Control of First-Order Time-Delay Plants Affected by Parametric Uncertainty

Radek Matušů, Roman Prokop, Jiří Vojtěšek, and Tomáš Dulík

***Abstract*—**This paper deals with an approach to control of single-input single-output first-order time-delay systems with parametric uncertainty. The presented method consists of an algebraic synthesis of control systems in the ring of proper and stable rational functions and subsequent graphical robust stability analysis. The applied robust stability tests are based on combination of plotting the value sets of a family of closed-loop characteristic quasi-polynomials with the zero exclusion principle. Three successive cases of controlled plants with uncertain gain, uncertain time constant and uncertain time-delay term are analyzed in the illustrative example. All of these systems are controlled using two differently tuned realistic PID controllers. Obtained results are visualized and discussed.

***Keywords*—**Robust stabilization, time-delay systems, parametric uncertainty, value set concept, zero exclusion condition.

I. Introduction

A time delay represents very frequent and significant phenomenon which affects many areas of control engineering [1] – [9]. Typically it causes serious complications during the synthesis process. Although it has been deeply studied during many decades, there are still number of attractive topics for future research [10].

The principal requirement for control loops is their stability. Provided there is an uncertainty in description of (typically) controlled plants, the robust stability should be investigated. An array of methods and tools for analysis of robust stability of systems with parametric uncertainty can be found e.g. in [11] or subsequently in [12], [13]. For the purpose of this contribution, the combination of very universal tools known as the value set concept and the zero exclusion condition [11] has been employed. Besides, the synthesis method, which is applied within this paper for controller design, is based on an algebraic approach adopted from [14], [15] and elaborated e.g. in [16] – [18].

The main aim of the contribution is to present a technique for robust stabilization of first-order time-delay systems with parametric uncertainty by means of an algebraic approach to control design, plotting the value sets for a family of closed-loop quasi-polynomials and applying the zero exclusion condition. A comparison of parametric and unstructured approach to uncertainty modelling and robust stability analysis for time-delay systems has been shown in [19], [20]. This paper deals only with parametric uncertainty case, but elaborates the problem in more detail. Actually, the contribution extends the previous works [21], [22] where the partial issues have been tentatively solved.

The paper is the improved version of the conference contribution [23].

The work is organized as follows. In section II, the essence of the problem is adumbrated. The section III then outlines the applied control synthesis method. The following section IV briefly presents the issue of robust stability. Further, the illustrative example with analyses of robust stability and control simulations can be found in the extensive section V, which consists of 3 partial subsections. Finally, section VI offers some conclusion remarks.

Radek Matušů, Roman Prokop, Jiří Vojtěšek, and Tomáš Dulík are with the Faculty of Applied Informatics, Tomas Bata University in Zlín, nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic. The email contact is: rmatusu@fai.utb.cz.

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II. Problem Formulation

As it has been already indicated, the principal problem discussed in the contribution is to analyze robust stability of the closed control loop with a fixed controller and a first-order time-delay plant with parametric uncertainty. Thus, the controlled system is supposed to be described by the transfer function:

$$G(s,K,T,\Theta)=\frac{K}{Ts+1}e^{-\Theta s} \tag{1}$$

where one of the parameters K (gain), T (time constant) or Θ (time-delay term) can vary within a given interval while the other two remain fixed. Three various combinations have been considered in the analyses within the future section V, i.e.

$$K\in\langle 1;3\rangle;\quad T=3;\quad \Theta=5 \tag{2}$$

$$K=2;\quad T\in\langle 1;5\rangle;\quad \Theta=5 \tag{3}$$

$$K=2;\quad T=3;\quad \Theta\in\langle 1;9\rangle \tag{4}$$

The nominal system (used for the controller design) is assumed as:

$$G_N(s) = \frac{K_N}{T_N s + 1} e^{-\Theta_N s} = \frac{2}{3s+1} e^{-5s} \tag{5}$$

More precisely, an approximation of this nominal system has been utilized in the controller design itself as will be shown later.

III. Outline of the Controller Design Method

The controllers utilized for the robust stability tests and control simulations in the section 5 are designed by means of an algebraic approach under assumption of the classical feedback control loop.

The technique is based on algebraic approach developed in [14], [15]. It applies general solutions of Diophantine equations in the ring of proper and (Hurwitz-)stable rational functions (R_{PS}), Youla-Kučera parameterization and conditions of divisibility. One of the main advantage consists in possibility to tune the final controllers though a single scalar parameter $m > 0$. The details of the methodology, specific equations for calculation of controller parameters and tuning recommendations can be found in [16] – [18], [22], [24], etc.

