$$= \frac{-z^{-\frac{1}{2}}.(-C_1).(-C_2)}{(-C_2).(-C_2) - z^{-\frac{1}{2}}.(-C_2).z^{-\frac{1}{2}}.(-C_2)} = \frac{z^{-\frac{1}{2}}.C_1}{z^{-\frac{1}{2}}.C_2 - C_2} \quad (7)$$

IV. Solving by a Matrix Method

A. Solving by Reduced Nodal Method

To compare the solution of a circuit with switched capacitors by the above mentioned purely graph method, we will present a calculation of the same circuit by the usually used method of nodal charge equations using matrix calculus in the conclusion. In doing so, this solution will be done in just as detailed steps as the graph method so that we can compare both methods. The circuit in Fig.5 has four nodes, so the partial capacitance matrix $\mathbf{C_O}$ will be of the fourth.

$$\mathbf{C_O} = \begin{array}{c} \\ 1. \\ 2. \\ 3. \\ 4. \end{array} \begin{array}{c} \begin{array}{cccc} 1. & 2. & 3. & 4. \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & -C_2 \\ 0 & 0 & -C_2 & C_2 \end{bmatrix} \end{array} \quad (8)$$

An ideal operational amplifier, connected by its input into the node 3 and by its output into the node 4, when applying a modification of the node voltage method (so called reduced nodal method [1], [2]), modifies this matrix, so the matrix will get the shape (9).

$$\begin{array}{c} \\ 1.=2. \\ 3. \end{array} \begin{array}{c} \begin{array}{cc} 1.=2. & 4. \end{array} \\ \begin{bmatrix} C_1 & 0 \\ 0 & -C_2 \end{bmatrix} \end{array} \quad (9)$$

The final capacitance matrix (10) of the circuit will be composed of four of those sub-matrices (9), while the sub-matrix lying in the adjacent diagonal of the main matrix will be multiplied by $-z^{-\frac{1}{2}}$.

$$\begin{bmatrix} C_{EE} & -z^{-\frac{1}{2}}.C_{EO} \\ -z^{-\frac{1}{2}}.C_{OE} & C_{OO} \end{bmatrix} \quad (10)$$

In the next step this matrix will be reduced due to closing the switches in the following way :

Closing the switch in the even phase will be manifested by connecting nodes 1 and 2 in the even phase, and thus by uniting the voltages V_{1E} and V_{2E}, which means that the matrix columns 1E and 2E can be summarized, and by summarizing charges, which means that the matrix rows 1E and 2E can be summarized. The switch in the odd phase connects the node 2 to the node 3, which is, though, the entry node of an ideal operational amplifier, which has however zero voltage due to the infinite voltage gain of the ideal operational amplifier, which is manifested by the possibility to leave out the 2O column from the matrix. The capacitance matrix will thus be of the three grade, and will have the following form (11).

$$\begin{array}{c} \\ 1E.=2E. \\ 3E. \\ 2O.=3O. \end{array} \begin{array}{c} \begin{array}{ccc} 1E.=2E. & 4E. & 4O. \end{array} \\ \left[\begin{array}{cc|c} C_1 & 0 & 0 \\ 0 & -C_2 & z^{-\frac{1}{2}}.C_2 \\ \hline -z^{-\frac{1}{2}}.C_1 & z^{-\frac{1}{2}}.C_2 & -C_2 \end{array}\right] \end{array} \quad (11)$$

As the circuit is stirred only in the even phase, the only non-zero input charge is the charge Q_{1E}, so only two transfers can be calculated, namely $\frac{V_{4E}}{V_{1E}}$ and $\frac{V_{4O}}{V_{1E}}$, which can be expressed by applying the algebraic complements theory like this

$$\frac{V_{4E}}{V_{1E}} = \frac{\Delta_{1E:4E}}{\Delta_{1E:1E}} = \frac{\begin{vmatrix} 0 & z^{-\frac{1}{2}}.C_2 \\ -z^{-\frac{1}{2}}.C_1 & -C_2 \end{vmatrix}.(-1)^{1+2}}{\begin{vmatrix} -C_2 & z^{-\frac{1}{2}}.C_2 \\ z^{-\frac{1}{2}}.C_2 & -C_2 \end{vmatrix}.(-1)^{1+1}} =$$

