Development of Robust Self-Tuning Control for MIMO Linear Systems with dead-zone approach

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***Abstract*—**The subject of this paper is the development of a robust self-tuning control for multi-input multi-output (MIMO) linear systems with unknown time-varying parameters, which can be described by auto-regressive exogenous mathematical models, in presence of unmodelled dynamics. We propose an explicit self-tuning control scheme which is based on the modified recursive least squares estimation algorithm with known dead zone. The considered system is represented by interconnected systems with multi-dead-zone. A recursive parametric estimation algorithm is developed, which can estimate the parameters intervening in the considered system. The stability conditions of the developed estimation scheme are established on the basis of the Lyapunov method. A simulation example is treated to test the performances of the developed explicit self-tuning control scheme.

***Keywords*—**ARX mathematical models, MIMO systems, Recursive parametric estimation algorithm, Robust self-tuning control, Stability.

I. Introduction

SEVERAL parametric estimation algorithms and robust adaptive control schemes were developed on the basis of monovariable ARX (Auto-Regressive eXogenous) mathematical models in presence of unmodelled dynamics see [1] and [2]. In this case, a relative dead zone is employed in the parametric estimation algorithm [3-7]. Here, the implementation of the dead zone depends on the upper bound of the unmodelled dynamics and of the disturbances.

Different robust adaptive control schemes of SISO (Single-Input Single Output) linear systems, which can be described by the ARX mathematical models with unknown time-varying parameters, were developed on the basis of the acknowledged information of the parameters of bounding function (the unmodelled dynamics and disturbances). Among those schemes are: robust adaptive pole placement control based on the recursive least squares estimation algorithm with dead zone (see [4]), robust model reference adaptive control (see [3]), robust self-tuning control based on the modified RLS estimation algorithm with dead zone, where the stability of adaptive control schema was been established (see [5]).

In this paper, we present a robust self-tuning control for MIMO (Multi-Inputs Multi-Outputs) linear time-varying systems, which are described by ARX mathematical models, in presence of unmodelled dynamics. The formulation problem of self-tuning control of MIMO systems is studied and published in the literature (see [8]).

Study of MIMO systems, which are constituted of interconnected systems, has attracted the attention of several researchers (see [9-11]). The key idea is to decompose the MIMO system in several interconnected systems. In each interconnected system, the unmodelled dynamic is present. Here, we present some relative dead zones. The parametric estimation of each interconnected system is required, by using the modified RLS estimation algorithm with dead zone. The stability of the estimation algorithm of the parameters of MIMO system depends on the stability of each estimation algorithm employed to estimate the parameter of the concerned interconnected system. The stability analysis of the proposed parametric estimation scheme was studied on the basis of the Lyapunov method.

The rest of this paper is organized as follows. In Section 2, we describe the considered class of MIMO systems by the ARX mathematical models in presence of unmodelled dynamics. A decomposition approach of a MIMO system into interconnected systems is considered. Section 3 presents modified Recursive Least Squares (RLS) estimation algorithm with a relative dead zone, which permits to estimate the parameters intervening of each interconnected system. The stability conditions of the developed recursive parametric estimation algorithm are established on the basis upon the Lyapunov method. In section 4, the robust self-tuning control problem, which can be applied to the considered class of MIMO systems, is solved. Section 5 treats a numerical simulation example for illustrative purposes, where the self-

This work was supported in part by the ministry of higher education and scientific research in Tunisia.

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tuning control explicit scheme on the basis of the RLS parametric estimation algorithm with forgetting factor has been used to compare different simulation results obtained by the two control scheme. We conclude in Section 6.

II. System Description

Let us consider a MIMO system, which is constituted by m inputs and m outputs. The considered system can be described by the following ARX mathematical model:

$$A_C(q^{-1},k)y(k) = q^{-d}B_C(q^{-1},k)u(k) + e(k) \tag{1}$$

where $u(k)$, $y(k)$ and $e(k)$ represent the inputs, the outputs and the noises of the system at the discrete-time k, respectively, such that:

$$u^T(k) = [u_1(k) \cdots u_m(k)] \tag{2}$$

$$y^T(k) = [y_1(k) \cdots y_m(k)] \tag{3}$$

$$e^T(k) = [e_1(k) \cdots e_m(k)] \tag{4}$$

d represents the time-delay, and $A_C(q^{-1},k)$ and $B_C(q^{-1},k)$ are polynomial matrices given by:

$$A_C(q^{-1},k) = A(q^{-1},k) + \varsigma_A(q^{-1},k) \tag{5}$$

$$B_C(q^{-1},k) = B(q^{-1},k) + \varsigma_B(q^{-1},k) \tag{6}$$

with

$$A(q^{-1},k) = I_{m\times m} + A_1(k)q^{-1} + \cdots + A_{na}(k)q^{-na} \tag{7}$$

$$B(q^{-1},k) = I_{m\times m} + B_1(k)q^{-1} + \cdots + B_{nb}(k)q^{-nb} \tag{8}$$

$$\varsigma_A(q^{-1},k) = \varsigma_{A_1}(k)q^{-1} + \cdots + \varsigma_{A_{na}}(k)q^{-na} \tag{9}$$

$$\varsigma_B(q^{-1},k) = \varsigma_{B_1}(k)q^{-1} + \cdots + \varsigma_{B_{nb}}(k)q^{-nb} \tag{10}$$

where na and nb represent the orders of the polynomial matrices $A(q^{-1},k)$ and $B(q^{-1},k)$, respectively, and $\varsigma_A(q^{-1},k)$ and $\varsigma_B(q^{-1},k)$ are the unmodelled dynamics present in the considered system.

In the following, let us suppose that the parameters d, na and nb are known.

The considered ARX mathematical model can be written as follows:

$$A(q^{-1},k)y(k) = q^{-d}B(q^{-1},k)u(k) + v(k) \tag{11}$$

where $v(k)$ is defined by:

$$v(k) = -\varsigma_A(q^{-1},k)y(k) + q^{-d}\varsigma_B(q^{-1},k)u(k) + e(k) \tag{12}$$

In order to simplify the description of the dead zone, we propose to decompose the considered MIMO system into m interconnected systems. We can write the following matrices, with: $t = 1,\dots,na$, $l = 1,\dots,nb$:

$$A_t(k) = \begin{bmatrix} a_{11,t}(k) \cdots a_{1m,t}(k) \\ \vdots \quad \ddots \quad \vdots \\ a_{m1,t}(k) \cdots a_{mm,t}(k) \end{bmatrix} \tag{13}$$

$$\varsigma_{A_t}(k) = \begin{bmatrix} \varsigma_{a_{11,t}}(k) \cdots \varsigma_{a_{1m,t}}(k) \\ \vdots \quad \ddots \quad \vdots \\ \varsigma_{a_{m1,t}}(k) \cdots \varsigma_{a_{mm,t}}(k) \end{bmatrix} \tag{14}$$

$$B_l(k) = \begin{bmatrix} b_{11,l}(k) \cdots b_{1m,l}(k) \\ \vdots \quad \ddots \quad \vdots \\ b_{m1,l}(k) \cdots b_{mm,l}(k) \end{bmatrix} \tag{15}$$

$$\varsigma_{B_l}(k) = \begin{bmatrix} \varsigma_{b_{11,l}}(k) \cdots \varsigma_{b_{1m,l}}(k) \\ \vdots \quad \ddots \quad \vdots \\ \varsigma_{b_{m1,l}}(k) \cdots \varsigma_{b_{mm,l}}(k) \end{bmatrix} \tag{16}$$

The output $y_i(k)$ of the i^{th} interconnected system, $i = 1,\dots,m$, is given as follows:

$$\begin{aligned} y_i(k) = &-\sum_{t=1}^{na}\sum_{f=1}^{m} a_{if,t}(k)y_f(k-t) \\ &-\sum_{t=1}^{na}\sum_{f=1}^{m} \varsigma_{a_{if,t}}(k)y_f(k-t) \\ &+\sum_{l=1}^{nb}\sum_{f=1}^{m} b_{if,l}(k)u_f(k-d-l) \\ &+\sum_{l=1}^{nb}\sum_{f=1}^{m} \varsigma_{b_{if,l}}(k)u_f(k-d-l) + e_i(k) \end{aligned} \tag{17}$$

or

$$y_i(k) = \sum_{l=1}^{nb} b_{i,l}^T(k)u(k-d-l) + \sum_{l=1}^{nb} \varsigma_{b_{i,l}}^T(k)u(k-d-l) - \sum_{t=1}^{na} a_{i,t}^T(k)y(k-t) - \sum_{t=1}^{na} \varsigma_{a_{i,t}}^T(k)y(k-t) + e_i(k) \tag{18}$$

with

$$a_{i,t}^T(k) = [a_{i1,t}(k) \cdots a_{im,t}(k)] \tag{19}$$

$$\varsigma_{a_{i,t}}^T(k) = [\varsigma_{a_{i1,t}}(k) \cdots \varsigma_{a_{im,t}}(k)] \tag{20}$$

$$b_{i,l}^T(k) = [b_{i1,l}(k) \cdots b_{im,l}(k)] \tag{21}$$

$$\varsigma_{b_{i,l}}^T(k) = [\varsigma_{b_{i1,l}}(k) \cdots \varsigma_{b_{im,l}}(k)] \tag{22}$$