In the paper, the nominal plant is given by (5) which is not suitable form for the controller design because the synthesis works with rational functions only. For that reason the time-delay term in (5) has been approximated by using the popular first order Padé approximation:

$$\begin{aligned} G_A(s) &= \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{-0.\overline{6}s + 0.2\overline{6}}{s^2 + 0.7\overline{3}s + 0.1\overline{3}} = \\ &= \frac{-5s+2}{7.5s^2 + 5.5s + 1} = \frac{2(1-2.5s)}{(3s+1)(1+2.5s)} \approx \frac{2}{3s+1} e^{-5s} = G_N(s) \end{aligned} \tag{6}$$

Assumption of the traditional feedback control loop with a step-wise reference signal and application of the method (see e.g. [16] – [18], [22], [24]) leads to the realistic PID controller with general structure:

$$C(s) = \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{s(s + \tilde{p}_1)} \tag{7}$$

where the parameters can be calculated by using rules:

$$\begin{aligned} \tilde{p}_1 &= p_0 + m - p_0 m \frac{b_1}{b_0} \\ \tilde{q}_2 &= q_1 + \frac{p_0 m}{b_0} \\ \tilde{q}_1 &= q_0 + q_1 m + a_1 \frac{p_0 m}{b_0} \\ \tilde{q}_0 &= q_0 m + a_0 \frac{p_0 m}{b_0} \end{aligned} \tag{8}$$

and:

$$\begin{aligned} p_0 &= \frac{3m^2 b_0 b_1 - a_0 b_0 b_1 - 3m b_0^2 + a_1 b_0^2 - b_1^2 m^3}{a_1 b_0 b_1 - b_0^2 - a_0 b_1^2} \\ q_1 &= \frac{3m - a_1 - p_0}{b_1} \\ q_0 &= \frac{m^3 - a_0 p_0}{b_0} \end{aligned} \tag{9}$$

The parameters of the controlled system are taken from (3). Usage of two choices for parameter $m > 0$ results in specific controller parameters:

$$\begin{aligned} m = 0.12 \Rightarrow \ & \tilde{q}_2 = 0.24736;\ \tilde{q}_1 = 0.072077; \\ & \tilde{q}_0 = 0.0007776;\ \tilde{p}_1 = -0.088425 \end{aligned} \tag{10}$$

$$\begin{aligned} m = 0.18 \Rightarrow \ & \tilde{q}_2 = 0.14336;\ \tilde{q}_1 = 0.056203; \\ & \tilde{q}_0 = 0.0039366;\ \tilde{p}_1 = 0.082237 \end{aligned} \tag{11}$$

Notice that the first controller (10) is unstable and thus not appropriate for practical application. However, it comes in useful for the purpose of the simulation examples in the section V (and especially subsection V-B).

IV. Robust Stability

The principal question is if the controller (10) or (11) robustly stabilizes the whole family of controlled plants (1) for the cases (2), (3) and (4), respectively. In other words, the main goal is to investigate robust stability of the family of closed-loop characteristic quasi-polynomials with the structure:

$$\begin{aligned} & p_{CL}(s, K, T, \Theta) = (Ts+1)(s^2 + \tilde{p}_1 s) + \cdots \\ & \cdots K e^{-\Theta s}(\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0) \end{aligned} \tag{12}$$

where two plant parameters are fixed while the third one is uncertain according to one of the scenario (2), (3) or (4), and where the controller parameters are given by either (10) or (11).

The answer can be found with the assistance of the universal graphical approach which combines the value set concept and the zero exclusion condition [11]. Very briefly speaking, the value set at one frequency ω can be obtained by substitution of s for $j\omega$ in the family (12), fixing ω and letting the relevant uncertain parameter (K, T or Θ) range over the prescribed interval. Then, the family (12) is robustly stable if and only if it contains at least one stable member and the zero point (the origin of the complex plane) is excluded from the value sets at all non-negative frequencies. Further information on the value set concept and the zero exclusion condition as well as more general issues of robustness for systems with parametric uncertainty can be found e.g. in [11] or also in [12], [13].

V. Analyses of Robust Stability and Control Simulations

This section is intended to provide tests of robust stability together with simulations of control outputs under assumption of the closed control loop with the plant family (1) and controller (10) or (11). Both controllers are applied to all three possible combinations of plant parameters (2), (3) or (4).

All the figures within this section were plotted in Matlab environment. More specifically, they were achieved under conditions as follows: The value sets of the quasi-polynomial families were plotted (according to description in the previous section) for the range of frequencies from 0 to 5 with step 0.1. From the control simulations point of view, some “representative” set of systems was selected by sampling the respective uncertain parameter ($K = 1:0.02:3$, $T = 1:0.05:5$ or $\Theta = 1:0.1:9$) and subsequently it was used for the simulation. Thus, there were e.g. 101 “representative” systems for the case of the uncertain gain. Moreover, the control response of the nominal system (5) is also included in the figures (red curve). Besides, the step load disturbance of the size –0.2 was injected into the input of the controlled plant during the last third of simulation time.