$$= \frac{-(-z^{-\frac{1}{2}}).C_1.z^{-\frac{1}{2}}.C_2.(-1)}{(-C_2).(-C_2) - z^{-\frac{1}{2}}.C_2.z^{-\frac{1}{2}}.C_2} = \frac{z^{-1}.C_1}{z^{-1}.C_2 - C_2} \quad (12)$$

and for the second one

$$\frac{V_{4O}}{V_{1E}} = \frac{\Delta_{1E:4O}}{\Delta_{1E:1E}} = \frac{\begin{vmatrix} 0 & C_2 \\ -z^{-\frac{1}{2}}.C_1 & -z^{-\frac{1}{2}}.C_2 \end{vmatrix}.(-1)^{1+3}}{\begin{vmatrix} -C_2 & z^{-\frac{1}{2}}.C_2 \\ z^{-\frac{1}{2}}.C_2 & -C_2 \end{vmatrix}.(-1)^{1+1}} =$$

$$= \frac{-z^{-\frac{1}{2}}.C_1.C_2}{(-C_2).(-C_2) - z^{-\frac{1}{2}}.C_2.z^{-\frac{1}{2}}.C_2} = \frac{z^{-\frac{1}{2}}.C_1}{z^{-1}.C_2 - C_2} \qquad (13)$$

B. Solving by Modified Nodal Method

This solution will be done in just as detailed steps as the graph method so that we can compare both methods, too.

The circuit in Fig.5 has four nodes, so the partial capacitance matrix $\mathbf{C_O}$ will be of the fourth.

$$\mathbf{C_O} = \begin{array}{r} \\ 1. \\ 2. \\ 3. \\ 4. \end{array} \begin{array}{c} \begin{array}{cccc} 1. & 2. & 3. & 4. \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & -C_2 \\ 0 & 0 & -C_2 & C_2 \end{bmatrix} \end{array} \qquad (14)$$

An ideal operational amplifier, connected by its input into the node 3 and by its output into the node 4, modifies this matrix, so the matrix will get the shape (15).

V. Solving Circuits Considering Operational Amplifier with Diffraction of its Frequency Characteristics of Amplification

A. Construction of the Transformation Graph with the Break Point Frequencies

Nevertheless, the voltage amplification decrease itself is caused by the input resistances h_{11} and the capacity between the base and the collector of transistors C_{BC} which the operational amplifier is made up of and which create the integrated RC cells.

The ratio of the output to input voltage of the integrated cell is (14).

$$\frac{V_2}{V_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1} \qquad (14)$$

For the threshold frequency ω_M it holds that $RC = \frac{1}{\omega_M}$, so after substituting we get $\frac{V_2}{V_1} = \frac{1}{j\frac{\omega}{\omega_M} + 1} = \frac{\omega_M}{j\omega + \omega_M}$ for a passive cell. The active cell has the amplification A, so $\frac{V_2}{V_1} = \frac{A.\omega_M}{j\omega + \omega_M}$ and for $\omega >> \omega_M$ this relation can be simplified to $\frac{V_2}{V_1} = \frac{A.\omega_M}{j\omega}$. At the transient frequency of the operational amplifier then $\frac{V_2}{V_1} = 1$ so it holds that $\frac{\omega_M}{\omega_T}.A = 1$, which implies that $\omega_M = \frac{\omega_T}{A}$. By introducing this frequency the relation for the ratio of the output and input voltages will get the shape

$$\frac{V_2}{V_1} = \frac{A.\omega_M}{j\omega} = \frac{A.\frac{\omega_T}{A}}{\frac{\omega_T}{A} + j\omega} = \frac{\omega_T}{\frac{\omega_T + j\omega A}{A}} = \frac{A.\omega_T}{\omega_T + sA} \qquad (15)$$

If both the break point frequencies on amplification are considered, the second frequency ω_2, coming after the frequency ω_T (when $\frac{V_2}{V_1} = 1$), is expressed by another integrated cell, for whose output to input voltage ratio it holds that $\frac{\omega_2}{\omega_2 + s}$, which together with the first frequency gives the resulting relation (16).

$$\frac{V_2}{V_1} = \frac{A.\omega_T}{\omega_T + sA}.\frac{\omega_2}{\omega_2 + s} \qquad (16)$$

These relations are then used as transfers of voltage of the branches of the transformation graph, where amplification occurs, for example the graph of an operational amplifier considering two break point frequencies for an inverting input in the node $\tilde{1}$ and an output in the node $\tilde{2}$ is in Fig. 8a, and the graph of a differential operational amplifier concerning a single break point frequency on the amplification characteristics in an open feedback loop in the Fig.8b.

