A Design Method and Algorithm for USBL Systems with Skew Three-element Arrays

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***Abstract*—** This paper presents a design method and coordinate determination algorithm for ultra-short baseline (USBL) systems where the coordinates of the underwater object are determined with utilization both the orthogonal and non-orthogonal (skew) elemental (three-element) USBL arrays. In the article a five-element USBL receiving array is studied. The proposed design of five-element receiving array allows to have six orthogonal (four horizontal and two vertical) three-element USBL arrays and the four inclined skew three-element USBL arrays. The case of calculation of the Cartesian coordinates of the object in the reference coordinate system bounded up with the USBL system's carrier is considered. The proposed design method and the algorithm are based on the determination of the object's position on the basic three-element USBL receiving arrays (orthogonal and non-orthogonal) with the following averaging these results by applying multiple rotations of the elemental three-element arrays around the horizontal and vertical axes associated with the carrier coordinate system. It is supposed that the spatial orientation of the receiving USBL array is controlled by the measurement of its pitch and roll angles. The coordinate determination algorithm for the proposed USBL system is designed and tested with the assumption that the object can have arbitrary position in the lower hemisphere and the USBL array can have significant inclination.

***Keywords*—**Ultra-short baseline (USBL) system, underwater object, transponder, carrier coordinate system, local coordinate system, skew coordinate system, pitch and roll angles.

I. INTRODUCTION

THE central problem of the USBL system design is the underwater object location determination with high accuracy in real marine conditions. The principle of operating of the USBL systems is well known and is described in detail in [1,2]. Object position determination with this method is realized by means of the measuring of the distance to the object and its angular position relative to the measuring system location. During the last decades numerous studies and investigations for improvements in accuracy and reliability of object position determination with use of the USBL systems were realized [3-7]. To improve the reliability of the USBL systems various special signal processing techniques were employed. In particular the chirp signals and greater inter-element array separation [3] were used. Also the acoustic digital spread spectrum [4] and modulated Barker-coded signals [5] were applied. In [6] the USBL system with frequency-hopped pulses was investigated. The problem of improvement of accuracy in the case of instability of the position of the receiving USBL array was studied in detail in [7]. In [8,9] the problem of low precision in coordinate determination for the case of the object found in the plane of receiving bases is studied. In [10,11] the design method of multi-element USBL systems is proposed. The idea of this design method was to increase the reliability and accuracy of coordinate determination with use of the supplementary receiving elements of the USBL array.

The idea of the design method in the present article is to use not only orthogonal three-element arrays of the receiving antenna but as well the non-orthogonal (skew) three-element arrays of the receiving antenna.

II. BASIC USBL SYSTEM

The measuring of the object coordinates is realized as the following. The USBL system transmitter sends an interrogation acoustical impulse in the propagation medium where the object is located at an accessible distance. The object must be equipped with a transponder that receives the interrogation impulse and sends an acoustical impulse in reply. The distance to the object is determined by the measurement of the values of propagation times of the interrogation impulse and the transponder response pulse. The angular position of the object is determined by the measurement of the phase difference of the transponder pulse carrier frequency on the receiving array outputs. The minimum number of receiving elements for the USBL system for object coordinate determination is three [1]. To improve the reliability and accuracy of coordinate determination the number of elements of receiving USBL array can be increased. In general the propagation medium is non-homogeneous. Furthermore, in this paper we assume that the propagation medium is homogeneous and the multipath interference is absent.

We consider briefly the principle of coordinate determination for the case of the three-element orthogonal USBL array.

Let $\Sigma=(0,x,y,z)$ be the sea-surface associated reference frame with the origin in the point O (see Fig.1). Let the plane xy coincide with the sea surface (it is supposed that the sea surface is not perturbed) and the z axis goes downward. It is supposed that the carrier coordinate system $\Sigma_{carrier}= (0,x_{carrier}, y_{carrier}, z_{carrier})$ coincides with the $\Sigma=(0,x,y,z)$ when the carrier does not have pitch and roll inclinations. Also it is supposed that the x-coordinate axis coincides with the carrier longitudinal axis *L-L'* (the positive direction coincides with the direction of the straight arrowed line), the *y*-coordinate coincides with the carrier lateral axis *B-B'*, and *z* axis goes downwards. Now we can define the USBL array orientation in the introduced carrier coordinate system. Let $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ be the local coordinate system for the considered USBL system (with receiving elements 1,2,3). It is assumed that the origin of the USBL coordinate system coincides with the origin of the carrier coordinate system, the angle between the y-axis and base 1-2 is 135°, and the angle between the y-axis and the base 3-2 is 45°. It is also supposed that the receiving USBL array is rigidly mounted on the carrier hull.

The geometry of the introduced sea-surface associated coordinate system $\Sigma=(0,x,y,z)$ and the receiving three-element USBL coordinate system $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ is presented in Fig.1. The reference grid is shown only for the Σ_{123} coordinate system.

In Fig.1 we specify the angles (α, β and γ) that define the position of the underwater object (located in point P) in the $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ coordinate system. We also assume that the USBL system is equipped with a special unit to measure the pitch and roll angles of the carrier (the pitch and roll angles of the USBL array are the same as for the carrier). Let angles ξ and ζ be pitch and roll angles corresponding to the receiving USBL antenna (in the figure these angles show the rotations relative to the carrier lateral B-B' axis and the carrier longitudinal *L-L'* axis).

Let that the interrogation impulse has been sent and the reply impulse is being received by antenna. The distance to the object is defined by measuring the propagation times of the interrogation and reply pulses.

Time delays on receiving elements define the object's angular position. The time delays τ_{12} and τ_{32} of the signal on the outputs of the receiving elements of the base 1-2 and the base 3-2 (it is supposed that $R>>d$) can be expressed in the following way: $\tau_{12}=(d/c)\cos\beta$ and $\tau_{32}=(d/c)\cos\alpha$, where *c* is the speed of the sound in the water.

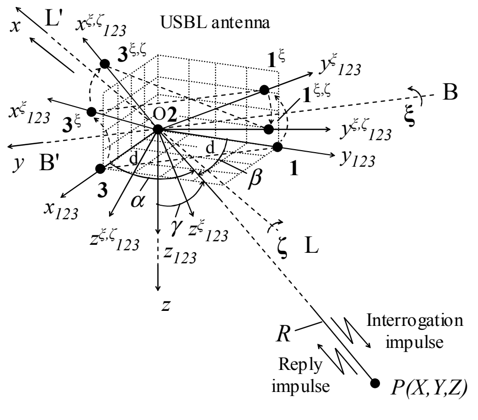

Fig.1. Geometry of the receiving antenna and the carrier longitudinal and lateral axes

With d/c defined as τ_d we can write the direction cosines $\cos\alpha=\tau_{32}/\tau_d$ and $\cos\beta=\tau_{12}/\tau_d$. The third direction cosine is defined as: $\cos\gamma=\sqrt{1-(\tau_{12}/\tau_d)^2-(\tau_{32}/\tau_d)^2}$. Cartesian coordinates of the point *P* in the $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ coordinate system are: $X_{123}=R\cos\alpha$, $Y_{123}=R\cos\beta$, $Z_{123}=R\cos\gamma$. To obtain the coordinates of point *P* in the $\Sigma=(0,x,y,z)$ coordinate system (point $P(X,Y,Z)$ in Fig.1) it is necessary to carry out the corresponding transformation of obtained coordinates X_{123}, Y_{123}, Z_{123}.