A. Uncertain Gain

In the first part, the uncertain gain, that means plant parameters according to (2), has been assumed. The tuning parameter $m = 0.12$ leads to the PID controller parameters (10). Putting all these numbers into the closed-loop characteristic quasi-polynomial structure (12) brings the main object of interest from the robust stability viewpoint. The value sets of the family of quasi-polynomials are plotted in fig. 1 and the closer view to the neighbourhood of the complex plane origin is zoomed in fig. 2. As can be seen the zero point is included in the value sets and thus the family is not robustly stable for this case. It is confirmed also by fig. 3 which shows the simulated control outputs. The nominal system (5) is stabilized but the some systems from the assumed family (2) are not. Besides, the set of corresponding manipulated variables (controller outputs) is depicted in fig. 4.

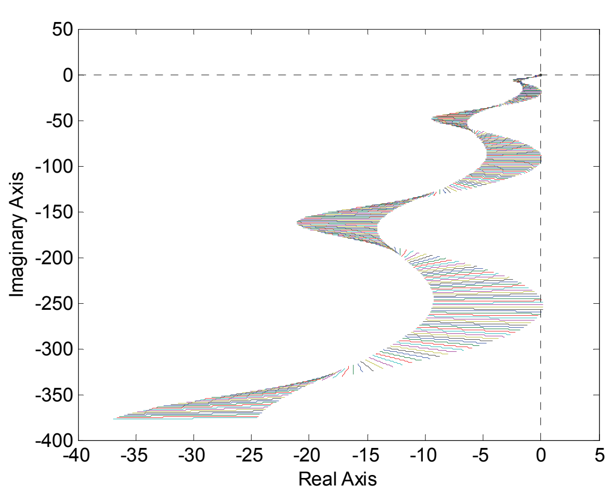

Fig. 1 value sets – plant (2), controller (10)

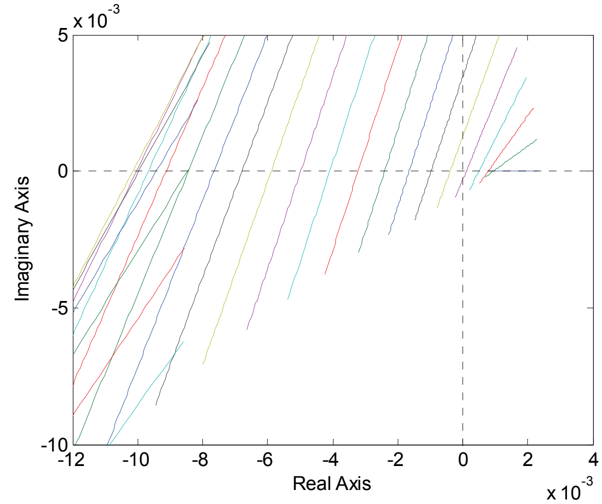

Fig. 2 zoomed value sets – plant (2), controller (10)

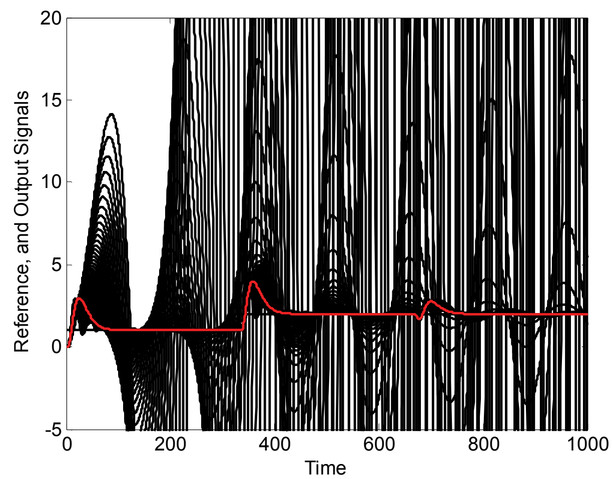

Fig. 3 control outputs – plant (2), controller (10)

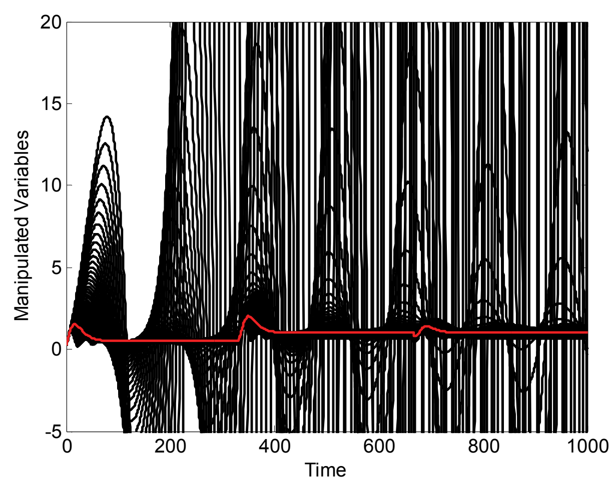

Fig. 4 manipulated variables – plant (2), controller (10)

Now, the same simulations are repeated but for the controller tuned by $m = 0.18$ with parameters (11). The value sets are depicted in fig. 5 and its zoomed version in fig. 6. Obviously, the family contains at least one stable member and the point zero in excluded from the value sets so the family must be robustly stable. Consequently, this fact can be seen also from control outputs given in fig. 7. The fig. 8 then shows the manipulated variables.