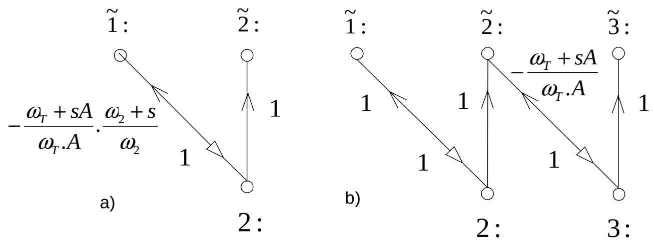

Fig. 8 transformation graphs concerning break point frequencies on the amplification characteristic

Since an operational amplifier in a switched circuit processes high frequencies, the frequency characteristics of its amplification is taken into account by considering either one frequency $A = \frac{A_0.\omega_T}{\omega_T + sA}$ or both frequencies $A = \frac{A_0.\omega_T}{\omega_T + sA}.\frac{\omega_2}{\omega_2 + s}$ of its diffraction, when A_0 is the maximum value of amplification in an open loop of feedback.

B. Example

The described way of calculation will be illustrated by an example of solution of a circuit with a switched capacitor whose schematic wiring diagram is shown in Fig.5.

The circuit in Fig.5 has four nodes; therefore the starting graph of the circuit in Fig. 9 has four nodes, too.

The operational amplifier is connected to the third node by its inverting input and into the fourth node by its output, and consequently the branch with the charge transfer of the transformation graph goes from the node 3, the branch with the voltage transfer of the transformation graph enters the node 4 and the branch with the voltage transfer expressing the final amplification of the operational amplifier by the value $\frac{1}{A}$ enters the node 3.

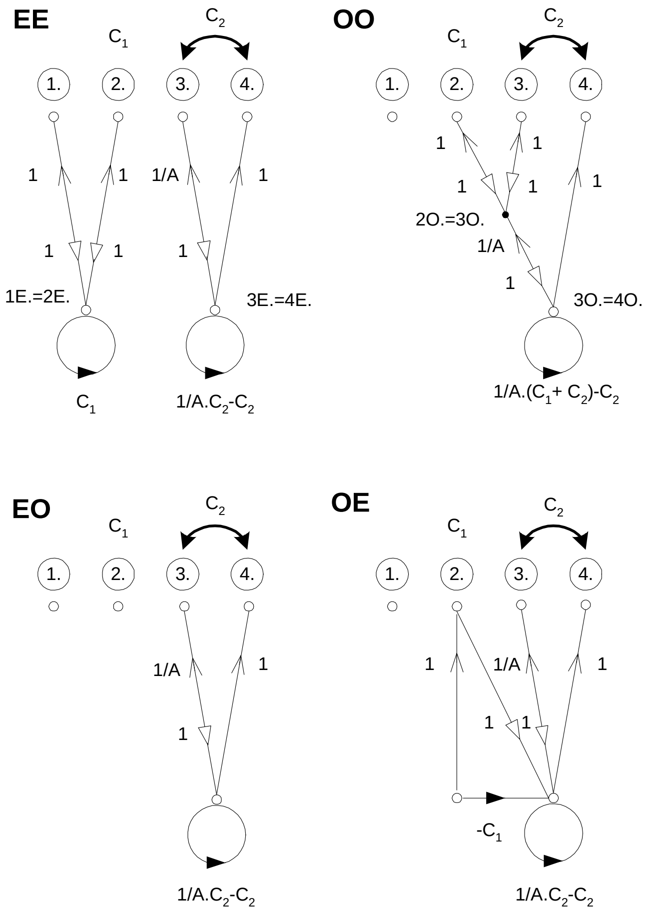

Fig.9 the transformations graphs for EE, OO, EO and OE phases

Following this transformation graph, the capacity C_2 connected between nodes 3 and 4 then transforms into the resulting capacity of the amount $\frac{1}{A}.C_2 - C_2$, as the capacitor C_2 is connected to the node 3 by one of its ends, therefore the inherent look at this node has the transfer C_2 and is transformed according to the equation $C = a^V.\tilde{C}.a^Q.\alpha$, to the inherent loop $\frac{1}{A}.C_2$, where $a^V = \frac{1}{A}$.