Let the pitch and roll rotations of the receiving antenna take place. The pitch and roll rotations of the elemental three-element USBL antenna are shown in Fig.1. After the first rotation on pitch angle ξ relative to B-B' axis the receiving elements are displaced to points 1^{ξ} and 3^{ξ} respectively. After the second rotation on the angle ζ relatively *L-L'* axis the receiving elements are displaced to point $1^{\xi,\zeta}$ and $3^{\xi,\zeta}$ respectively (see Fig.1). With the rotations of receiving bases the corresponding transformations of coordinate systems from $\Sigma_{123}=(0,x_{123},y_{123},z_{123})$ to $\Sigma^{\xi}_{123}=(0,x^{\xi}_{123},y^{\xi}_{123},z^{\xi}_{123})$ and to $\Sigma^{\xi,\zeta}_{123}=(0,x^{\xi,\zeta}_{123}, y^{\xi,\zeta}_{123}, z^{\xi,\zeta}_{123})$ have taken place.

Calculation expressions for the case of the pitch and roll of the three-element receiving antenna with introduced orientation relative to the carrier have been obtained in [8,9]. So we describe the calculation procedure here very briefly.

If we introduce vectors: $\boldsymbol{p}^{\xi,\zeta}_{123}=[X^{\xi,\zeta}_{123},\ Y^{\xi,\zeta}_{123},\ Z^{\xi,\zeta}_{123}]^T$ ($[X^{\xi,\zeta}_{123},\ Y^{\xi,\zeta}_{123},\ Z^{\xi,\zeta}_{123}]^T$ represents the transpose of the vector $[X^{\xi,\zeta}_{123},\ Y^{\xi,\zeta}_{123},\ Z^{\xi,\zeta}_{123}]$) and $\boldsymbol{p}^{\xi}_{123}=[X^{\xi}_{123},\ Y^{\xi}_{123},\ Z^{\xi}_{123}]^T$ (vector $\boldsymbol{p}^{\xi,\zeta}_{123}$ represents the coordinates of the object in $\Sigma^{\xi,\zeta}_{123}$ coordinate system and vector $\boldsymbol{p}^{\xi}_{123}$ represents the coordinates of the object in Σ^{ξ}_{123} coordinate system) and the transformation matrix $\boldsymbol{B}=\boldsymbol{B}[\zeta,,\eta_{123},,\chi_{123},,\nu_{123}]$ with direction cosines $\eta_{123}=\cos<(x^{\xi}_{123},L)$, $\chi_{123}=\cos<(y^{\xi}_{123},L)$, $\nu_{123}.=\cos<(z^{\xi}_{123},L)$ (matrix $\boldsymbol{B}$ transforms vector $\boldsymbol{p}^{\xi}_{123}$ to vector $\boldsymbol{p}^{\xi,\zeta}_{123}$) we can write the equation: $\boldsymbol{p}^{\xi}_{123}=\boldsymbol{B}^{-1}\boldsymbol{p}^{\xi,\zeta}_{123}$.

If we introduce vector $\boldsymbol{p}_{123}=[X_{123},\ Y_{123},\ Z_{123}]^T$ (vector $\boldsymbol{p}^{\xi}_{123}$ represents the coordinates of the object in Σ_{123} coordinate system) and the transformation matrix $\boldsymbol{A}=\ \boldsymbol{A}[\xi,\eta_{123},\chi_{123},\nu_{123}]$ with direction cosines $\eta_{123}=\cos<(x_{123},B)$, $\chi_{123}=\cos<(y_{123},B)$, $\nu_{123}=\cos<(z_{123},B)$ (matrix $\boldsymbol{A}$ transforms vector $\boldsymbol{p}_{123}$ to vector $\boldsymbol{p}^{\xi}_{123}$) we can write the equation: $\boldsymbol{p}_{123}=\boldsymbol{A}^{-1}\boldsymbol{p}^{\xi}_{123}$. These two transformations can be combined in the equation $\boldsymbol{p}_{123}=\ \boldsymbol{A}^{-1}\boldsymbol{B}^{-1}\boldsymbol{p}^{\xi,\zeta}_{123}$.

In order to obtain the coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate system (we will have the same coordinates of the object in the carrier coordinate system $\Sigma_{carrier}=(0,x_{carrier},y_{carrier},z_{carrier})$ and in the sea-surface associated coordinate system $\Sigma=(0,x,y,z)$ if the carrier is not inclined) it is necessary to make one more rotation of the coordinate system Σ_{123} around the axis *z* on the angle of 135° (see Fig.1). For the $\Sigma=(0,x,y,z)$ coordinate system, we have the following direction cosines for the z-axis: $\cos(x,z)=0$, $\cos(y,z)=0$, $\cos(z,\ z)=1$. If we introduce vector $\boldsymbol{p}=[X,\ Y,\ Z]^T$ (vector $\boldsymbol{p}$ represents the coordinates of the object in $\Sigma=(0,x,y,z)$ coordinate system) and the transformation matrix $\boldsymbol{C}$ (matrix $\boldsymbol{C}$ transforms vector $\boldsymbol{p}$ to vector $\boldsymbol{p}_{123}$) the final equation to find vector $\boldsymbol{p}$ will be the following: $\boldsymbol{p}=\boldsymbol{C}^{-1}\boldsymbol{A}^{-1}\boldsymbol{B}^{-1}\boldsymbol{p}^{\xi,\zeta}{}_{123}$. Thus to find the coordinates in the $\Sigma=(0,x,y,z)$ coordinate system we write the final matrix equation:

$$\boldsymbol{p}=\boldsymbol{C}^{-1}\boldsymbol{A}^{-1}\boldsymbol{B}^{-1}\boldsymbol{p}^{\xi,\zeta}{}_{123}. \tag{1}$$

III. FIVE-ELEMENT USBL SYSTEM

Articles [8,9] noted the problem of the low precision coordinate determination for the cases when the controlled object is found in planes of receiving bases of elemental USBL systems. To resolve this problem a five-element USBL system with different spatial orientation of receiving bases was proposed. Also the special algorithm was designed and investigated and significant accuracy improvement was obtained.

In [10,11] the additional modernization of the antenna construction and improvement of the coordinate determination algorithm were proposed. In particular the number of transducers of the receiving array was increased to nine. As a result the new USBL array had a larger number of elemental USBL arrays with various spatial orientations and a higher reliability of system was obtained.

The main goal of present investigation is to check the possibility of obtaining the equivalent results with a smaller number of receiving array elements. If we analyze the construction of the five-element USBL array investigated in [8],[9] we can observe that besides having orthogonal basic three-element arrays this USBL antenna also has a non-orthogonal basic three-element arrays. We can also observe that these non-orthogonal USBL arrays have different spatial orientations similar to the orientations of the additional orthogonal three-element USBL arrays of the nine-element array (mentioned above and have been investigated in [10] and [11]).

IV. SKEW THREE-ELEMENT USBL SYSTEM

We will investigate the proposed design method for the five-element USBL array that has been studied in [8,9]. The construction of the foregoing five-element USBL array is shown in Fig.2. The USBL array has the following orthogonal three-element USBL systems: $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{153}$, $USBL_{254}$. The additional non-orthogonal three-element USBL systems presented in this array are: $USBL_{152}$, $USBL_{253}$, $USBL_{354}$, $USBL_{451}$ (in this particular case all these non-orthogonal three-element arrays form equilateral triangles). Our proposal is to utilize these non-orthogonal (skew) three-element arrays as a part of the USBL measuring system. We will demonstrate the proposed method in detail only for the case of the skew $USBL_{152}$ system. For the rest of the skew three-element USBL systems the sequence of necessary operational steps will be similar.