We can write the following expressions:

$$\theta_i^T(k) = [a_{i,1}^T(k) \cdots a_{i,na}^T(k)\ b_{i,1}^T(k) \cdots b_{i,nb}^T(k)] \tag{23}$$

$$\varsigma_i^T(k) = [\varsigma_{a_{i,1}}^T(k) \cdots \varsigma_{a_{i,na}}^T(k)\ \varsigma_{b_{i,1}}^T(k) \cdots \varsigma_{b_{i,nb}}^T(k)] \tag{24}$$

$$\varphi^T(k) = [-y^T(k-1) \cdots -y^T(k-na)\ u^T(k-d-1) \cdots u^T(k-d-nb)] \tag{25}$$

$$v_i(k) = \varsigma_i^T(k)\varphi(k) + e_i(k) \tag{26}$$

The output $y_i(k)$ given in equation (18) becomes:

$$y_i(k) = \theta_i^T(k)\varphi(k) + v_i(k) \tag{27}$$

We suppose that the upper bound ρ_i of $\varsigma_i(k)$ is known and the noise $e_i(k)$, with zero mean and variance σ_i^2, is bounded with upper bound m_{oi}.

The upper bound of $v_i(k)$ is given by the parameter $d_i(k)$, such that:

$$d_i(k) = \rho_i \|\varphi(k)\| + m_{oi} \tag{28}$$

Noting that the parameter $d_i(k)$ corresponds to a relative dead zone, which is used in the parametric estimation algorithm of each interconnected system.

III. Parametric estimation

In this section, we treat the parametric estimation problem of a linear stochastic MIMO system, which can be described by the considered ARX mathematical model.

A. Recursive Parametric Estimation Algorithm

The parameters intervening in the vector $\theta_i(k)$ of the mathematical model (27), as described by (23), can be estimated by using the following modified least squares parametric estimation algorithm with a dead zone:

$$\begin{aligned} \hat{\theta}_i(k) &= \hat{\theta}_i(k-1) + \delta_i(k)P(k-1)\varphi(k)\zeta_i(k)/L(k) \\ P(k) &= P(k-1) - \delta_i(k)[P(k-1)\varphi(k)\varphi^T(k)P(k-1)/L(k)] \\ \zeta_i(k) &= y_i(k) - \varphi^T(k)\hat{\theta}_i(k-1) \\ L(k) &= \mu_i + \varphi^T(k)P(k-1)\varphi(k) \end{aligned} \tag{29}$$

where $\zeta_i(k)$ represents the prediction error, μ_i is a constant parameter and $\delta_i(k)$ is a time-varying parameter, such that:

$$\delta_i(k) = \begin{cases} 0 \text{ if } \|\zeta_i(k)\| \le \beta_i d_i(k) \\ \gamma_i \text{ otherwise }, \gamma_i \in [\sigma_{0i}, 1/\beta_i - \sigma_{0i}] \end{cases} \tag{30}$$

with: $\sigma_{0i} \in [0,1]$, $\beta_i > 2/(1+\sigma_{0i})$, $\mu_i \in [0,1]$.

Noting that the estimation of the parameters of the considered MIMO system described by (1) corresponds to the estimation of the parameters of the m interconnected systems, which are described by (27), using the modified RLS parametric estimation algorithm with a dead zone.

B. Stability Analysis of the Parametric Estimation Scheme

In this subsection, we establish the stability conditions of the developed parametric estimation scheme, as given by (29), on the basis of the Lyapunov method.

The prediction error $\zeta_i(k)$ given in (29) can be rewritten as follow:

$$\zeta_i(k) = v_i(k) - \varphi^T(k)\tilde{\theta}_i(k-1) \tag{31}$$

where $\tilde{\theta}_i(k-1) = \hat{\theta}_i(k-1) - \theta_i(k-1)$ represents the vector of the parameter estimation error.