The branch between the nodes 3 and 4 with the transfer C_2 is transformed to the inherent loop with the transfer $-C_2$, because in the relation $C = a^V.\tilde{C}.a^Q.\alpha$ is now $\alpha = -1$, as the branch of the original graph converts to the inherent loop in the resulting transformed graph. In the odd phase OO by closing the switch the nodes 2 and 3 will be connected, which will demonstrate in the graph by their transformation – uniting into a single node 2O.=3O., and this resulting node is at the same time the input node of the operational amplifier. Therefore a branch with the charge transfer a^Q of the transformation graph of the operational amplifier issues from this node.

In the remaining phases EO and OE, we start, according to the equation $C = a^V.\tilde{C}.a^Q.\alpha$, along the branch with the voltage transfer a^V from the resulting node to the original node and we enter back to the resulting node along the branch with the charge transfer a^Q. The transformation graphs for all the four cases are in Fig. 5.

The summary graph obtained from the partial transformed graphs from the Fig. 5 by the above mentioned procedure is then shown in Fig. 10. First the results of the transformed graphs for EE and OO phases are plotted (in case of this example only) as nodes.

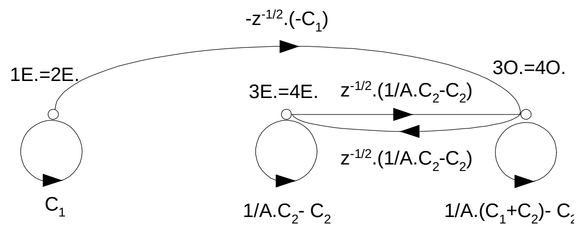

Fig.10 the summary graph of the SC circuit from Fig.5 for real operational amplifier

In the next step, the results of the transformed graph for the EO and OE phases multiplied by $-z^{-\frac{1}{2}}$ or $z^{-\frac{1}{2}}$ are then drawn between these nodes as branches, i.e. the branch with the transfer $-z^{-\frac{1}{2}}.(-C_1)$ between the nodes 1E.=2E. and 3O.=4O. and the branches with the transfers $z^{-\frac{1}{2}}.(\frac{1}{A}C_2 - C_2)$ between the nodes 3E.=4E. and 3O.=4O.

By evaluating this summary graph which is done by substitution into the Mason's formula $T = \frac{\sum p_{(i)}.\Delta_{(i)}}{V - \sum S^{(K)}.V^{(K)}}$ we get the following final results this way:

From the graph it is obvious that the entry node is: 1E or the first node in the even phase, therefore there will only be transfers from the even phase of the first node. It is further evident from the graph that the exit (i.e. fourth) node exists here both in the even phase as: 4E (4E.=3E.) and in the odd phase as: 4O (4O.=3O.). It is thus possible to express in numbers the two following transfers $\frac{V_{4E}}{V_{1E}}$ and $\frac{V_{4O}}{V_{1E}}$, for which it holds that:

$$\frac{V_{4E}}{V_{1E}} = \frac{\sum p_{(i)}\Delta_{(i)}}{V - \sum S^{(K)}V^{(K)}} =$$

$$= \frac{-z^{-\frac{1}{2}}(-C_1).z^{-\frac{1}{2}}(\frac{1}{A}C_2 - C_2)}{(\frac{1}{A}C_2 - C_2)\left[\frac{1}{A}(C_1 + C_2) - C_2\right] - \left[z^{-\frac{1}{2}}(\frac{1}{A}C_2 - C_2)z^{-\frac{1}{2}}(\frac{1}{A}C_2 - C_2)\right]} =$$

where V_{IJ} is the voltage and V or $V^{(K)}$ is transfer of the loop, by canceling out and removing the complex fractions