Let $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ be the local coordinate system for the considered $USBL_{152}$ system (with receiving elements 1,5,2). The origin of the Σ_{152} coordinate system is located at the point of the placement of receiving element number 5.

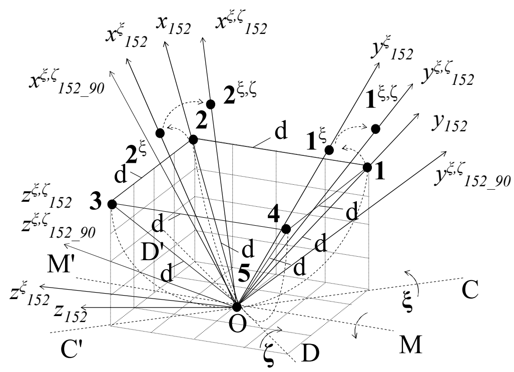

Fig.2. Five-element USBL array

Before representing the sequence of algorithm steps we will introduce two new axes to obtain easier examination of $USBL_{152}$ array pitch and roll inclinations. Let the *C-C′* be the axis that is parallel to the lateral axis *B-B′* and let the axis *C-C′* pass through the point 5 (the point of location of the receiving element 5). Let the *D-D′* be the axis that is parallel to the longitudinal axis *L-L′* and let the axis *D-D′* pass through the point 5 (the point of location of the receiving element 5). After the introduction of these axes the rotation of the $USBL_{152}$ array around the lateral axis *B-B′* on the pitch angle can be replaced by the rotation around the axis *C-C′* and the rotation around the longitudinal axis *L-L′* on the roll angle can be replaced by the rotation around the axis *D-D′*. For further consideration all rotations of the $USBL_{152}$ coordinate system will take place around its origin.

V. CALCULATION EXPRESSIONS FOR SKEW THREE-ELEMENT USBL SYSTEM

We will consider the derivation of the calculation expressions for $USBL_{152}$ system in detail. Then we will present the expressions for the other skew three-element USBL systems ($USBL_{253}$, $USBL_{354}$, $USBL_{451}$).

For convenience to get the calculation expressions we will firstly consider the case of location of the three-element $USBL_{152}$ system on a horizontal plane. Let $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ be the local skew coordinate system for the considered USBL system. The angle between the axis x_{152}, and axis y_{152} is defined by the elected design of receiving antenna. We will designate this angle as ν (for the proposed antenna design ν=60°). The origin of the coordinate system coincides with the receiving element 5. We also assume that the axis z forms a right angle with the axes x_{152}, and y_{152}. We will suppose that the object is located in the point P. Geometry of the receiving antenna and location of the object are represented in Fig.3.

Direction cosines $\cos\alpha$ and $\cos\beta$ are defined in the same way as in the case of orthogonal axes:

$$\cos\alpha=\tau_{25}/\tau_d\ ; \quad \cos\beta=\tau_{15}/\tau_d. \tag{2}$$

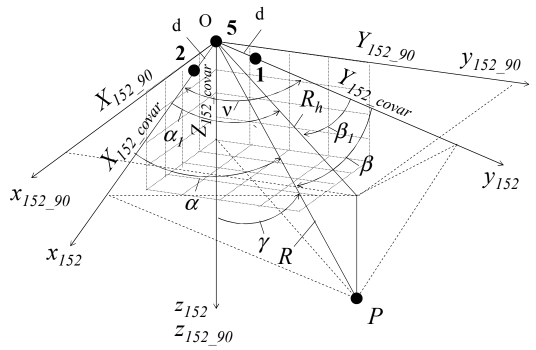

Fig.3. Skew three-element USBL array

To obtain the third direction cosine *cosγ* (or angle *γ*) we need to solve the next system of equations:

$$R_h = R\cos\alpha/\cos\alpha_1;$$
$$R_h = R\cos\beta/\cos\beta_1;$$
$$R_h = R\sin\gamma;$$
$$\alpha_1+\beta_1=\nu, \tag{3}$$

where R_h is the distance to the object in horizontal plane; α_1 and β_1 are the angles that define the angular position of the object in the horizontal plane.

For the solution of (3) we can write the expression for α_1 and β_1 as follows:

$$\alpha_1 = arctg\left[\frac{1}{\sin\nu}\left(\frac{\cos\beta}{\cos\alpha}-\cos\nu\right)\right];$$
$$\beta_1 = arctg\left[\frac{1}{\sin\nu}\left(\frac{\cos\alpha}{\cos\beta}-\cos\nu\right)\right]. \tag{4}$$

The angle γ is defined as follows:

$$\gamma = arcsin\frac{cos\alpha}{cos\alpha_1} = arcsin\frac{cos\beta}{cos\beta_1}. \tag{5}$$

The covariant coordinates X_{152_covar} , Y_{152_covar} and Z_{152_covar} in skew coordinate system $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ are defined by the expressions:

$$X_{152\_covar} = R\cos\alpha;$$
$$Y_{152\_covar} = R\cos\beta;$$
$$Z_{152\_covar} = R\cos\gamma. \tag{6}$$

The contravariant coordinates X_{152_ccntr} , Y_{152_ccntr} and Z_{152_contr} in skew coordinate system $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ are defined by the expressions:

$$X_{152\_ccntr} = X_{152\_covar} - R\cos\alpha\ tg\alpha_1\ ctg\nu;$$
$$Y_{152\_ccntr} = Y_{152\_covar} - R\cos\beta\ tg\beta_1\ ctg\nu;$$

$$Z_{152\_contr} = Z_{152\_covar} = R\cos\gamma\ . \tag{7}$$

After measuring the coordinates of the object in the skew coordinate system (in this particular case in the $\Sigma_{152}=(0,x_{152},y_{152},z_{152})$ coordinate system) we must transform these coordinates to regular Cartesian coordinates. We can do this transformation in this stage. It will allow us to make the further coordinate transformations by using the already designed procedures (the rotations of Cartesian coordinate systems around earlier defined axes by pitch and roll angles). So now we define the coordinates of the object in the Cartesian coordinate system $\Sigma_{152\_90}=(0,x_{152\_90},y_{152\_90},z_{152\_90})$. The *z*-axis in the new coordinate system will not change. The Cartesian coordinates in the new coordinate system will be defined with the expressions:

$$X_{152\_90} = X_{152\_ccntr}\cos 345° + Y_{152\_ccntr}\cos 285°;$$
$$Y_{152\_90} = X_{152\_ccntr}\cos 75° + Y_{152\_ccntr}\cos 15°;$$
$$Z_{152\_90} = Z_{152\_contr} = R\cos\gamma\ . \tag{8}$$

To obtain the coordinates of point *P* in the $\Sigma=(0,x,y,z)$ coordinate system (point $P(X,Y,Z)$ in Fig.1) it is necessary perform the corresponding transformations of obtained coordinates X_{152_90}, Y_{152_90}, Z_{152_90} .