We can rewrite the estimated vector $\hat{\theta}_i(k)$ as follows:

$$\hat{\theta}_i(k) = \hat{\theta}_i(k-1) + \delta_i(k)P(k-1)\varphi(k)\zeta_i(k)/L(k) \tag{32}$$

Using (31) and (32), the vector of the parameter estimation error $\tilde{\theta}_i(k)$ can be given by:

$$\begin{aligned}\tilde{\theta}_i(k) = \tilde{\theta}_i(k-1) - \delta_i(k)[P(k-1)\varphi(k)\varphi^T(k)/L(k)]\tilde{\theta}_i(k-1) \\ + \delta_i(k)P(k-1)\varphi(k)/L(k)v_i(k)\end{aligned} \quad (33)$$

or

$$\begin{aligned}\tilde{\theta}_i(k) = P(k)P^{-1}(k-1)\tilde{\theta}_i(k-1) \\ + \delta_i(k)P(k-1)\varphi(k)v_i(k)/L(k)\end{aligned} \quad (34)$$

In the following, we propose to analyze the stability conditions of the developed recursive parametric estimation scheme on the basis of the Lyapunov method. Thus, let us consider the following Lyapunov function $V_i(k)$:

$$V_i(k) = \tilde{\theta}_i^T(k)P^{-1}(k)\tilde{\theta}_i(k) \quad (35)$$

Using (32), the quantity $P^{-1}(k)\tilde{\theta}_i(k)$ can be writing as follows:

$$\begin{aligned}P^{-1}(k)\tilde{\theta}_i(k) = P^{-1}(k-1)\tilde{\theta}_i(k-1) \\ + \delta_i(k)[P^{-1}(k)P(k-1)\varphi(k)/L(k)]v_i(k)\end{aligned} \quad (36)$$

where the vector of the parameter estimation error $\tilde{\theta}_i(k)$ is given by:

$$\begin{aligned}\tilde{\theta}_i^T(k) = \tilde{\theta}_i^T(k-1)P(k)P^{-1}(k-1) \\ + \delta_i(k)\varphi^T(k)P(k-1)v_i(k)/L(k)\end{aligned} \quad (37)$$

Using (36) and (37), the Lyapunov function $V_i(k)$ becomes:

$$\begin{aligned}V_i(k) = 2\delta_i(k)[\varphi^T(k)\tilde{\theta}_i(k-1)/L(k)]v_i(k) \\ + \tilde{\theta}_i^T(k-1)P(k)P^{-1}(k-1)P^{-1}(k-1)\tilde{\theta}_i(k-1) + \delta_i^2(k) \\ .[\varphi^T(k)P(k-1)P^{-1}(k)P(k-1)\varphi(k)v_i^2(k)/L^2(k)]\end{aligned} \quad (38)$$

with

$$\begin{aligned}P(k)P^{-1}(k-1)P^{-1}(k-1) = P^{-1}(k-1) \\ - \delta_i(k)\varphi(k)\varphi^T(k)/L(k)\end{aligned} \quad (39)$$

Thus, we can write the Lyapunov function $V_i(k)$ in the following form:

$$\begin{aligned}V_i(k) = \tilde{\theta}_i^T(k-1)P^{-1}(k-1)\tilde{\theta}_i(k-1) \\ - \delta_i(k)\tilde{\theta}_i^T(k-1)\varphi(k)\varphi^T(k)\tilde{\theta}_i(k-1)/L(k) \\ + 2\delta_i(k)[\varphi^T(k)\tilde{\theta}_i(k-1)/L(k)]v_i(k) + \delta_i(k)^2 \\ .[\varphi^T(k)P(k-1)P^{-1}(k)P(k-1)\varphi(k)v_i(k)^2/L^2(k)]\end{aligned} \quad (40)$$

Using the matrix inversion lemma, the matrix $P^{-1}(k)$ can be expressed as:

$$P^{-1}(k) = P^{-1}(k-1) + \delta_i(k)\varphi(k)\varphi^T(k)/L(k) \quad (41)$$

Thus, we can write:

$$\begin{aligned}\varphi^T(k)P(k-1)P^{-1}(k)P(k-1)\varphi(k)/L(k) \\ = \varphi^T(k)P(k-1)\varphi(k)/(L(k)L1(k))\end{aligned} \quad (42)$$

such that:

$$L1(k) = (\mu_i + (1-\delta_i(k))\varphi^T(k)P(k-1)\varphi(k)) \quad (43)$$

The variation of the considered Lyapunov function $V_i(k)$, noted by: $\Delta V_i(k) = V_i(k) - V_i(k-1)$, can be expressed as follows:

$$\begin{aligned}\Delta V_i(k) = \delta_i^2(k)\varphi^T(k)P(k-1)\varphi(k) \\ .[v_i^2(k)/(L(k)L1(k))] + \delta_i(k)\varphi^T(k)\tilde{\theta}_i(k-1) \\ .[(2v_i(k) - \varphi^T(k)\tilde{\theta}_i(k-1))/L(k)]\end{aligned} \quad (44)$$

with

$$V_i(k-1) = \tilde{\theta}_i^T(k-1)P^{-1}(k-1)\tilde{\theta}_i(k-1) \quad (45)$$

or

$$\begin{aligned}2v_i(k)\varphi^T(k)\tilde{\theta}_i(k-1) - [\varphi^T(k)\tilde{\theta}_i(k-1)]^2 = \\ -[v_i(k) - \varphi^T(k)\tilde{\theta}_i(k-1)]^2 \\ + v_i^2(k) - \zeta_i^2(k) + v_i^2(k)\end{aligned} \quad (46)$$

Therefore, the variation of the considered Lyapunov function $V_i(k)$ can be defined by:

$$\Delta V_i(k) = -\delta_i(k)\zeta_i^2(k)/L(k) + \delta_i(k)v_i^2(k)/L1(k) \quad (47)$$

Multiplying and dividing by β_i the second member on the right of equation (47), we found:

$$\begin{aligned}\Delta V_i(k) &= -\delta_i(k)[(\zeta_i^2(k) - \beta_i v_i^2(k))/(\beta_i L(k))] \\ &- \delta_i(k)[(\zeta_i(k))^2/(\beta_i L(k))] - [(\beta_i - 1)\mu_i / L1(k)] \\ &+ [(\beta_i - 1 - \beta_i\delta_i(k))\varphi^T(k)P(k-1)\varphi(k)/L1(k)]\end{aligned} \tag{48}$$

If we have $\beta_i > 1$, $\delta_i(k) \in [\sigma_{0i}, 1/\beta_i - \sigma_{0i}]$, $\sigma_{0i} \in [0,1]$ and , $i = 1, \ldots, m$, then we get:

$$\zeta_i^2(k) - \beta_i v_i^2(k) > 0 \tag{49}$$

$$[\beta_i - 1] - \beta_i\delta_i(k) \geq \beta_i[1 + \sigma_{0i}] - 2 > 0 \tag{50}$$

We conclude that the derivate of the Lyapunov function is negative, such that:

$$\Delta V_i(k) < 0 \tag{51}$$

So, the stability condition of the developed parametric estimation scheme, which corresponds to the convergence condition of the modified RLS parametric estimation algorithm with dead zone (29), is established.

Then, the convergence condition of the developed modified RLS parametric estimation algorithm with dead zone (29) is ensured.

IV. Robust self-tuning control

In this section, we develop a robust explicit self-tuning control scheme, which can be applied to the considered MIMO systems in presence of unmodelled dynamic, as described by (11).

The formulation of the robust explicit self-tuning control scheme can be indeed by the minimization of the following criterion:

$$\begin{aligned}J(k+d+1) &= [Q(q^{-1})u(k)]^2 \\ &+ \left[S(q^{-1})[y(k+d+1) - y_r(k+d+1)]\right]^2\end{aligned} \tag{52}$$

where $y_r(k+d+1)$ represents the vector of the desired output signals, $u(k)$ is the control law and $S(q^{-1})$ and $Q(q^{-1})$ are polynomial matrices of the order ns and nq , respectively, such that:

$$S(q^{-1}) = I + S_1 q^{-1} + \cdots + S_{ns} q^{-ns} \tag{53}$$

$$Q(q^{-1}) = Q_0 + Q_1 q^{-1} + \cdots + Q_{nq} q^{-nq} \tag{54}$$

where the matrices S_{ts} and Q_{tq} , $ts = 1, \ldots, ns$, $tq = 1, \ldots, nq$, are defined as:

$$S_{ts} = \begin{bmatrix} S_{ts,11} & 0 \\ 0 & S_{ts,mm} \end{bmatrix} \tag{55}$$

$$Q_{tq} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{mm} \end{bmatrix} \tag{56}$$

Noting that the orders ns and nq of the polynomial matrices $S(q^{-1})$, and $Q(q^{-1})$, respectively, are chosen by the designer.

