Noise Reduction using Adaptive Singular Value Decomposition

Somkait Udomhunsakul

Abstract— Noise reduction is one of the most essential processes for image processing. The goal of the noise reduction is how to remove noise while keeping the important image features as much as possible. In this paper, a novel method to remove additive noise from digital image, based on the combination of Gaussian filter and the singular value decomposition, is proposed. Firstly, Gaussian filter is used to classify noisy image into two parts, which are its blur and noisy edge images. Next, the noise on noisy edge image, obtained from the difference between the original noisy image and its blur image, is reduced by using an adaptive block-based singular value decomposition filtering (BSVD). Finally, the reconstruction images are obtained from combining between noisy edge image, filtered by an adaptive BSVD filtering, and its original blur image. From the experiments, the objective and subjective measurements prove that the proposed approach compared with traditionally methods can suppress noise, preserve the significant image features as well as effectively smooth in the homogeneous area. Therefore, the proposed method leads to a practical method to be used for noise reduction.

Keywords— Denoising, Noise reduction, Block based SVD Filtering, Gaussian noise, Gaussian Filter

I. Introduction

REMOVING, noise from a signal is an essential issue in the field of digital image processing. In general, when filtering random noise from a noisy image, there are two main issues of noise reduction that need to be considered, which are how much noise had been removed and how well edges are preserved. Hence, the difficult problem of this issue is how to get rid of noise without losing important signal. Nowadays, there are several simple techniques for noise suppression such as Moving average filter and Gaussian filter. However, they can effectively suppress noise but fail to preserve many useful details, being merely a low pass filter [1]. This leads to search for nonlinear filtering alternatives.

In the past decades, there were some researchers introduced an adaptive filter to remove Gaussian noise [2-3] and impulse noise. For instance a new algorithm using the combination of fuzzy logic and unsymmetric trimmed median filter was introduced to remove the impulse noise [4]. Other alternatives, a combination of the applied kFill algorithm and the median filter were used to remove impulse noise in binary, gray scale and color images [5]. In 2005, an algorithm was developed for image noise removal based on local adaptive window size/shape filtering. It can be applied to several problems, including image restoration and visual correspondence [6]. In addition, there had been considerably interest in using the Wavelet Transform as a powerful technique for recovering signal from noisy data. This method is commonly referred to as Wavelet Shrinkage technique. In 1995, a soft thresholding for denoising in 1-D signal was proposed [7]. S. Chang, B. Yu and M. Vetterli introduced an adaptive Wavelet thresholding for image denoising and compression [8]. They proposed a new Shringkage method, BaeyShrink, which outperformed Donoho and Johnstone's Sureshrink. However, Wavelet denoising method has two main drawbacks [9], which are the choice of the threshold and the specific distributions of the signal and noise may not be well matched at different scales. Moreover, Singular Value Decomposition (SVD) is a technique that can be used for noise reduction. In 1997, noise estimation and filtering technique using block-based singular value decomposition filtering (BSVD) was introduced [10,11]. In this technique, the noising image is divided into each 8x8 block size in order to consume quite less computation time. This method was proven to be outperformed soft thresholding. However, this method can perform well to preserve image edge but fail to smooth in the homogeneous region [12].

In this research study, a noise reduction technique based on the combination of Gaussian Filter and BSVD filtering is proposed. The goal of this research study is to present an adaptive BSVD filtering for noise reduction. The proposed approach, instead of applying BSVD filter on the noisy image directly, applies BSVD filtering on the noisy edge image obtained from the difference of the original noisy image and its blur image. The Gaussian filter is applied to original noisy image to get its blur image. Finally, the reconstructed image is obtained from the combination between noisy edge image, filtered by an adaptive BSVD filtering, and its original blur image. From the experiments, the proposed approach compared with traditionally methods can suppress noise, preserve the significant image features as well as effectively smooth in the homogeneous area.

The paper is organized as follows. Section 2 provides the implementation strategies. In section 3, the methodology of proposed approach is provided. Section 4 gives some experimental results to demonstrate the effectiveness of the proposed method compared with the performance of other existing methods applied on a number of test images. Finally, concluding remarks are given in section 5.