Let the pitch and roll rotations of the receiving antenna take place. The pitch and roll rotations of the three-element USBL arrays are shown in Fig.1,2. The difference of the consideration of the rotation of the $USBL_{152\_90}$ coordinate system from the rotation of $USBL_{123}$ coordinate system is in the following: in the case of the $USBL_{123}$ system the rotation is realized around the origin that is located in point 2 and in the case of the $USBL_{152\_90}$ system the rotation is realized around the origin that is located in point 5. With the first rotation on pitch angle ξ relative to *C-C′* axis the receiving elements are displaced to points 1^{ξ} and 2^{ξ} respectively. After a second rotation on the angle ζ relatively *D-D′* axis the receiving elements are displaced to point $1^{\xi,\zeta}$ and $2^{\xi,\zeta}$ respectively (see Fig.2). In our algorithm we will carry out the rotations with orthogonal coordinate systems. With this assumption the corresponding transformations of coordinate systems from $\Sigma_{152\_90}=(0,x_{152\_90},\ y_{152\_90},\ z_{152\_90})$ to $\Sigma^{\xi}_{152\_90}=(0,x^{\xi}_{152\_90},\ y^{\xi}_{152\_90},\ z^{\xi}_{152\_90})$ and to $\Sigma^{\xi,\zeta}_{152\_90}=(0,x^{\xi,\zeta}_{152\_90},y^{\xi,\zeta}_{152\_90},z^{\xi,\zeta}_{152\_90})$ have taken place. Calculation expressions for the case of the pitch and roll of the orthogonal three-element receiving antenna with different orientation relative to the carrier have been obtained in [8,9]. After the transformation of the skew coordinates to the orthogonal coordinates in the $\Sigma^{\xi,\zeta}_{152\_90}$ the further procedure of the calculation of the coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate system is accomplished in the same way as it is made for the other orthogonal three-element USBL systems.

We will describe the calculation procedure in detail only for the $USBL_{152}$ system. For the other three skew USBL systems ($USBL_{253}$, $USBL_{354}$, $USBL_{451}$) the calculation procedures will be similar.

We will consider the general case supposing that the receiving array has pitch and roll inclinations. In this case firstly we find the covariant coordinates of the object $X^{\xi,\zeta}_{152\_covar}$, $Y^{\xi,\zeta}_{152\_covar}$, $Z^{\xi,\zeta}_{152\_covar}$ in the $\Sigma^{\xi,\zeta}_{152}$ skew coordinate system utilizing the formulas (2), (4), (5) and (6). We can represent these coordinates as a row vector and then transform it to the column vector $\boldsymbol{p}^{\xi,\zeta}_{152\_covar}=[X^{\xi,\zeta}_{152\_covar}, Y^{\xi,\zeta}_{152\_covar}, Z^{\xi,\zeta}_{152\_covar}]^T$ ($[X^{\xi,\zeta}_{152\_covar}, Y^{\xi,\zeta}_{152\_covar}, Z^{\xi,\zeta}_{152\_covar}]^T$ represents transpose of the vector $[X^{\xi,\zeta}_{152\_covar}, Y^{\xi,\zeta}_{152\_covar}, Z^{\xi,\zeta}_{152\_covar}]$). The contravariant coordinates of the object we can represent as a vector $\boldsymbol{p}^{\xi,\zeta}_{152\_contr}$. The vector $\boldsymbol{p}^{\xi,\zeta}_{152\_contr}$ is calculated with the applying the formulas (7). The next step is the calculation of the coordinate vector $\boldsymbol{p}^{\xi,\zeta}_{152\_90}$ that represents the object's coordinates in the orthogonal coordinate system $\Sigma^{\xi,\zeta}_{152\_90}= (0,x^{\xi,\zeta}_{152\_90},, y^{\xi,\zeta}_{152\_90},, z^{\xi,\zeta}_{152\_90})$. Vector $\boldsymbol{p}^{\xi,\zeta}_{152\_90}$ can be found by applying the formulas (8).

In general case the Cartesian coordinate system $\Sigma^{\xi,\zeta}_{152\_90}$ has inclination that defined by pitch and roll angles. From this step we can utilize the approach that was successfully applied to USBL arrays where only the orthogonal three-element USBL arrays were utilized [8,9]. In accordance with this approach the coordinate system $\Sigma^{\xi,\zeta}_{152\_90}$ can be considered as the coordinate system obtained by rotation (by the roll angle ζ) of the coordinate system $\Sigma^{\xi}_{152\_90}$ that has only pitch inclination. The coordinate system $\Sigma^{\xi}_{152\_90}$ can be considered as the coordinate system obtained by rotation (by the pitch angle) of the coordinate system Σ_{152_90} that does not have any inclination.

For the coordinate system $\Sigma_{152\_90}=(0,x_{152\_90}, ,y_{152\_90},, z_{152\_90})$ and for introduced earlier axis *CC′* the direction cosines will be defined by next formulas:

$$\cos\angle(x_{152\_90},C) = -\frac{\sqrt{3}-1}{2\sqrt{3}};$$
$$\cos\angle(y_{152\_90},C) = \frac{\sqrt{3}+1}{2\sqrt{3}};$$
$$\cos\angle(z_{152\_90},C) = \frac{1}{\sqrt{3}}. \tag{9}$$

For the $\Sigma^{\xi}_{152\_90}=(0,x^{\xi}_{152\_90},,y^{\xi}_{152\_90},,z^{\xi}_{152\_90})$ coordinate system and axis *DD′* the direction cosines will be defined by next formulas:

$$\cos\angle(x^{\xi}_{152\_90},D) = -\frac{\sqrt{3}+1}{2\sqrt{3}};$$
$$\cos\angle(y^{\xi}_{152\_90},D) = \frac{\sqrt{3}-1}{2\sqrt{3}};$$
$$\cos\angle(z^{\xi}_{152\_90},D) = -\frac{1}{\sqrt{3}}. \tag{10}$$

If we introduce vector $\boldsymbol{p}_{152\_90}=[X_{152\_90}, Y_{152\_90}, Z_{152\_90}]^T$ (vector $\boldsymbol{p}_{152\_90}$ represents the coordinates of the object in Σ_{152_90} coordinate system), vector $\boldsymbol{p}^{\xi}_{152\_90}= [X^{\xi}_{152\_90},,Y^{\xi}_{152\_90},Z^{\xi}_{152\_90}]^T$ (vector $\boldsymbol{p}^{\xi}_{152\_90}$ represents the coordinates of the object in $\Sigma^{\xi}_{152\_90}$ coordinate system) and the transformation matrix $\boldsymbol{A}_{152\_90}$ with direction cosines (9) and pitch angle ξ (matrix $\boldsymbol{A}_{152\_90}$ transforms vector $\boldsymbol{p}_{123}$ to vector $\boldsymbol{p}^{\xi}_{123}$) we can write the equation: $\boldsymbol{p}_{152\_90} = \boldsymbol{A}^{-1}_{152\_90}\boldsymbol{p}^{\xi}_{152\_90}$.

If we introduce transformation matrix $\boldsymbol{B}_{152\_90}$ with direction cosines (10) and roll angle ζ (matrix $\boldsymbol{B}_{152\_90}$ transforms vector $\boldsymbol{p}^{\xi}_{152\_90}$ to vector $\boldsymbol{p}^{\xi,\zeta}_{152\_90}$) we can write the equation: $\boldsymbol{p}^{\xi}_{152\_90} = \boldsymbol{B}^{-1}_{152\_90}\boldsymbol{p}^{\xi,\zeta}_{152\_90}$. These two transformations can be combine in the equation $\boldsymbol{p}_{152\_90} = \boldsymbol{A}^{-1}_{152\_90}\boldsymbol{B}^{-1}_{152\_90}\boldsymbol{p}^{\xi,\zeta}_{152\_90}$.