The derivate of the criterion $J(k+d+1)$, which is described by (52), is given by:

$$\begin{aligned}\partial J(k+d+1)/\partial(u(k)) &= 2Q_0 Q(q^{-1})u(k) \\ &+ 2B_1(k)S(q^{-1})[y(k+d+1) - y_r(k+d+1)]\end{aligned} \tag{57}$$

with

$$\begin{aligned}S(q^{-1})y(k+d+1) &= qB(q^{-1},k)F(q^{-1},k)u(k) \\ &+ G(q^{-1},k)y(k) + F(q^{-1},k)v(k+d+1)\end{aligned} \tag{58}$$

where $F(q^{-1},k)$ and $G(q^{-1},k)$ are solutions of the following polynomial matrix equation:

$$S(q^{-1}) = A(q^{-1},k)F(q^{-1},k) + q^{-d-1}G(q^{-1},k) \tag{59}$$

The polynomial matrices $F(q^{-1},k)$ and $G(q^{-1},k)$ are given by, respectively:

$$F(q^{-1},k) = 1 + F_1(k)q^{-1} + \cdots + F_d(k)q^{-d} \tag{60}$$

$$G(q^{-1},k) = G_0(k) + G_1(k)q^{-1} + \cdots + G_{na-1}(k)q^{1-na} \tag{61}$$

Thus, the optimal control law $u(k)$ can be written by

$$\begin{aligned}u(k) &= -G(q^{-1},k)Z^{-1}(q^{-1},k)y(k) \\ &+ B_1^{-1}(k)S(q^{-1})Z^{-1}(q^{-1},k)y_r(k+d+1)\end{aligned} \tag{62}$$

where the polynomial matrices $H(q^{-1},k)$ and $Z(q^{-1},k)$ are given by, respectively:

$$Z(q^{-1},k) = H(q^{-1},k) + B_1^{-1}(k)Q_0 Q(q^{-1}) \tag{63}$$

$$H(q^{-1},k) = qB(q^{-1},k)F(q^{-1},k) \tag{64}$$

A Robust explicit self-tuning control scheme

The proposed robust explicit self-tuning control scheme, which can be applied to the considered MIMO systems, is defined by the following three steps:

Step 1: estimation of parameters intervening in the *ARX* mathematical model (27) using the modified parametric estimation algorithm RLS with relative dead zone (29);

Step 2: determination of the parameters intervening in the two polynomial matrices $F(q^{-1},k)$ and $G(q^{-1},k)$ by resolving the following polynomial equation:

$$S(q^{-1}) = \hat{A}(q^{-1},k)F(q^{-1},k) + q^{d-1}G(q^{-1},k) \tag{65}$$

Step 3: computation the control law $u(k)$ given by (62).

V. Simulation results

To illustrate the advantages of our robust explicit self-tuning control scheme, we consider a MIMO system, which can be described by an ARX mathematical model of type (1), with: $d=1$, $m=2$.

The two outputs $y_1(k)$ and $y_2(k)$ of the considered MIMO system are given by the following expressions, respectively:

$$\begin{aligned} y_1(k) = & -[a_{11}(k) + \varsigma_{a_{11}}(k)]y_1(k-1) \\ & -[a_{12} + \varsigma_{a_{12}}(k)]y_2(k-1) \\ & +[b_{11} + \varsigma_{b_{11}}(k)]u_1(k-2) \\ & +[b_{12} + \varsigma_{b_{12}}(k)]u_2(k-2) + e_1(k) \end{aligned} \tag{66}$$

$$\begin{aligned} y_2(k) = & -[a_{21} + \varsigma_{a21}(k)]y_1(k-1) \\ & -[a_{22} + \varsigma_{a22}(k)]y_2(k-1) \\ & +[b_{21}(k) + \varsigma_{b_{21}}(k)]u_1(k-2) \\ & +[b_{22} + \varsigma_{b22}(k)]u_2(k-2) + e_2(k) \end{aligned} \tag{67}$$

where the parameters intervening in these expressions are supposed unknown and slowly time-varying.

In this section, we will be interested to test the performances and the effectiveness of the proposed robust explicit self-tuning control scheme, by this numerical simulation example.

Thus, the relative data of the numerical implementation of the proposed robust explicit self-tuning control scheme are the following ones:

1. the various values of the parameters intervening in the expression (66) are chosen, such as: $a_{11}(k) = -0.10 + 0.02\sin(0.12k)$, $a_{12} = 0.13$, $b_{11} = 0.24$, $\varsigma_{b11}(k) = 0.01\sin(0.02k)$, $b_{12} = 0.27$, $\varsigma_{a11}(k) = 0.01\cos(0.02k)$, $\varsigma_{a12}(k) = 0.01\cos(0.02k)$, $\varsigma_{b12}(k) = 0.01\sin(0.02k)$;
2. the reference signal $y_{r1}(k)$ is given by: $y_{r1}(k) = 5\sin(0.125\,k)$;
3. the noise sequence $\{e_1(k)\}$ consists of independent random variables with zero mean and variance $\sigma_1^{\,2} = 0.02$;
4. the various values of the parameters intervening in the expression (67) are chosen, such as: $a_{21} = -0.12$, $b_{21}(k) = 0.26 + 0.01\sin(0.12k)$, $a_{22} = 0.10$, $\varsigma_{a22}(k) = 0.01\cos(0.02k)$, $b_{22} = -0.31$, $\varsigma_{a21}(k) = 0.01\cos(0.02k)$, $\varsigma_{b21}(k) = 0.01\sin(0.02k)$, $\varsigma_{b22}(k) = 0.01\sin(0.02k)$;
5. the reference signal $y_{r2}(k)$ is given by: $y_{r2}(k) = 4\sin(0.125\,k)$;
6. the noise sequence $\{e_2(k)\}$ consists of independent random variables with zero mean and variance $\sigma_2^{\,2} = 0.01$;
7. the polynomial matrix $S(q^{-1})$ is given by:

$$S(q^{-1}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix} q^{-1} \tag{68}$$

We define the tracking errors $h_1(k)$ and $h_2(k)$ of the two interconnected systems of the considered MIMO system by:

$$h_1(k) = y_{r1}(k) - y_1(k) \tag{69}$$

$$h_2(k) = y_{r2}(k) - y_2(k) \tag{70}$$

The simulation results of the proposed robust self-tuning control explicit scheme on the basis of the RLS parametric estimation algorithm with dead zone (Control scheme 1 ·········) and the simulation results of the self-tuning control explicit scheme on the basis of the RLS parametric estimation algorithm with forgetting factor (Control scheme 2 - - - -) are shown in Fig. 1-6.

The evolution curves of the desired output $y_{r1}(k)$ and the output of the system $y_1(k)$ are shown in Fig. 1. The evolution curves of the desired output $y_{r2}(k)$ and the output of the system $y_2(k)$ are shown in Fig. 2. In Fig. 3. and 4., we present the evolution curves of the variance $\sigma^2_{\zeta 1}(k)$ and $\sigma^2_{\zeta 2}(k)$ of the prediction errors $\zeta_1(k)$ and $\zeta_2(k)$.

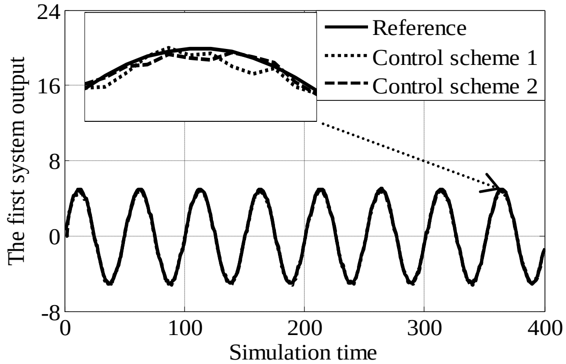

Fig. 1 Evolution curves of the desired output $y_{r1}(k)$ and the output $y_1(k)$.

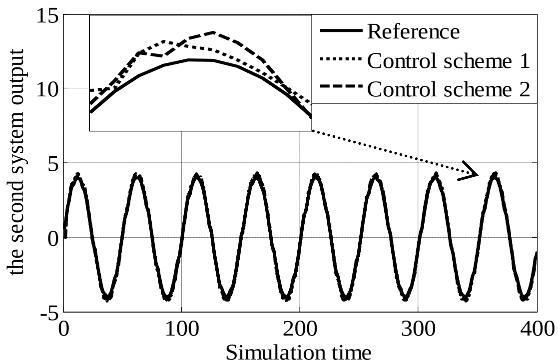

Fig. 2 Evolution curves of the desired output $y_{r2}(k)$ and the output $y_2(k)$.

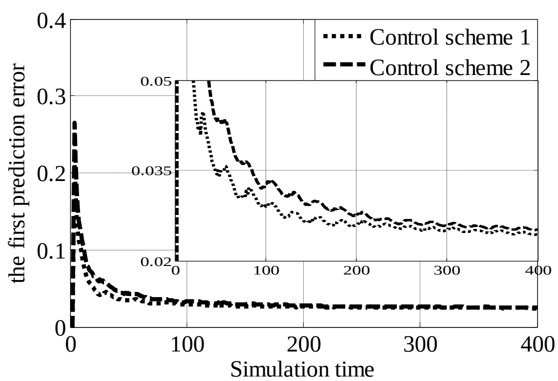

Fig. 3 Evolution curve of the variance $\sigma^2_{\zeta 1}(k)$ of the tracking error $\zeta_1(k)$.