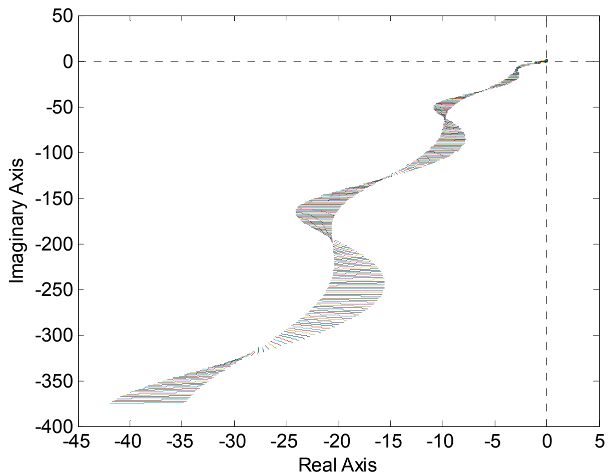

Fig. 5 value sets – plant (2), controller (11)

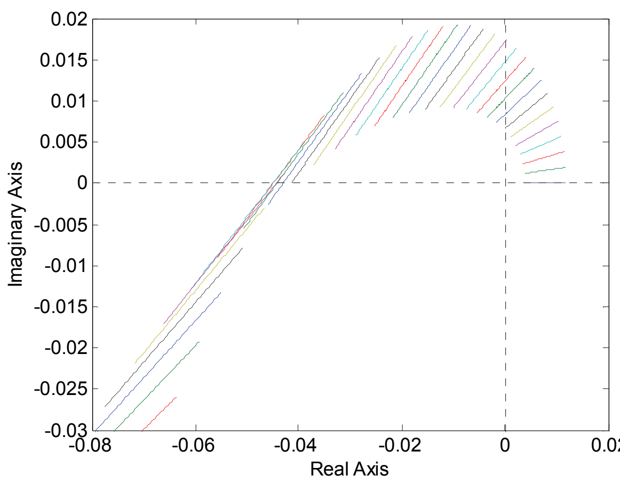

Fig. 6 zoomed value sets – plant (2), controller (11)

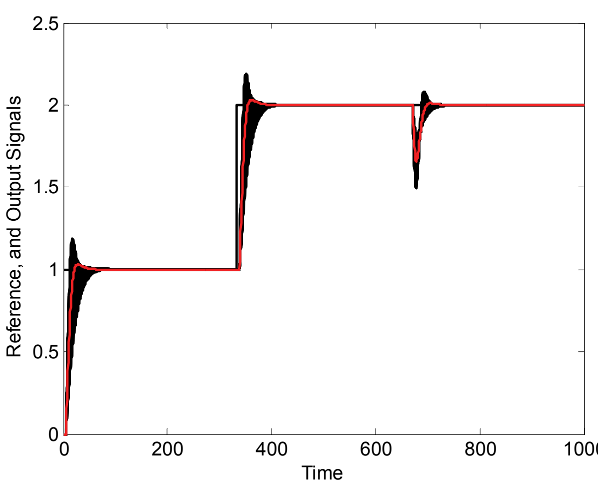

Fig. 7 control outputs – plant (2), controller (11)

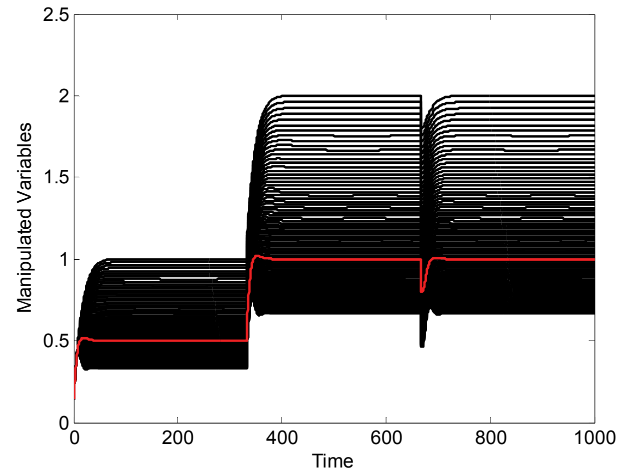

Fig. 8 manipulated variables – plant (2), controller (11)

B. Uncertain Time Constant

The second part focuses on the case of uncertain time constant, i.e. (3).