In order to obtain the coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate system (we will have the same coordinates of the object in the carrier coordinate system $\Sigma_{carrier}=(0,x_{carrier},y_{carrier},z_{carrier})$ and in the sea-surface associated coordinate system $\Sigma=(0,x,y,z)$ if the carrier is not inclined) it is necessary to make firstly one rotation of the coordinate system Σ_{152_90} around the *M-M′* axis on the angle of 35.2644° (see Fig.2) to obtain coordinates of the object in the vertical orientated coordinate system Σ_{152_90}. After that we have to accomplish two more rotation. First rotation is carry out on the angle of 45° around the vertical axis, the second rotation is carryout on the angle 90° around the *C-C′* axis. Finally we have the coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate system.

These three rotations we can represent with the three corresponding transformation matrixes. We introduce these matrixes in the following way: transformation matrix $\boldsymbol{C}_{152\_90}$ rotate the coordinate system $\Sigma=(0,x,y,z)$ around the *C-C′* axis on the angle of 90°; transformation matrix $\boldsymbol{D}_{152\_90}$ rotate the coordinate system derived in the previous step around the vertical axis on the angle of 45°; transformation matrix $\boldsymbol{F}_{152\_90}$ rotate the coordinate system derived in the previous step around the *M-M′* axis on the angle of 35.2644° (see Fig.2). As a result of these three rotations we have introduced earlier Σ_{152_90} coordinate system. If we introduce vector $\boldsymbol{p}_{\mathrm{USBL}152}$ (vector $\boldsymbol{p}_{\mathrm{USBL}152} = [X_{\mathrm{USBL}152}, Y_{\mathrm{USBL}152}, Z_{\mathrm{USBL}152}]^T$ represents the Cartesian coordinates of the object in $\Sigma=(0,x,y,z)$ coordinate system obtained with the USBL_{152} system) we can write the final matrix equation as follows:

$$\boldsymbol{p}_{\mathrm{USBL}152} = \boldsymbol{C}^{-1}_{152\_90}\boldsymbol{D}^{-1}_{152\_90}\boldsymbol{F}^{-1}_{152\_90}\boldsymbol{A}^{-1}_{152\_90}\boldsymbol{B}^{-1}_{152\_90}\boldsymbol{p}^{\xi,\varsigma}_{152\_90}. \tag{11}$$

VI. CALCULATION EXPRESSIONS FOR FIVE-ELEMENT USBL SYSTEM

In the case of the five-element USBL array, the system consists of ten basic USBL systems with ten different spatial orientations: USBL_{123}, USBL_{234}, USBL_{341}, USBL_{412}, USBL_{153}, USBL_{254}, USBL_{152}, USBL_{253}, USBL_{354}, and USBL_{451}.

The coordinates of the object are determined individually in each elemental USBL system. The USBL_{234}, USBL_{341} and USBL_{412} systems differ from the USBL_{123} system in their own values of the pitch and roll angles and in their own angles of rotation of each USBL antenna around the *z*-axis. The coordinate determination for the $USBL_{153}$ systems is almost the same as for the $USBL_{123}$ system, the difference is that one additional step is required to reduce the $USBL_{153}$ system coordinates to a horizontal plane (by rotation the $USBL_{153}$ system on a 90° angle). We have to do the same with coordinates obtained with the $USBL_{254}$ system. To accomplish these additional rotations for the $USBL_{153}$ and $USBL_{254}$ arrays we introduce for each system the transformation matrix (matrix $\boldsymbol{D}$). The $USBL_{253}$, $USBL_{354}$ and $USBL_{451}$ systems differ from the $USBL_{152}$ in their own direction cosines, in their own values of the pitch and roll angles and in their own angles of rotation of each USBL array around the *z*-axis (see Fig.2). So for the $USBL_{253}$, $USBL_{354}$ and $USBL_{451}$ systems we have to accomplish the all steps that have been implemented for $USBL_{152}$ system. In order to distinguish the results of the measured coordinates by different basic USBL systems we introduce the consequent designations for the vectors and transformation matrixes for each particular USBL system:

$$
\begin{aligned}
\boldsymbol{p}_{\mathrm{USBL}_{123}} &= \boldsymbol{C}^{-1}_{123}\boldsymbol{A}^{-1}_{123}\boldsymbol{B}^{-1}_{123}\boldsymbol{p}^{\xi\varsigma}_{123};\\
\boldsymbol{p}_{\mathrm{USBL}_{234}} &= \boldsymbol{C}^{-1}_{234}\boldsymbol{A}^{-1}_{234}\boldsymbol{B}^{-1}_{234}\boldsymbol{p}^{\xi\varsigma}_{234};\\
\boldsymbol{p}_{\mathrm{USBL}_{341}} &= \boldsymbol{C}^{-1}_{341}\boldsymbol{A}^{-1}_{341}\boldsymbol{B}^{-1}_{341}\boldsymbol{p}^{\xi\varsigma}_{341};\\
\boldsymbol{p}_{\mathrm{USBL}_{412}} &= \boldsymbol{C}^{-1}_{412}\boldsymbol{A}^{-1}_{412}\boldsymbol{B}^{-1}_{412}\boldsymbol{p}^{\xi\varsigma}_{412};\\
\boldsymbol{p}_{\mathrm{USBL}_{153}} &= \boldsymbol{C}^{-1}_{153}\boldsymbol{D}^{-1}_{153}\boldsymbol{A}^{-1}_{153}\boldsymbol{B}^{-1}_{153}\boldsymbol{p}^{\xi\varsigma}_{153};\\
\boldsymbol{p}_{\mathrm{USBL}_{254}} &= \boldsymbol{C}^{-1}_{254}\boldsymbol{D}^{-1}_{254}\boldsymbol{A}^{-1}_{254}\boldsymbol{B}^{-1}_{254}\boldsymbol{p}^{\xi\varsigma}_{254};\\
\boldsymbol{p}_{\mathrm{USBL}_{152}} &= \boldsymbol{C}^{-1}_{152\_90}\boldsymbol{D}^{-1}_{152\_90}\boldsymbol{F}^{-1}_{152\_90}\boldsymbol{A}^{-1}_{152\_90}\boldsymbol{B}^{-1}_{152\_90}\boldsymbol{p}^{\xi\varsigma}_{152\_90};\\
\boldsymbol{p}_{\mathrm{USBL}_{253}} &= \boldsymbol{C}^{-1}_{253\_90}\boldsymbol{D}^{-1}_{253\_90}\boldsymbol{F}^{-1}_{253\_90}\boldsymbol{A}^{-1}_{253\_90}\boldsymbol{B}^{-1}_{253\_90}\boldsymbol{p}^{\xi\varsigma}_{253\_90};\\
\boldsymbol{p}_{\mathrm{USBL}_{354}} &= \boldsymbol{C}^{-1}_{354\_90}\boldsymbol{D}^{-1}_{354\_90}\boldsymbol{F}^{-1}_{354\_90}\boldsymbol{A}^{-1}_{354\_90}\boldsymbol{B}^{-1}_{354\_90}\boldsymbol{p}^{\xi\varsigma}_{354\_90};\\
\boldsymbol{p}_{\mathrm{USBL}_{451}} &= \boldsymbol{C}^{-1}_{451\_90}\boldsymbol{D}^{-1}_{451\_90}\boldsymbol{F}^{-1}_{451\_90}\boldsymbol{A}^{-1}_{451\_90}\boldsymbol{B}^{-1}_{451\_90}\boldsymbol{p}^{\xi\varsigma}_{451\_90}.
\end{aligned}
\tag{12}
$$