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II. Implementation Strategies

A. Gaussian Filter

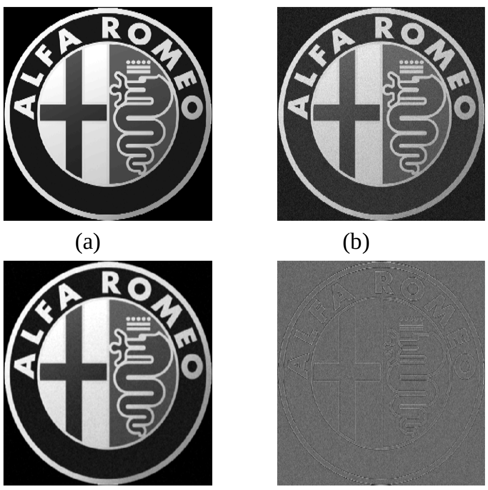
Gaussian filter is a class of low-pass filter based on the Gaussian probability distribution function [1]. Its coefficients can be written as this following equation,

$$f(x,y)=\frac{e^{-\frac{x^2+y^2}{2\sigma_g{}^2}}}{2\pi\sigma_g{}^2} \tag{1}$$

where (x,y) is the position on mask of Gaussian filter and σ_g is the standard deviation of Gaussian filter.

B. Effect of Gaussian Filter

Noise reduction using Gaussian filter is one of the simplest ways to remove noise from signals. However, because this filter is a kind of low pass filters, the signals on low frequency were preserved where as all of the high frequency signals were cut off. The recovered image from this filter definitely loss information on high frequency signals, which are detail or image edges. As a result, the recovered image will lack of sharpness. Therefore, blurring effect is one of the significant problems of this scheme. As can be seen from figure 1, the result of recovered image is smooth, Fig. 1 (c), but quite blur because a lot of noise and image edges are discarded by Gaussian filter, Fig. 1 (d).



(a) (b)

(c) (d)

Fig. 1, Recovered image using Gaussian filter
(a) Original image (b) Noisy image (c) Recovered image using Gaussian filter and (d) Discarded image

C. Singular Value Decomposition

In the theory of singular value decomposition (SVD), every matrix A of $m \times n$ size $(m \geq n)$ can be decomposed into a product of three matrices,

$$A = USV^T = \sum_{i=1}^{n} s_i \vec{u}_i \vec{v}_i^t \tag{2}$$

where U and V are orthogonal matrices with column vectors $\vec{u}_i$ and $\vec{v}_i$, and $S = diag(s_i, s_2, ..., s_n)$, which is a diagonal matrix. The diagonal elements of S can be arranged in a descending order called the singular values of A and [10],

$$s_1 \geq s_2 \geq ... \geq s_n \tag{3}$$

To apply this transformation for noise reduction process, any image is considered as a matrix A and decomposed into three matrices U, S and V. The matrix S consists of singular value on diagonal line by ordering from maximum value, on left-top corner, to minimum values, on right-bottom corner of the matrix. Energy of signal is compressed on large singular values. The idea of this scheme is to preserve only singular values of signal and eliminate the rest of its, which can be assumed as noise. However, the time consumption is needed to carefully consider because the larger size of original matrix, the much more computation time to process. Furthermore, the efficiency to reduce noise is depended on the block size chosen. This causes lead to an idea to separate the whole image into sub-block before performing, this algorithm known as BSVD filtering [10, 11].

D. Effect of Singular Value Decomposition

For Block-based SVD filtering, noisy image is divided into sub-block images and then each sub-block image is transformed to singular value domain. Then each singular value is thresholded by hard-thresholding to eliminate singular value that is less than threshold value and remain singular value that is higher than it. As can be seen from figure 2, recovered image from BSVD filtering is looked not smooth in the homogeneous area. It can be shown that this method cannot preserve the good quality of image in term of smoothness.

(a) (b)

Fig. 2, Recovered image using BSVD filter
(a) Original image (b) Zoom recovered image

III. METHODOLOGY

The proposed method was designed by using the combination of Gaussian and BSVD filters. The advantage of each method are combined in order to get the best performance of noise reduction in terms of keeping detail and reducing noise on smooth area as shown in figure 3. In contrast to apply BSVD on the noisy image directly [10], the proposed algorithm applies BSVD on the noisy edge image obtained from the difference between the original noisy image and its blur image. Finally, the reconstructed image is performed by the combination of the BSVD noise reduction image result with its blur image. However, in order to get the best recovered images, a few parameters should be carefully considered. Those are thresholding value function, optimal variance of Guassian and suitable block size.