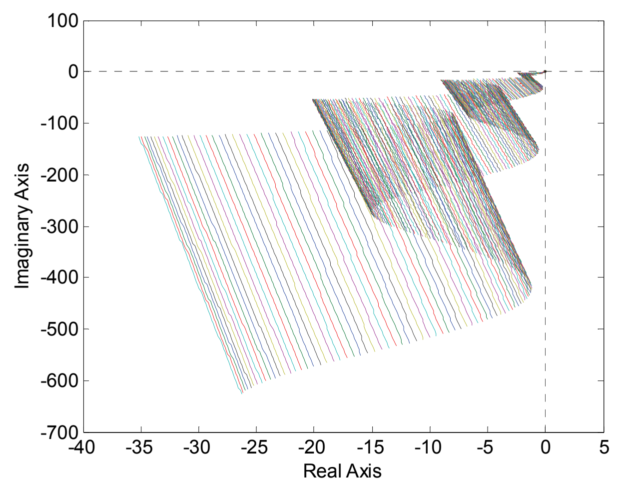

Fig. 9 value sets – plant (3), controller (10)

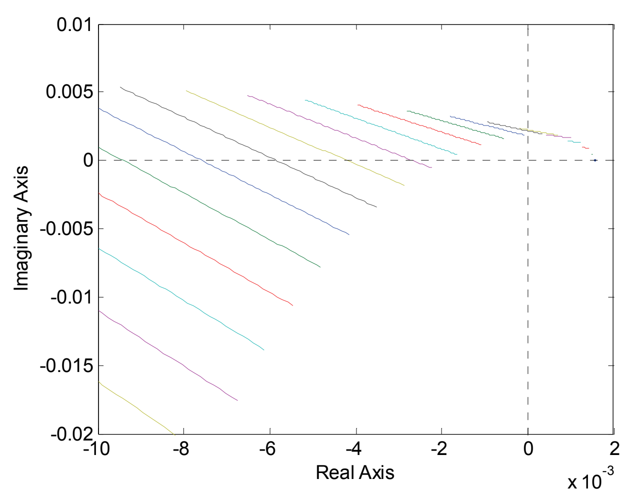

Fig. 10 zoomed value sets – plant (3), controller (10)

The analysis for the controlled plant with parameters (3) and the first controller (10) is analogical to the one described in the subsection 5-A. The value sets are shown in fig. 9, zoomed value sets for better perspective of the situation near the zero point in fig. 10, the set of simulated control outputs in fig. 11, and corresponding set of manipulated variables in fig. 12.

Notice that even the unstable controller (10) is able not only to stabilize the nominal system, but also to robustly stabilize the control loop with all possible values of controlled plant time constant.

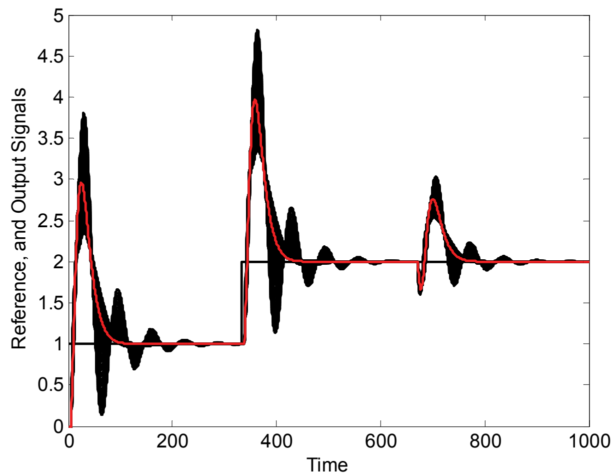

Fig. 11 control outputs – plant (3), controller (10)

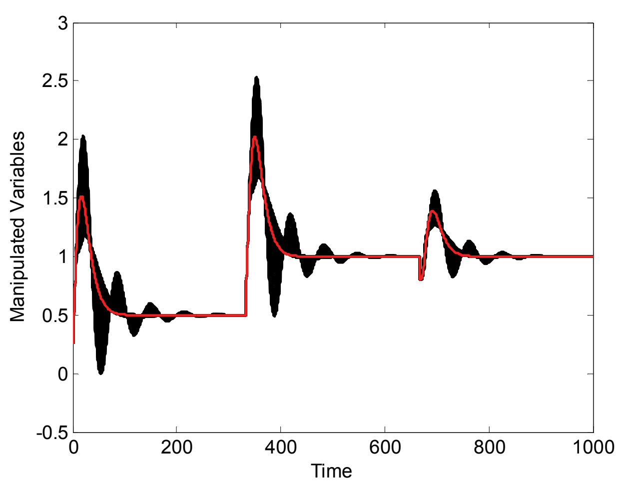

Fig. 12 manipulated variables – plant (3), controller (10)

The results for the case of controller (11) provided again by means of value sets, their zoomed version, control responses, and manipulated variables can be found in figs. 13 – 16. The control loop is robustly stable again, but the performance was improved.