VII. ALGORITHM DESCRIPTION

It is assumed that the measured values are: ξ and ζ – pitch and roll angles of the receiving nine-element antenna (see Fig.2); t – interrogation and response pulse separation; τ_{12}, τ_{32}, τ_{23}, τ_{43}, τ_{34}, τ_{14}, τ_{41}, τ_{21}, τ_{15}, τ_{35}, τ_{25}, τ_{45} - time delays for receiving bases of the corresponding $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{153}$, $USBL_{452}$, $USBL_{152}$, $USBL_{253}$, $USBL_{354}$, and $USBL_{451}$ systems (twelve time delays are measured, to provide the positive values of time delays the second-indexed outputs are inverted). First the vector $\boldsymbol{p}^{\xi,\zeta}_{153}=[X^{\xi,\zeta}_{153},,Y^{\xi,\zeta}_{153},,Z^{\xi,\zeta}_{153}]^T$) is calculated. The sign of the $Z^{\xi,\zeta}_{153}$ coordinate is defined by utilizing the time delay values obtained for the $USBL_{452}$ system (values τ_{25}, τ_{45}). If $\tau_{45}>\tau_{25}$ the sign of the $Z^{\xi,\zeta}_{153}$ coordinate is assumed to be negative. If $\tau_{45}\leq\tau_{25}$ is assumed the $Z^{\xi,\zeta}_{153}$ coordinate is positive. With the obtained values of the vector $\boldsymbol{p}^{\xi,\zeta}_{153}=[X^{\xi,\zeta}_{153}, Y^{\xi,\zeta}_{153}, Z^{\xi,\zeta}_{153}]^T$ the values of τ'_{25} y τ'_{45} are calculated for the $USBL_{452}$ system. Then the modules of the differences $\delta1= |\tau'_{25}-\tau_{25}|$ and $\delta2=|\tau'_{45}-\tau_{45}|$ are calculated (values τ_{25} y τ_{45} are obtained through measurement). Then the value $\Delta1=(\delta1^2+\delta2^2)^{0.5}$ is calculated. The same procedure is repeated with the opposite sign of $Z^{\xi,\zeta}_{153}$ coordinate with calculation of the corresponding value $\Delta2$. If $\Delta1>\Delta2$ the latter sign is assumed as correct. Otherwise the initial sign value of $Z^{\xi,\zeta}_{153}$ coordinate is assumed as correct. Then the same procedure is applied to the $USBL_{254}$ system vector $\boldsymbol{p}^{\xi,\zeta}_{254}=[X^{\xi,\zeta}_{254}, Y^{\xi,\zeta}_{254},,Z^{\xi,\zeta}_{254}]^T$) in order to define, the sign of the $Z^{\xi,\zeta}_{254}$ coordinate.

Further the values of the Cartesian coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate system are calculated. Through these calculations the values of the angles between the plane of the measuring antenna and the direction to the object (latitude angle to object) are evaluated.

For that the coordinate vectors in the spherical coordinate can be expressed as follows:

$$
\begin{aligned}
\boldsymbol{q}^{\xi,\varsigma}_{153} &= (R_{153},\psi^{\xi,\varsigma}_{153},\varphi^{\xi,\varsigma}_{153})^T;\\
\boldsymbol{q}^{\xi,\varsigma}_{254} &= (R_{254},\psi^{\xi,\varsigma}_{254},\varphi^{\xi,\varsigma}_{254})^T,
\end{aligned}
\tag{13}
$$

where $\psi^{\xi,\zeta}_{153}$, $\varphi^{\xi,\zeta}_{153}$ are the polar and azimuth angles in the $USBL_{153}$ spherical coordinate system and $\psi^{\xi,\zeta}_{254}$, $\varphi^{\xi,\zeta}_{254}$ are the polar and azimuth angles in the $USBL_{254}$ spherical coordinate system. The values of latitude angle for each USBL system define the decision to utilize or no utilize this system in the calculation of coordinate means. Analysis of values of latitude angle for elemental USBL systems shows that for reliable calculations the modulus of latitude angle must be more than 10° [9]. So if the values of the corresponding polar angles of both systems are found outside [80°, 100°] diapason, the values of both systems are utilized. If the value of the polar angle of one of the USBL systems lies outside the diapason [80°, 100°] and the value of the polar angle of the other system belongs to the [80°, 100°] then the value of the first system is utilized and the value obtained from another system is not taken into account. If the values of the polar angles of both systems are found within [80°, 100°] diapason, the polar angle value of the system with the greater latitude angle (latitude angle to object originating from the corresponding *x-y* plane) should be kept for further consideration. The final step in the coordinate determination of this part of the algorithm is the calculation of the Cartesian coordinates of the object in the $\Sigma=(0,x,y,z)$ coordinate (calculation of the vectors $\boldsymbol{p}_{\mathrm{USBL}_{153}}$ and $\boldsymbol{p}_{\mathrm{USBL}_{254}}$). These calculations for the $USBL_{153}$ and $USBL_{254}$ systems are carried out according to the formulas (12).

In the second stage of the algorithm the object coordinates are calculated using the time delays obtained by the four horizontal measuring systems ($USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$) and four inclined skew measuring systems ($USBL_{152}$, $USBL_{253}$, $USBL_{354}$, and $USBL_{451}$). It is also supposed that the receiving USBL array can have pitch and roll inclinations.

In order to resolve the Z-coordinate sign ambiguity problem for horizontal and inclined systems a special procedure is applied. We consider the skew $USBL_{152}$ system in order to describe this procedure (for the other three-element USBL systems this part of the algorithm functions in the same way). Suppose we determine the sign of the coordinate $Z^{\xi,\zeta}_{152\_90}$ ($Z^{\xi,\zeta}_{152}$). First we calculate the coordinates of the object based on the delays measured with the $USBL_{152}$ system ($\boldsymbol{p}^{\xi,\zeta}_{152\_90}=[X^{\xi,\zeta}_{152\_90}, Y^{\xi,\zeta}_{152\_90}, Z^{\xi,\zeta}_{152\_90}]^T$). The sign of the $Z^{\xi,\zeta}_{152\_90}$ coordinate is obtained using the values τ_{15}, τ_{35} of the $USBL_{153}$ system and the values τ_{25}, τ_{45} of the $USBL_{254}$ system. The values of the components of the vector $\boldsymbol{p}^{\xi,\zeta}_{152\_90}=[X^{\xi,\zeta}_{152\_90}, Y^{\xi,\zeta}_{152\_90}, Z^{\xi,\zeta}_{152\_90}]^T$ allow us to calculate the values of the delays τ'_{15} and τ'_{35} for the $USBL_{153}$ system and value of delays τ'_{25} and τ'_{45} for the $USBL_{254}$ system. Next, we calculate the absolute values of the following differences: $\delta 1=|\tau'_{15}-\tau_{15}|$, $\delta 2=|\tau'_{35}-\tau_{35}|$, $\delta 3=|\tau'_{25}-\tau_{25}|$ and $\delta 4=|\tau'_{45}-\tau_{45}|$ (the values τ_{15}, τ_{35}, τ_{25} and τ_{45} are obtained through measurement). Now we calculate the geometric mean $\delta_{geom\_mean\_1}$ of the δ1, δ2, δ3 and δ4. The same procedure is repeated with opposite sign designation for the coordinate $Z^{\xi,\zeta}_{152\_90}$. The geometric mean in this case will be $\delta_{geom\_mean\_2}$. The coordinates corresponding to a less geometric mean are taken as correct. The same procedure is applied to the $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{253}$, $USBL_{354}$ and $USBL_{451}$ systems.