A. Thresholding

BSVD filtering was proposed and proved that it can effectively suppress noise while preserving edge details [10]. However, the selection of the threshold value is directly influenced to how much of noise can be removed. Therefore, the optimum threshold value is derived as shown below. From equation (2), it can be spread into

$$A = s_1 \vec{u}_1 \vec{v}_1^t + s_2 \vec{u}_2 \vec{v}_2^t + \dots + s_l \vec{u}_l \vec{v}_l^t \tag{4}$$

where l is the rank of matrix A.

$$|A|^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2 = \sum_{i=1}^{l} s_i^2 \tag{5}$$

where A is the additive zeros mean Gaussian noise of image with σ_n, which can be derived this equation into

$$\sigma_n^2 = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} \left(a_{ij} - \mu_A\right)^2}{m \times n} \tag{6}$$

$$\sigma_n^2 = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2}{m \times n} \quad at\ \mu_A = 0 \tag{7}$$

$$m \times n \times \sigma_n^2 = \sum_{i=1}^{l} s_i^2 \tag{8}$$

From equation (8), it can be concluded that

$$s_1 \le \sqrt{mn}\sigma_n \tag{9}$$

In equation (9), it is shown that every s_i of noise is less than $\sqrt{mn}\sigma_n$. So, we propose to remain all of s_i that higher than $\sqrt{mn}\sigma_n$ and cut off all of s_i that lower than $\sqrt{mn}\sigma_n$. The hard-thresholding function (HBSVD) is adopted for threshold process, which is shown in equation (10), to cut off s_i of noise.

$$HBSVD(s) = \begin{cases} 0 & ,|s| \le T \\ s & ,|s| > T \end{cases} \tag{10}$$

Where

$$T = \sqrt{mn}\sigma_n \tag{11}$$

To estimate σ_N, the median of absolute difference (MAD) is used, [7]

$$\hat{\sigma} = \frac{MAD}{0.6745} = \frac{Median\left(\left|x - Median(x)\right|\right)}{0.6745} \tag{12}$$

The noise variance on the noisy edge image can be expressed as,

$$\sigma_n = k \times \sigma_N \tag{13}$$

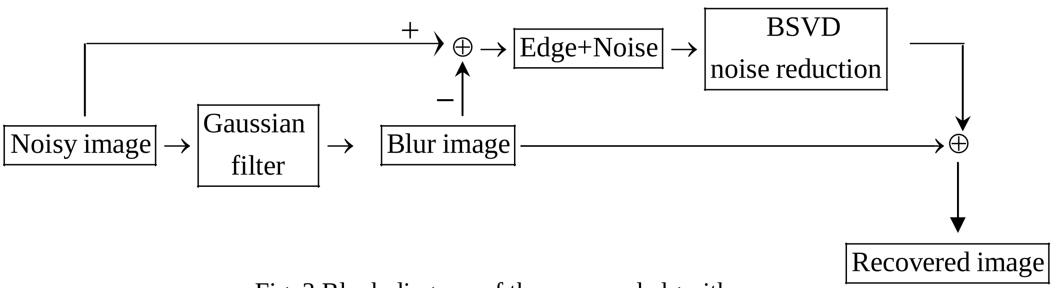


Fig. 3 Block diagram of the proposed algorithm

And

$$k=\sqrt{\frac{\sum_{i=1}^{p}\sum_{j=1}^{q}\left|1-Mask\left(i,j\right)\right|^{2}}{p\times q}} \tag{14}$$

Where $Mask\left(i,j\right)$ is the coefficient of Fourier Transform of Gaussian mask obtained from equation (14) at i^{th} column and j^{th} row. In this case, we use $\hat{\sigma}$ in terms of σ_N. We can estimate σ_n as,

$$\sigma_n = k\hat{\sigma} = \frac{MAD}{0.6745}\times k \tag{15}$$

Finally, the optimal threshold value can be defined as,

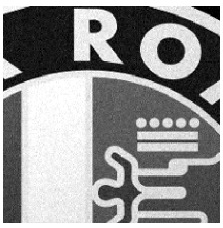
$$T=\left(\frac{k\times MAD}{0.6745}\right)\sqrt{mn} \tag{16}$$

B. *Optimal variance of Gaussian Filter*

Figure 4 shows the results of Gaussian filter at different variances. It can be seen that the image results are looked difference. Therefore, it is essential to select an optimum variance of Gaussian filter in order to get the best performance. The experiment to select the optimal variance of Gaussian filter is demonstrated in section 4.