$$= \frac{C_1 A}{z^1(C_1 + C_2 - AC_2) - C_2 + AC_2} \tag{17}$$

and

$$\frac{V_{4O}}{V_{1E}} = \frac{\sum p_{(i)}\Delta_{(i)}}{V - \sum S^{(K)}V^{(K)}} =$$

$$= \frac{-z^{-\frac{1}{2}}(-C_1).(\frac{1}{A}C_2 - C_2)}{(\frac{1}{A}C_2 - C_2)\left[\frac{1}{A}(C_1 + C_2) - C_2\right] - \left[z^{-\frac{1}{2}}(\frac{1}{A}C_2 - C_2)z^{-\frac{1}{2}}(\frac{1}{A}C_2 - C_2)\right]} =$$

$$= \frac{z^{-\frac{1}{2}}C_1 A}{z^1(C_1 + C_2 - AC_2) - C_2 + AC_2} \tag{18}$$

By substituting $A = \frac{A_0.\omega_T}{\omega_T + sA}$ or $A = \frac{A_0.\omega_T}{\omega_T + sA}.\frac{\omega_2}{\omega_2 + s}$ we can calculate the frequency dependence.

VI. Solving by a Matrix Method

To compare the solution of a circuit with switched capacitors by the above mentioned purely graph method, we will present a calculation of the same circuit by the usually used method of nodal charge equations using matrix calculus in the conclusion. In doing so, this solution will be done in just as detailed steps as the graph method so that we can compare both methods.

The circuit in Fig. 5 has four nodes, so the partial capacitance matrix $\mathbf{C_O}$ will be of the fourth grade and after recording by an algorithm analogous to the node voltage method it will have the following form (19).

$$\mathbf{C_O} = \begin{array}{c c} & \begin{array}{cccc} 1. & 2. & 3. & 4. \end{array} \\ \begin{array}{c} 1. \\ 2. \\ 3. \\ 4. \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & -C_2 \\ 0 & 0 & -C_2 & C_2 \end{bmatrix} \end{array} \tag{19}$$

An operational amplifier, connected by its input into the node 3 and by its output into the node 4, when applying a modification of the node voltage method (so called forbidden line method), modifies this matrix so that it replaces its line with the index corresponding to the index of the exit node of the operational amplifier (here the fourth line) by its coupling equation $V_4 = AV_3$, rewritten to the shape $AV_3 - 1.V_4 = 0$, so the matrix will get the shape (20).

$$\mathbf{C_O} = \begin{array}{c c} & \begin{array}{cccc} 1. & 2. & 3. & 4. \end{array} \\ \begin{array}{c} 1. \\ 2. \\ 3. \\ 4. \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 \\ 0 & 0 & C_2 & -C_2 \\ 0 & 0 & A & -1 \end{bmatrix} \end{array} \tag{20}$$

The capacitance matrix of the circuit will be composed of four of those sub-matrices, while the sub-matrix lying in the adjacent diagonal of the main matrix will be multiplied by $-z^{-\frac{1}{2}}$, which means that all elements of these sub-matrices will be multiplied by $-z^{-\frac{1}{2}}$. The capacitance matrix will thus be of the eighth grade and will have the following form (21).

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & -z^{-\frac{1}{2}}.C_1 & 0 & 0 \\ 0 & 0 & C_2 & -C_2 & 0 & 0 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 \\ 0 & 0 & A & -1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -z^{-\frac{1}{2}}.C_1 & 0 & 0 & 0 & C_1 & 0 & 0 \\ 0 & 0 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 & 0 & 0 & C_2 & -C_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & A & -1 \end{array}\right] \tag{21}$$

In the next step this matrix will be reduced due to closing the switches in the following way:

Closing the switch in the even phase will be manifested by connecting nodes 1 and 2 in the even phase, and thus by uniting the voltages V_{1E} and V_{2E}, which means that the matrix columns 1E and 2E can be summarized, and by summarizing charges, which means that the matrix rows 1E and 2E can be summarized.

The switch in the odd phase connects the node 2 to the node 3, which is, though, the entry node of an ideal operational amplifier, which has however zero voltage due to the infinite voltage gain of the ideal operational amplifier, which is manifested by the possibility to leave out the 2O column from the matrix.