The final stage of the algorithm examines the reliability of the calculation of the Z-coordinate for the USBL systems under consideration. For that we calculate the spherical coordinates:

$$\boldsymbol{q}^{\xi,\varsigma}_{123}=(R_{123},\psi^{\xi,\varsigma}_{123},\varphi^{\xi,\varsigma}_{123})^T;\quad \boldsymbol{q}^{\xi,\varsigma}_{234}=(R_{234},\psi^{\xi,\varsigma}_{234},\varphi^{\xi,\varsigma}_{234})^T;$$
$$\boldsymbol{q}^{\xi,\varsigma}_{341}=(R_{341},\psi^{\xi,\varsigma}_{341},\varphi^{\xi,\varsigma}_{341})^T;\quad \boldsymbol{q}^{\xi,\varsigma}_{412}=(R_{412},\psi^{\xi,\varsigma}_{412},\varphi^{\xi,\varsigma}_{412})^T;$$
$$\boldsymbol{q}^{\xi,\varsigma}_{152}=(R_{152},\psi^{\xi,\varsigma}_{152},\varphi^{\xi,\varsigma}_{152})^T;\quad \boldsymbol{q}^{\xi,\varsigma}_{253}=(R_{253},\psi^{\xi,\varsigma}_{253},\varphi^{\xi,\varsigma}_{253})^T;$$
$$\boldsymbol{q}^{\xi,\varsigma}_{354}=(R_{354},\psi^{\xi,\varsigma}_{354},\varphi^{\xi,\varsigma}_{354})^T;\quad \boldsymbol{q}^{\xi,\varsigma}_{451}=(R_{451},\psi^{\xi,\varsigma}_{451},\varphi^{\xi,\varsigma}_{451})^T. \tag{14}$$

If the values of the polar angle of some USBL systems lie within the diapason [80°, 100°], the calculated values are discarded. If the values of the polar angle of the analyzed USBL systems lay outside of the diapason [80°, 100°] the corresponding Cartesian coordinates of these systems are considering as reliable. The values of the object coordinates in the carrier coordinate system are calculated according to the formulas (12). The last step of the algorithm implies the calculation of the arithmetic means of the object coordinates in the carrier coordinate system with reliable data obtained by the elemental USBL systems:

$$\boldsymbol{p}_{\mathrm{USBL}}=(X_{\mathrm{USBL}},Y_{\mathrm{USBL}},Z_{\mathrm{USBL}})^T=$$
$$\text{arithmetic mean}\,(\boldsymbol{p}_{\mathrm{USBL}153},\boldsymbol{p}_{\mathrm{USBL}254},\boldsymbol{p}_{\mathrm{USBL}123},\boldsymbol{p}_{\mathrm{USBL}234},$$
$$\boldsymbol{p}_{\mathrm{USBL}341},\boldsymbol{p}_{\mathrm{USBL}412},\boldsymbol{p}_{\mathrm{USBL}152},\boldsymbol{p}_{\mathrm{USBL}253},\boldsymbol{p}_{\mathrm{USBL}354},\boldsymbol{p}_{\mathrm{USBL}451}). \tag{15}$$

VIII. SIMULATION RESULTS

For the algorithm simulation a special computer program was designed. During the simulation of the algorithm, it was assumed that the distance to the object, and the pitch and roll angles were being measured precisely. We assume that the measurement of the time delays is provided by utilizing the binary counters and the signal-to-noise ratio (SNR) on the inputs of receiving elements and signal reception conditions allow us to measure the time delays without errors. It is also supposed that the accuracy of measurement of the time delays is limited by the clock drive frequency of the time delay counters. The different values of the horizontal distance to the object, the depth of the object, the azimuth angle, and the pitch and roll angles were utilized for modeling the difficult conditions to measure the object coordinates with high accuracy (cases when an object is found in the plane of the horizontal receiving bases or near to the plane of the horizontal receiving bases). The computer simulation of algorithm will estimate the instrumental precision of the USBL system.

Let *X, Y, Z* be the true values of the coordinates of the object in the coordinate system $\Sigma=(0,x,y,z)$. Let *R* be the true incline distance to the object. Let X_{USBL}, Y_{USBL}, Z_{USBL} be the values of the coordinates obtained by applying of the developed algorithm (the coordinates of the object are calculated utilizing the expression (12), (13) and (14) for $USBL_{153}$, $USBL_{254}$ $USBL_{123}$, $USBL_{234}$, $USBL_{341}$, $USBL_{412}$, $USBL_{152}$, $USBL_{253}$, $USBL_{354}$, and $USBL_{451}$ systems). The values of the true errors of determination of the coordinates are: $\Delta X=X_{USBL}-X$; $\Delta Y=Y_{USBL}-Y$, $\Delta Z= Z_{USBL}-Z$. The values of the relative true errors of the coordinates are: $\Delta X/R$; $\Delta Y/R$; $\Delta Z/R$. In process of the simulation the azimuth angle φ is changing clockwise (if looking down on the horizontal plane, see Figs.1,2) from 0° to 360° in the *(x,y)* coordinate plane (zero reading is coincided with x-axis of the $\Sigma=(0,x,y,z)$ coordinate system, see Fig.1). The other parameters of algorithm simulation have following values: the speed of the sound in the water *c*=1500m/s; the size of receiving bases *d*=0.056m; transponder pulse carrier frequency *f*=11KHz (operating frequency of USBL system); the frequency of the time delay counter f_C=25MHz.

The results of the simulation of the designed algorithm are shown in Figs.4-6. First we will consider the case when the receiving antenna does not have any inclination and the object is located in the horizontal distance of 100 meters and the relative depth is 5 meters. The angular position of the object (azimuth angle *φ*) is changing with the step of 1°.

The results of the algorithm simulation for the examining case (R=100m; Z=5 m; ξ=0°; ζ= 0°) are shown in Fig.4. In the absence of the pitch and roll the modulus of the latitude angle to the object (in graphs this angle is designated as $|\psi^{\xi,\zeta}_{1234}$ - 90°|) should be invariable and the value of the latitude angle is approximately 2.86°. It means that the horizontal USBL systems are not participated in the calculation of coordinate arithmetic means and the maximum number of systems that are taken into account in this case is 6 (N=6). We can observe this on the Fig.4, - the maximum number of utilized USBL systems is 6, and we can also observe that the number of utilized USBL systems is depend on azimuth angle and varied from 4 to 6.

We will consider in detail the behavior of the function $N=N(\varphi)$ only in the diapason of changing of the angle φ from 0° to 90°. In the other diapasons of φ ([90°-180°], [180°-270°], [270°-360°]) the behavior of the function $N=N(\varphi)$ will be analogous. In the azimuth angle ranges from 0° to 10° the number of the utilized USBL system is 5 (N=5) and the systems that are utilized in the calculation of the coordinate arithmetic means are: $USBL_{153}$, $USBL_{152}$, $USBL_{253}$, $USBL_{354}$ and $USBL_{451}$ (see Figs.2,4). For the $USBL_{452}$ system (the case: ζ= 0°) the azimuth angle φ can be interpreted as the altitude angle to the object relative to the plane of the $USBL_{452}$ three-element array. So the utilization of the $USBL_{452}$ system is began from φ >10°. In the interval [11°- 30°] the number of used systems is 6 (N=6, two vertical orthogonal USBL systems and all skew USBL systems are in use). From the φ >30° firstly one skew system and then two skew systems ($USBL_{152}$, $USBL_{451}$) have the latitude angles less than 10°. In the diapason [35°-45°] the number of USBL systems in use is 4. The behavior of the function $N=N(\varphi)$ is in the interval [45°-90°] is the same as in the just considered interval [0°-45°]. For the case of ξ=0°; ζ= 0° the behavior of the function $N=N(\varphi)$ is repeated in the other three 90°-sectors.