(a)


(b)

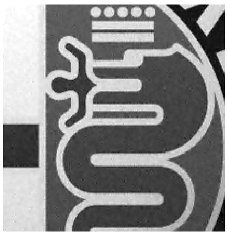

(c)


(d)

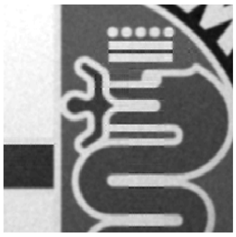
Fig. 4 Image results using diverse variances of Gaussian Filter (a) 0.5 (b) 0.8 (c) 1.2 and (d) 1.6

C. *Optimal block size of BSVD*

Figure 5 shows the results of noise reduction using BSVD at different block sizes. It can be seen that the image results are looked difference. Therefore, it is essential to select an optimum block size of BSVD. The experiment to select the optimal block size of BSVD is demonstrated in section 4.


(a)


(b)


(c)

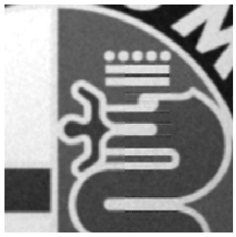

(d)

Fig. 5 Image results using different block sizes of BSVD (a) 4 (b) 8 (c) 16 and (d) 32

D. *Objective image quality assessment*

In general, image quality evaluation can be classified into two methods, which are subjective measurement and objective measurement. For objective measurement, it is save time more than subjective quality measurement [13]. The three objective measurements are selected and used for this research work study.

1. Peak signal to noise ratio (PSNR)

PSNR is the ratio between the maximum possible power of a clean signal and the power of corrupting noise on that clean image. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale. It is most easily defined via the mean squared error (MSE) defined as

$$MSE = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}e^{2}(m,n) \tag{17}$$

Where $m\times n$ is the dimension of image and $e^2(m,n)$ is the error or noisy signal on the position (m,n). The PSNR is defined as:

$$PSNR = 10\log\left(\frac{MAX_i^{\,2}}{MSE}\right) \quad dB \tag{18}$$

Here, MAX_i is the maximum possible pixel value of the image. When the pixels are represented using 8 bits per sample, MAX_i is equivalent to 255. In fact, the higher PSNR value, the better quality of recovered image.

2. Edge measurement

Edge measurement is used to evaluate the efficient of detail preservation of reconstructed image. The equation of edge measurement can be written as

$$Edge = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(Q(i,j)-\hat{Q}(i,j)\right)^2 \tag{19}$$

Where $Q(i,j)$ and $\hat{Q}(i,j)$ are Edge Gradients of original image and noisy image by using Sobel operator, respectively. In fact, the lower edge measurement value, the better edge preservation quality of recovered image.

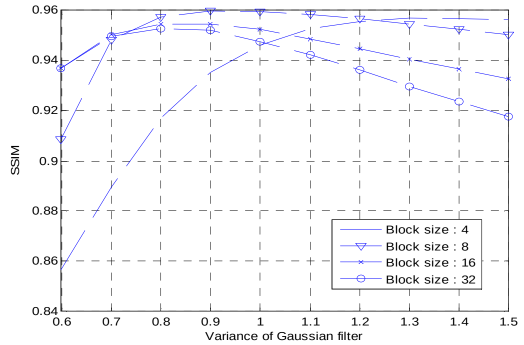
3. Structural Similarity Index

Since PSNR cannot be correlated well with the human perception, we also adopt to use Structural Similarity Index (SSIM), proven to be more correlated well with MOS (Mean Opinion Score), to further evaluate the performance of our proposed approach [14]. The SSIM index can be viewed as a quality measure of any noisy image by considering original image as a perfect image and comparing these two images in terms of luminance contrast and structure aspects. The SSIM was measured by using mean value of these three comparisons.