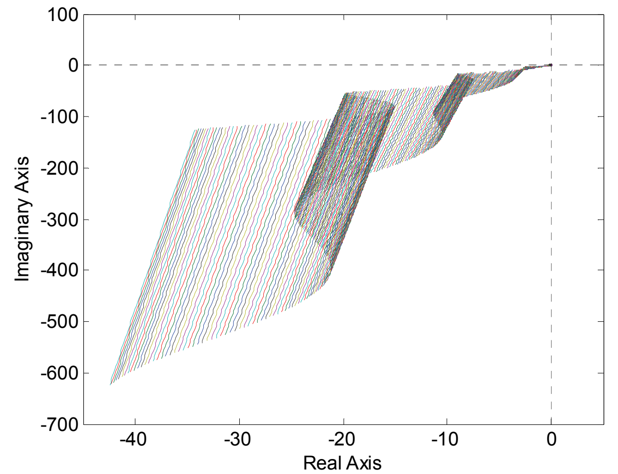

Fig. 13 value sets – plant (3), controller (11)

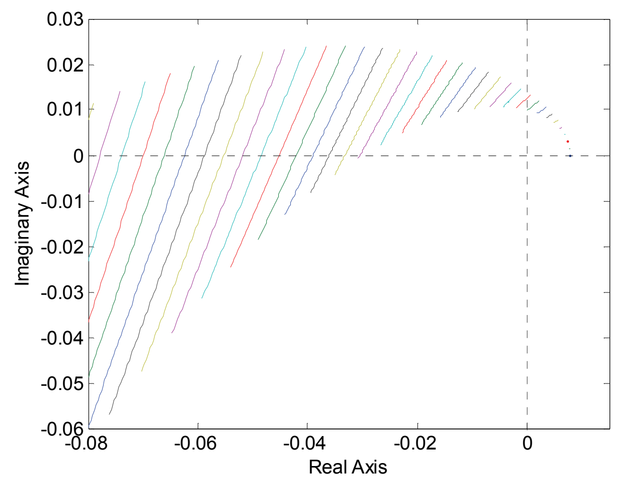

Fig. 14 zoomed value sets – plant (3), controller (11)

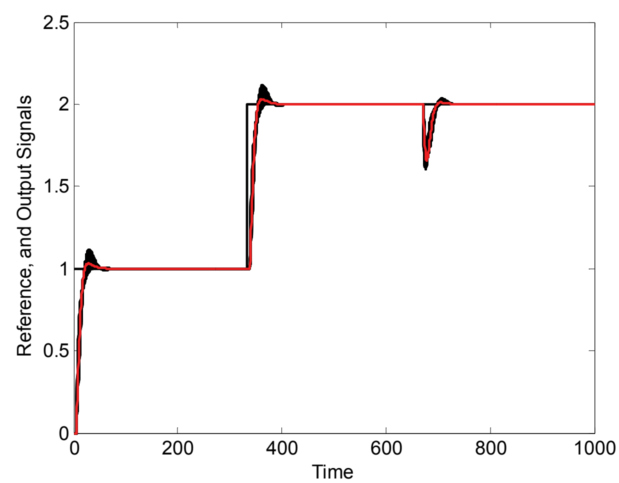

Fig. 15 control outputs – plant (3), controller (11)

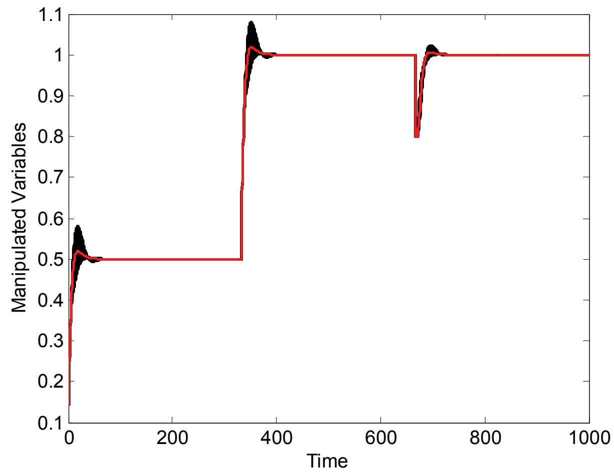

Fig. 16 manipulated variables – plant (3), controller (11)

C. *Uncertain Time-Delay Term*

The final part deals with the case of uncertain time-delay term regarding to (4).

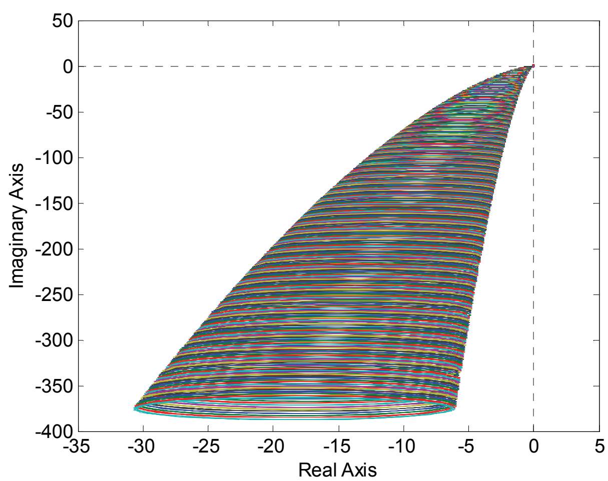

Fig. 17 value sets – plant (4), controller (10)