In the odd phase the circuit is also disconnected by the switch from the entry node 1 and that will be shown in the matrix by the possibility to leave out both the row and the column 1O.

After the above described reductions, the resulting matrix **C** will get the following form (22).

$$
\begin{array}{c} \\ 1E=2E \\ 3E \\ 4E \\ \\ 2O=3O \\ 4O \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1E=2E & 3E & 4E & 2O=3O & 4O \end{array} \\
\left[\begin{array}{ccc|cc}
C_1 & 0 & 0 & -z^{-\frac{1}{2}}.C_1 & 0 \\
0 & C_2 & -C_2 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 \\
0 & A & -1 & 0 & 0 \\
\hline
-z^{-\frac{1}{2}}.C_1 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 & C_1+C_2 & -C_2 \\
0 & 0 & 0 & A & -1
\end{array}\right]
\end{array} \quad (22)
$$

As the circuit is stirred only in the even phase, the only non-zero input charge is the charge Q_{1E}, so only two transfers can be calculated, namely $\frac{V_{4E}}{V_{1E}}$ and $\frac{V_{4O}}{V_{1E}}$, which can be expressed by applying the algebraic complements theory like this

$$
\frac{V_{4E}}{V_{1E}} = \frac{\Delta_{1:3}}{\Delta_{1:1}} = \frac{\begin{vmatrix} 0 & C_2 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 \\ 0 & A & 0 & 0 \\ -z^{-\frac{1}{2}}.C_1 & -z^{-\frac{1}{2}}.C_2 & C_1+C_2 & -C_2 \\ 0 & 0 & A & -1 \end{vmatrix}.(-1)^{1+3}}{\begin{vmatrix} C_2 & -C_2 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 \\ A & -1 & 0 & 0 \\ -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 & C_1+C_2 & -C_2 \\ 0 & 0 & A & -1 \end{vmatrix}.(-1)^{1+1}} =
$$

after numeration by an expansion along the item of the last row and quite an elaborate simplification the result will be

$$
= \frac{C_1.A}{z.(C_1+C_2-A.C_2)-C_2+A.C_2} \quad (23)
$$

and for the second one

$$
\frac{V_{4O}}{V_{1E}} = \frac{\Delta_{1:5}}{\Delta_{1:1}} = \frac{\begin{vmatrix} 0 & C_2 & -C_2 & -z^{-\frac{1}{2}}.C_2 \\ 0 & A & -1 & 0 \\ -z^{-\frac{1}{2}}.C_1 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 & C_1+C_2 \\ 0 & 0 & 0 & A \end{vmatrix}.(-1)^{1+5}}{\begin{vmatrix} C_2 & -C_2 & -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 \\ A & -1 & 0 & 0 \\ -z^{-\frac{1}{2}}.C_2 & z^{-\frac{1}{2}}.C_2 & C_1+C_2 & -C_2 \\ 0 & 0 & A & -1 \end{vmatrix}.(-1)^{1+1}} =
$$

after numeration by an expansion along the item of the last row and quite an elaborate simplification the result will be

$$
= \frac{z^{-\frac{1}{2}}.C_1.A}{z.(C_1+C_2-A.C_2)-C_2+A.C_2} \quad (24)
$$

i.e. identical with the result obtained by the transformation graph method, but rather more difficult.

By substituting $A = \frac{A_0.\omega_T}{\omega_T + sA}$ or $A = \frac{A_0.\omega_T}{\omega_T + sA}.\frac{\omega_2}{\omega_2 + s}$ the frequency dependence can be calculated again.

VII. Conclusion

While in case of using the graph method a graph was indicated, a transformation graph was plotted and from its results a summary graph was drawn and evaluated by the Mason's rule, after which the result was obtained by an easy simplification, in case of solving by the matrix calculus the procedure was much more complicated. First a partial capacitance matrix had to be composed, in the next step it was modified by an operational amplifier. From four matrices obtained by this a capacitance matrix was constructed and was reduced by the activity of switches; from the reduced matrix three algebraic complements were made up and they had to be expressed by means of an expansion because they were of a higher grade than 3. After an elaborate simplification in four steps, the same result was reached. In case of using the graph method a graph was solving, but modified nodal method is rather difficult.

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