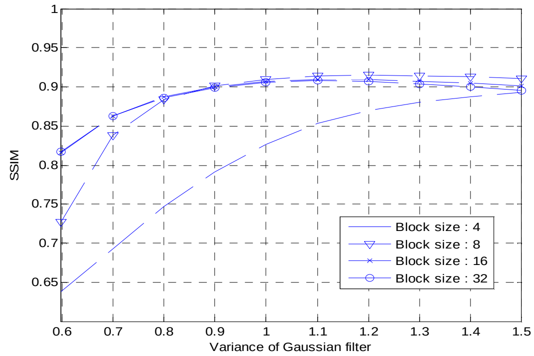
IV. EXPERIMENTAL RESULTS

A. Optimal variance of Gaussian Filter and block size

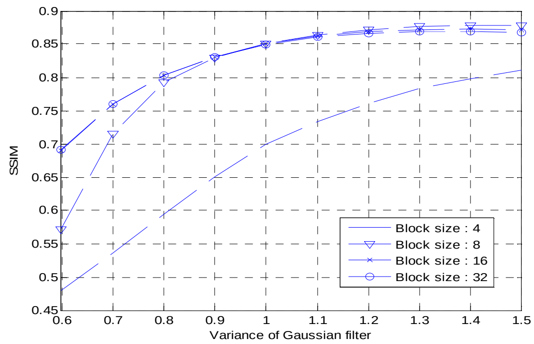
Variance of Gaussian filter and block size affect to efficiency of proposed algorithm, not only in terms of the complexity but the recovered image. In this experiment, the block sizes were varied from 4, 8, 16 and 32, which have less complexity than others. Also, variances of Gaussian filter were varied from 0.6 to 1.5 at noise variances 25, 100 and 225, respectively, testing on a cameraman image, size 256x256, figure 7 (b). As can be seen from figure 6(a), 6(b) and 6(c), the optimum block sizes for each variance of Gaussian filter and noise are different. However, the best performance among every noise variance on the optimal block size of BSVD filter is equivalent to 8. Also, the optimal variance for Gaussian filter at block size 8 for several noise variances is about 1 as shown in figure 6 (d). Therefore, these values were used to perform and test on proposed algorithm in the next experiment.



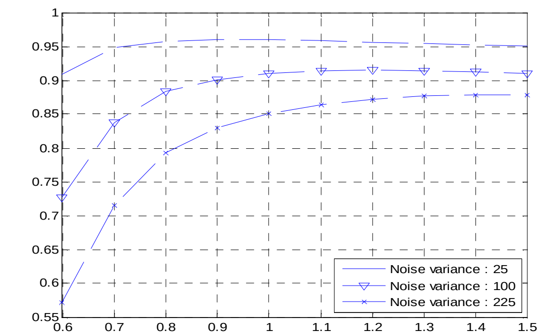
(a)



(b)

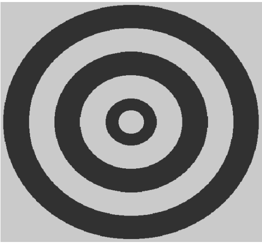


(c)



(d)

Fig. 6 SSIM values of Cameraman image on several noise variances (a) Noise variance = 25 (b) Noise variance = 100 (c) Noise variance = 255 (d) Noise variance 25, 100 and 225 at block size 8



(a)

(b)

Fig. 7 Original test images
(a) Simulated circle image (b) Cameraman image

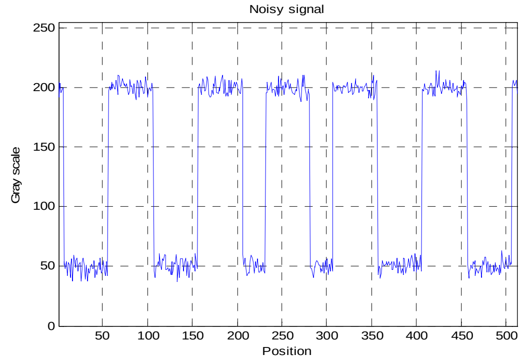
B. Proposed method testing

Firstly, simulated circle image, figure 7 (a), was used to empirical evaluate proposed algorithm compared with three traditional methods, Gaussian filter, Block-based SVD with block size 8 and thresholding value 30, 60 and 85 for noise variance 25, 100 and 225, respectively and Discrete Wavelet Transform (DWT) with db4 thresholded by universal soft-thresholding. The result in Table 1 and figure 8, it can be concluded that proposed method leads to the competitive algorithm among any other methods.

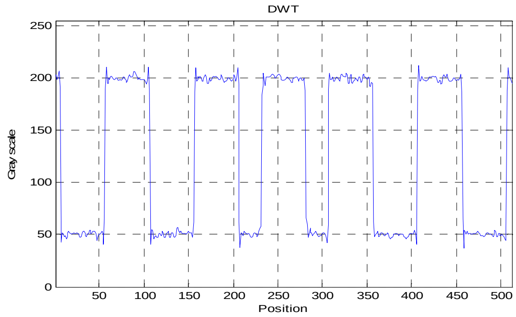
TABLE I
THE EFFICIENCY OF PROPOSED METHOD ON CIRCLE IMAGE, COMPARED WITH PSNR, EDGE MEASUREMENT AND SSIM