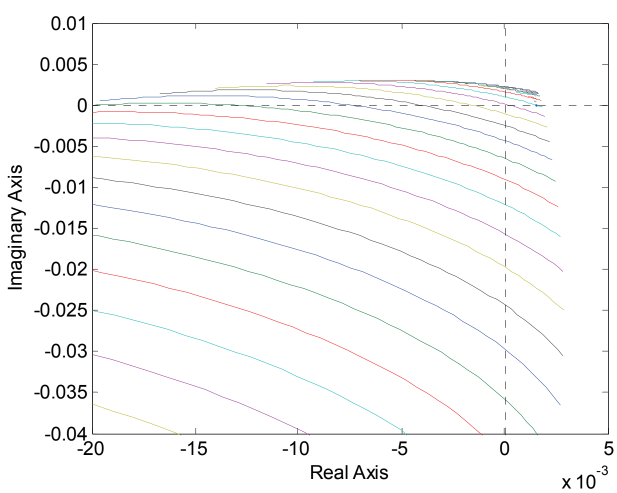

Fig. 18 zoomed value sets – plant (4), controller (10)

For the controller (10), the value sets, zoomed version of the value sets, output signals, and manipulated variables are visualized in figs. 17-20, respectively. The control circuit is robustly unstable.

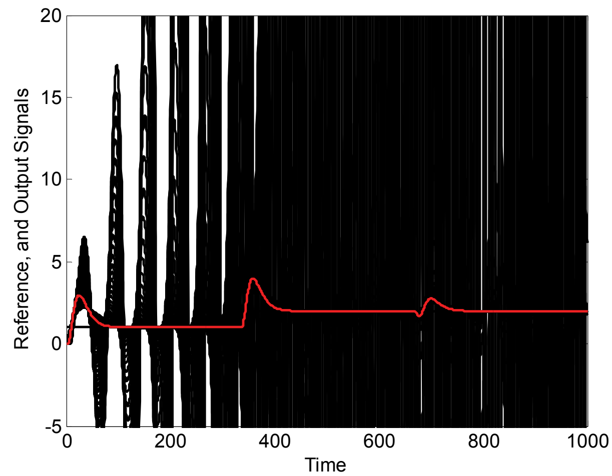

Fig. 19 control outputs – plant (4), controller (10)

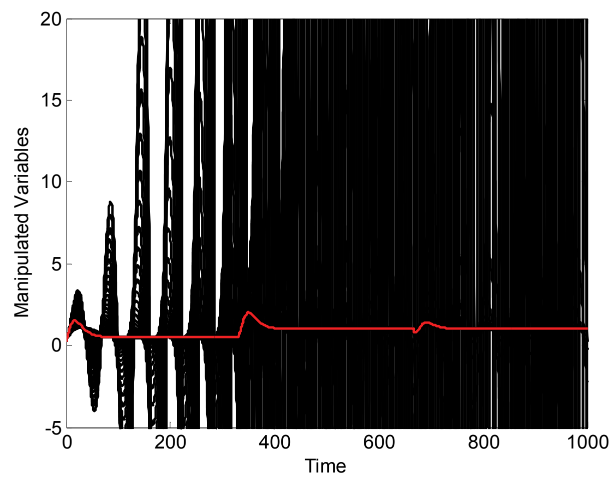

Fig. 20 manipulated variables – plant (4), controller (10)

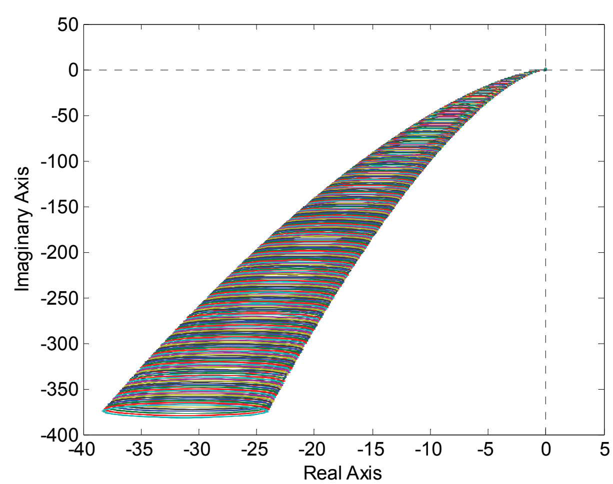

Fig. 21 value sets – plant (4), controller (11)

Analogically, the results for robustly stable scenario (both full view and zoomed versions of the value sets as well as "representative" control responses and manipulated variables), obtained by using the controller (9), are presented in figs. 21-24.