The utilization of the different elemental USBL arrays is defined according to algorithm described above. The relative errors of all three object coordinates (ΔX/R, ΔY/R, ΔZ/R) have not exceeded the threshold of 0.12% of inclined distance to the object. The behavior of the relative errors is also similar within each 90°-sector. The greatest errors take place for the Z-coordinate of the object. The relative errors of X- and Y-coordinates not exceeded the values of 0.1%.

Let that the USBL array has some inclination (pitch and roll angles are not zeros). The simulation results for these cases are shown in Figs.5-6.

From Fig.5 (R=100m; Z=15m; ξ=5°; ζ= –6°) it is seen that the relative errors (ΔX/R, ΔY/R, ΔZ/R) of all three coordinates have not exceeded the threshold of 0.2% of inclined distance to the object. The values of the relative errors ΔX/R and ΔY/R are less than 0.12% in all examined diapason of φ. The number of the utilizing elemental systems (N) is varying from 4 to 10.

From the graphs it is seen that if the $|\psi^{\xi,\zeta}_{1234}$ -90°|≤10° the number of the utilized system is less or equal to 6. If the $|\psi^{\xi,\zeta}_{1234}$ -90°|>10° the number of the utilized systems (N) is varying from 8 to 10 and the exact number is defined by the values of altitude angle for the other three-element array planes.

The graphs in Fig.6 illustrate the variation of relative errors of object coordinates for the case when R=100m; Z=40m; ξ= –20°; ζ=10°. The relative location of the measuring system and the object (with predetermined spatial orientation of receiving antenna) defines the case of significant inclination of receiving antenna and when the object can be found in the plane of horizontal receiving bases of USBL system. It is seen that the relative errors do not exceed the threshold of 0.15% of incline

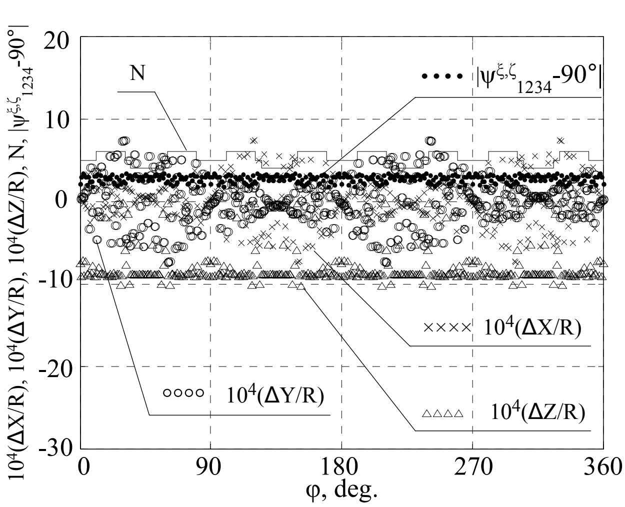

Fig.4. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relative to the $USBL_{1234}$ plane; R=100m; Z=5 m; ξ=0°; ζ= 0°.

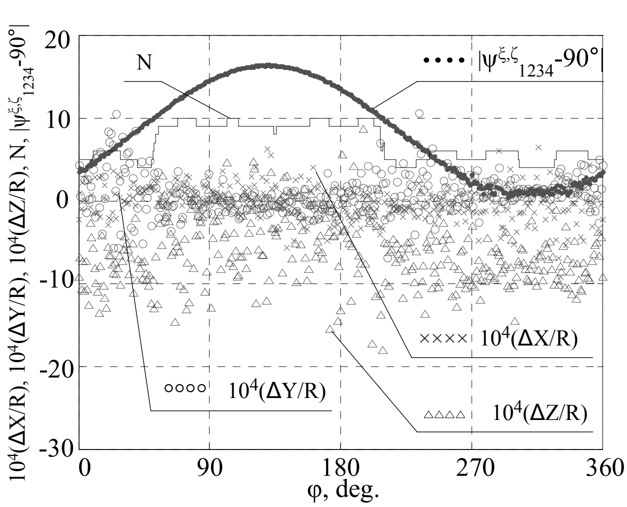

Fig.5. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relatively the $USBL_{1234}$ plane; R=100m; Z=15 m; ξ=5°; ζ= -6°.

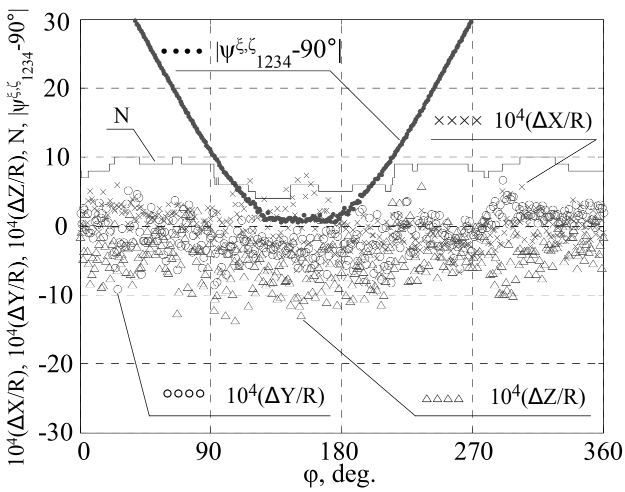

Fig.6. Variation of the relative errors of the object coordinates, number of the elemental USBL systems N that are used for calculation of coordinate means and modulus of the latitude angle to the transponder (object) relatively the $USBL_{1234}$ plane; R=100m; Z=40 m; ξ= -20°; ζ= 10°.

distance in all diapason of changing azimuth angle φ.

Also it is seen that when the modulus of altitude angle $|\psi^{\xi,\zeta}_{1234}$ -90°| is more than 10° the number of the utilized three-element systems is found in interval between 7 and 10 (7≤N≤10).

The designed algorithm has been examined for various relative locations of the measuring system and object (object location in the lower hemisphere was considered, the maximum horizontal distance was assumed to be 100m). The values of pitch and roll angles ξ and ζ are assumed to be in the range from –40° to +40°. The computer simulation demonstrated the reliable operation of the designed algorithm for all tested angular antenna positions and verified locations of object. The results of the calculation of the errors of the determination of the coordinates of the object show that the relative true errors of the coordinates have values less than 0.2% of the slant distance to the object. It is also necessary to mention, that in a wide range of distances, depths, pitch and roll angles the values of true relative errors are less than 0.1%.

IX. Conclusion

In this article we have considered the method of design of USBL system utilizing both orthogonal and non-orthogonal (skew) elemental three-element arrays. A case for the design of a five-element USBL system is presented. The paper focused on the problem of exploiting the latent resources of the receiving USBL array (skew three-element arrays) for accurate coordinate determination in conditions when the receiving antenna can have significant inclinations and the location of the object is arbitrary. The proposed algorithm accomplishes the selection of reliable elemental USBL arrays (orthogonal and non-orthogonal) utilizing the analysis of the values of latitude angles to the object for each elemental array. The presented algorithm is a significantly modified version of the algorithm designed for the five-element USBL system where only orthogonal elemental three-element arrays have been utilized [9]. Algorithm simulation was realized for a variety of USBL system and object mutual positions in a wide range of pitch and roll of receiving arrays (pitch and roll angles ξ and ζ are assumed to be in the range from -40° to +40°). For all tested angular receiving array positions and object locations the designed algorithm showed reliable operation. The accuracy of coordinate determination for the proposed five-element USBL system can be evaluated as the 0.2% of slant distance to the object.

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