Noise variances	Methods	PSNR	Edge	SSIM
25	Gaussian	28.386	0.034	0.934
	DWT	35.135	0.007	0.919
	BSVD	39.288	0.003	0.926
	Proposed method	**40.733**	**0.002**	**0.973**
100	Gaussian	27.984	0.038	0.860
	DWT	30.585	0.025	0.765
	BSVD	33.192	0.013	0.769
	Proposed method	**34.473**	**0.008**	**0.905**
225	Gaussian	27.422	0.045	0.765
	DWT	28.014	0.049	0.613
	BSVD	29.422	0.032	0.612
	Proposed method	**30.663**	**0.02**	**0.82**

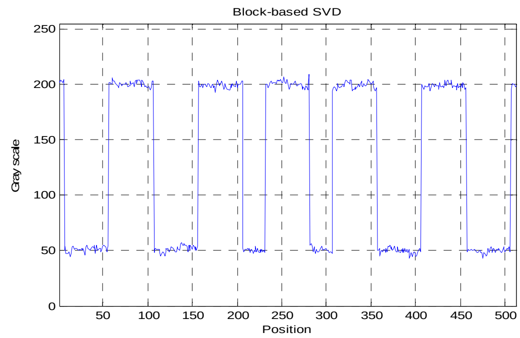
Secondly, the realistic images on different contents were used to further evaluate the efficiency of each algorithm in terms of subjective measurement. The proposed algorithm was used to test on two sample images, butterfly and boat images, to evaluate reconstructed images comparing with other methods. The results were shown in figure 9 and 10. As can be seen, Gaussian filter can produce smooth images but much detail was discarded. For Block-based SVD, although the detail of image is merely complete, the output seems to be looked not smooth in the homogenous area. As for DWT, the output was quite blurring. Therefore, the proposed method outperforms others in terms of sharpness and smoothness.



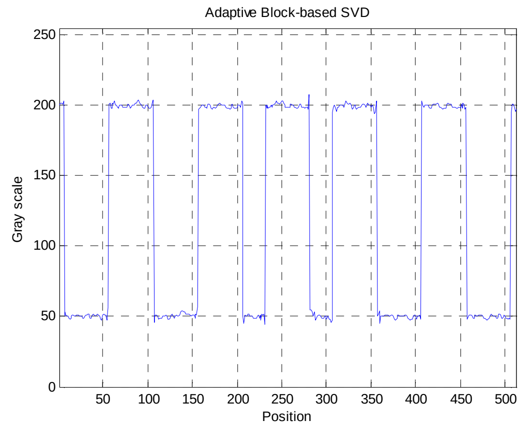
(a)



(b)



(c)



(d)

Fig. 8 Horizontal line image profile at the centre of recovered image on each method (a) Signal profile at noise variance 25 (b) Recovered signal using DWT (c) Recovered signal using BSVD (d) Recovered signal using proposed method