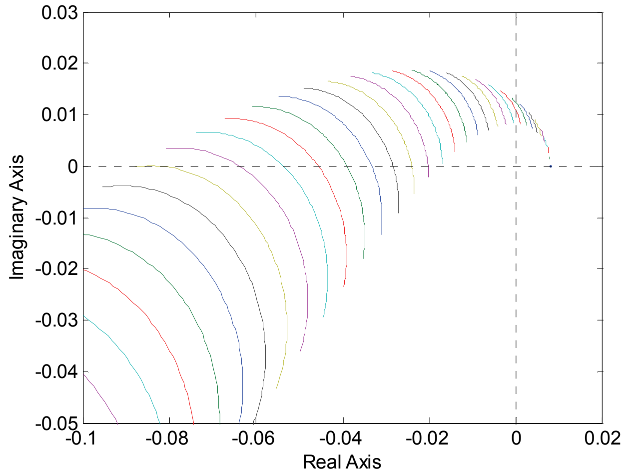

Fig. 22 zoomed value sets – plant (4), controller (11)

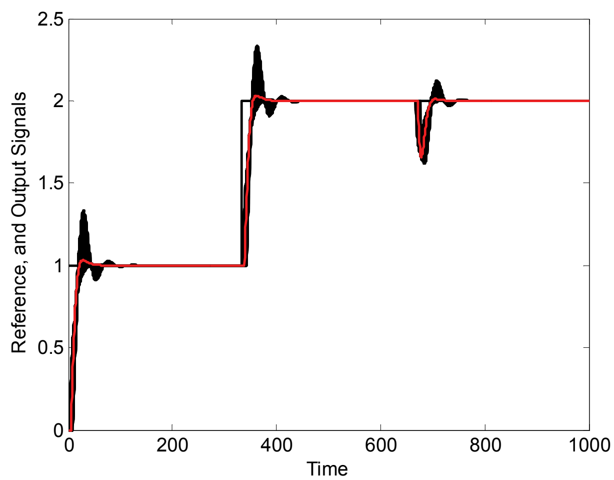

Fig. 23 control outputs – plant (4), controller (11)

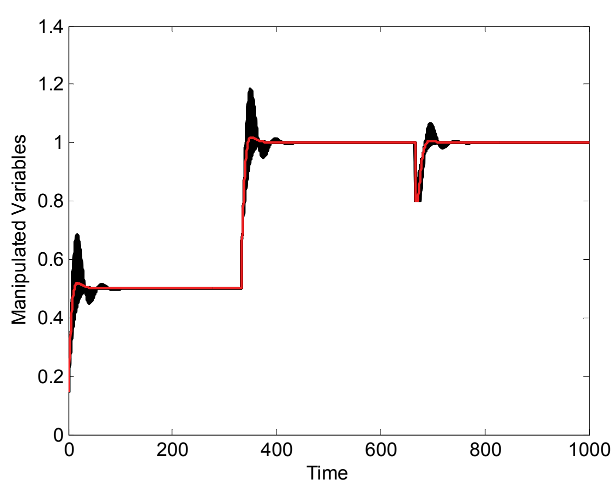

Fig. 24 manipulated variables – plant (4), controller (11)

VI. Conclusion

The contribution has dealt with possible method for robust stabilization of single-input single-output first-order time-delay plants affected by parametric uncertainty. The emphasis was laid especially on the robust stability analysis which was performed by means of plotting the value sets of a family of closed-loop characteristic quasi-polynomials and subsequent application of the zero exclusion condition. The continuous-time controllers utilized in the paper were designed via the general solutions of Diophantine equations in the R_{PS} and then tuned by the single parameter. The set of illustrative examples has been focused on three successive cases of controlled systems with uncertain gain, uncertain time constant and uncertain time-delay term. Robust stability/instability of closed loops containing these plants and two differently tuned PID controllers was investigated and then demonstrated also through the control simulations.

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Radek Matušů was born in Zlín, Czech Republic in 1978. He is a Researcher at Faculty of Applied Informatics of Tomas Bata University in Zlín, Czech Republic. He graduated from Faculty of Technology of the same university with an MSc in Automation and Control Engineering in 2002 and he received a PhD in Technical Cybernetics from Faculty of Applied Informatics in 2007. He worked as a Lecturer from 2004 to 2006. The main fields of his professional interest include robust systems and application of algebraic methods to control design.

Roman Prokop, born in 1952, is a Vice-Dean and a Full Professor at Faculty of Applied Informatics of Tomas Bata University in Zlín, Czech Republic. He graduated from Czech Technical University in Prague in 1976 and received a PhD from Slovak University of Technology in Bratislava in 1983. He was an Associate Professor from 1996 and a Full Professor from 2004. He aims his pedagogical and research work to automatic control theory, algebraic methods in control design and optimization. The main interests of the latest period are uncertain and robust systems, autotuning of controllers and time-delay systems.

Jiri Vojtesek (Ph.D.) was born in Zlin, Czech Republic in 1979 and studied at the Tomas Bata University in Zlin. where he got his master degree in chemical and process engineering in 2002. He has finished his Ph.D. focused on Modern control methods for chemical reactors in 2007. He now works as a Senior Lecturer at Department of Process Control, Faculty of Applied Informatics, Tomas Bata University in Zlin. His research interest are modeling, simulation and control of technological processes.

Tomáš Dulík (Ph.D.) is a Senior Lecturer at Department of Informatics and Artificial Intelligence, Faculty of Applied Informatics, Tomas Bata University in Zlín, Czech Republic. His research interests are digital signal processing, HW/SW codesign, communications, data and mobile networks, etc.