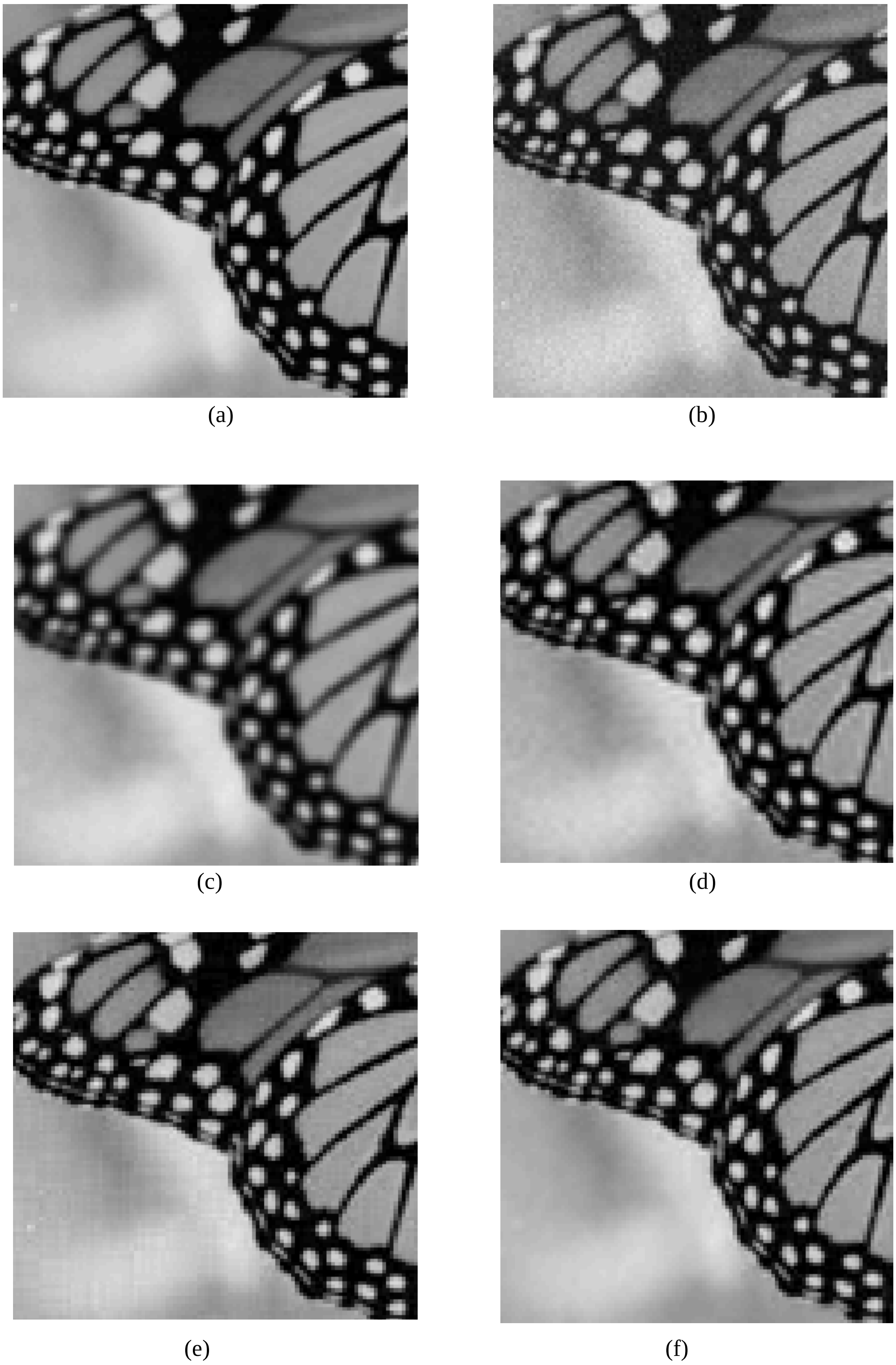


Fig. 9 Reconstructed image from each method
(a) Original image (b) Noisy image (c) Gaussian filter
(d) DWT (e) Block-based SVD and (f) proposed method

(a)

(b)

(c)

(d)

(e)

(f)

Fig. 10 Reconstructed image from each method
(a) Original image (b) Noisy image (c) Gaussian filter
(d) DWT (e) Block-based SVD and (f) proposed method

Finally, the simulation results obtained from three images of size 256x256, 8-bpp (Woman, Cameraman and Peppers) and four images of size 512x512, 8-bpp (Lenna, Barbara, Flower and Plane). The original images are shown in figure 11. Each image was corrupted with additive Gaussian noise of zero mean and $\sigma = 5$ and 10. Further, Gaussian filter, Wavelet denoising level 1 (db4 Soft thresholding function $\hat{\sigma} = \frac{MAD(HH_1)}{0.6745}$, $T = \hat{\sigma}\sqrt{2\log_{10}(m \times n)}$), BSVD algorithm (block size 8x8) are used to compared with the proposed approach. The threshold of BSVD was determined by using equation 13. In the experiments, Gaussian filter with $\sigma_g = 1$, so k for this experiment is equal to 0.8725. The image quality assessment in this section, only SSIM is adopted due to its proven to be more correlated well with MOS (Mean Opinion Score). The block size of BSVD 8x8 is chosen. The results are tabulated in table II and III. From the experimental results, the proposed method performs the best in term of SSIM.

TABLE II
SSIM VALUES OF RECONSTRUCTION IMAGES AT $\sigma = 5$

Images	Moving Average	Gaussian Filter	BSVD Filtering	Soft-Thresholding	Proposed method
Cameraman	0.83339	0.83475	0.92111	0.88296	**0.92868**
Peppers	0.92665	0.92719	0.92803	0.93098	**0.95206**
Girl	0.9183	0.91884	0.92463	0.92492	**0.94816**
Airplane	0.92852	0.92906	0.91077	0.91593	**0.94729**
Barbara	0.75577	0.75794	0.92752	0.88732	**0.93641**
Flowers	0.94548	0.94583	0.92872	0.94084	**0.95315**
Lenna	0.89889	0.89945	0.8968	0.89837	**0.92478**

TABLE III
SSIM VALUES OF RECONSTRUCTION IMAGES AT $\sigma = 10$

Images	Moving Average	Gaussian Filter	BSVD Filtering	Soft-Thresholding	Proposed method
Cameraman	0.78619	0.78633	0.7933	0.77033	**0.85892**
Peppers	0.8889	0.88898	0.83101	0.84643	**0.91032**
Girl	0.87974	0.87978	0.82186	0.83833	**0.90949**
Airplane	0.8742	0.87436	0.78412	0.80372	**0.89805**
Barbara	0.72143	0.7217	0.82844	0.78038	**0.88356**
Flowers	0.90739	0.9074	0.81441	0.86208	**0.91586**
Lenna	0.85218	0.85222	0.77703	0.7997	**0.87967**

V. CONCLUSION AND FUTURE WORK

This research presents an effective method for noise reduction using an adaptive block based Singular value decomposition (BSVD), which can preserve edge (detail information) as well as effectively smooth in the homogeneous region. The effectiveness of the proposed approach depends on the accuracy of the threshold value. Then, the optimal threshold value of the proposed approach is derived and defined. In addition, the proposed approach is compared with other traditional methods. The comparison suggests that the proposed method is competitive and outperform with other methods. Therefore, our method leads to an effective method that can be used for noise reduction. In the future work, presently, work is on to extend the work for another noise type even on color image processing.

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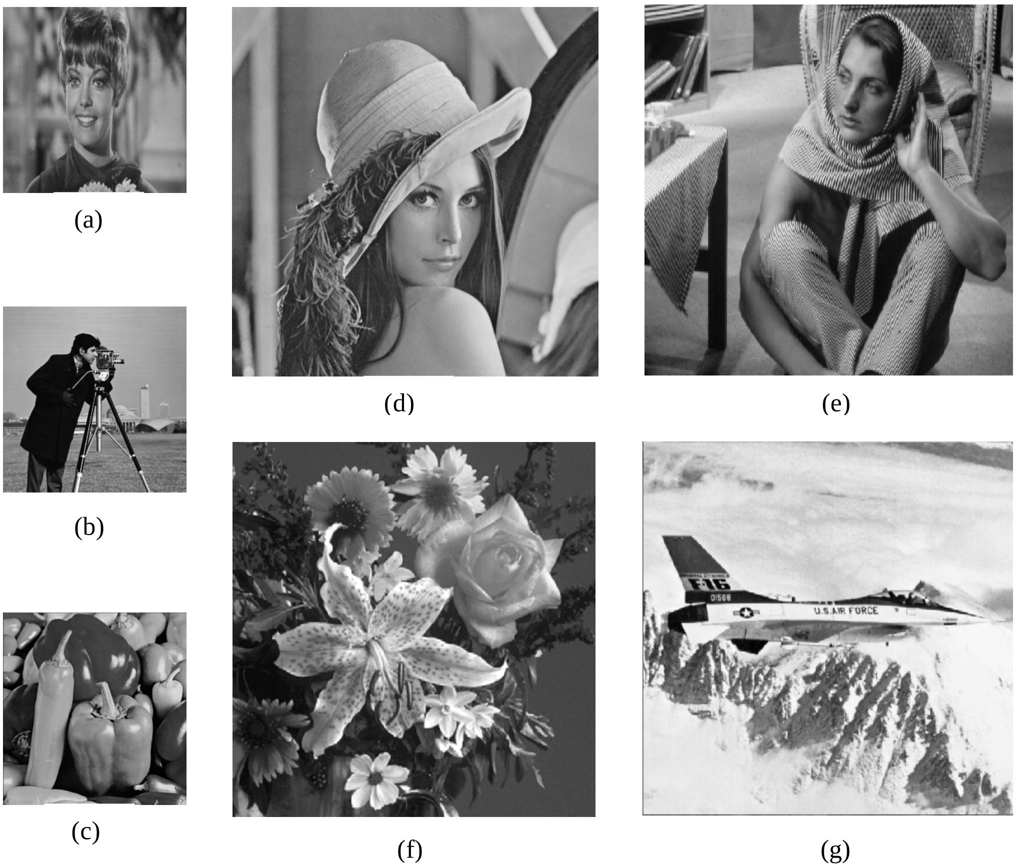

Fig. 11 (a), (b) and (c) are Woman, Cameraman and Peppers images, size 256X256
(d), (e), (f) and (g) are Lenna, Barbara, Flowers and Airplane images, size 512X512