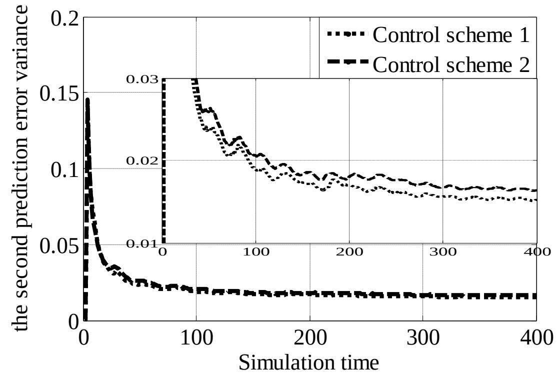

Fig. 4 Evolution curve of the variance $\sigma^2_{\zeta 2}(k)$ of the prediction error $\zeta_2(k)$.

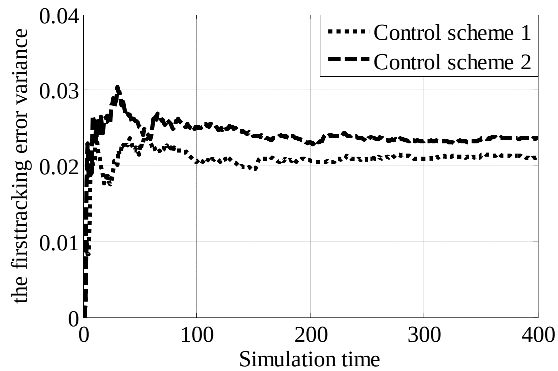

Fig. 5 Evolution curve of the variance $\sigma^2_{h1}(k)$ of the tracking error $h_1(k)$.

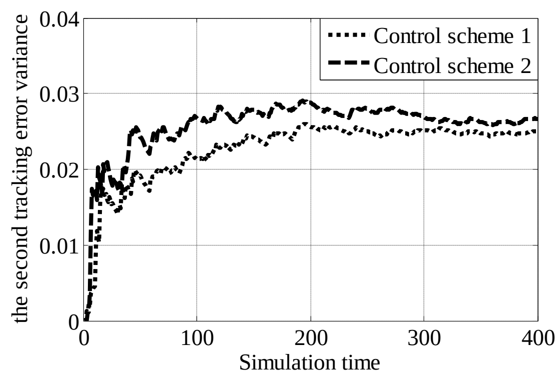

Fig. 6 Evolution curve of the variance $\sigma^2_{h2}(k)$ of the tracking error $h_2(k)$.

The calculated values of the prediction error variances $\sigma^2_{\zeta 1}$ and $\sigma^2_{\zeta 2}$, and the tracking error variances σ^2_{h1} and σ^2_{h2}, are given in Table 1, where:

$$\sigma_x^2 = \frac{\sum_{k=251}^{400} \left[x(k) - \overline{m}_x\right]^2}{100} \quad , \quad \overline{m}_x = \frac{\sum_{k=251}^{400} x(k)}{100}$$

Table I: Values of the prediction error variances and the tracking error variances

	Prediction Error Variance		Tracking Error Variance	
	$\sigma_{\zeta 1}^2$	$\sigma_{\zeta 2}^2$	σ_{h1}^2	σ_{h2}^2
Control scheme 1	0.0252	0.0149	0.0224	0.0217
Control scheme 2	0.0253	0.0164	0.0237	0.0266

From Fig. 1-6 and Table I, we can get the following conclusions:

1. the outputs $y_1(k)$ and $y_2(k)$ of the considered system track the desired signals $y_{r1}(k)$ and $y_{r2}(k)$, respectively; in the two control scheme.
2. the prediction errors $\zeta_1(k)$ and $\zeta_2(k)$, and the tracking errors $h_1(k)$ and $h_2(k)$ becomes small in the control scheme 1 than control scheme 2, in the discrete-time k;
3. the proposed robust explicit self-tuning control scheme 1, which is applied to the considered MIMO system, has given good results than the other control scheme.

VI. Conclusion

In this paper, we have proposed a robust self-tuning control explicit scheme for MIMO systems, which can be described by ARX mathematical models in presence of unmodelled dynamics. The problem formulation of this control scheme is conducted by the decomposition of the MIMO systems into interconnected systems.

A recursive parametric estimation algorithm is developed on the basis of the modified RLS parametric estimation algorithm with dead zone. The convergence condition of this developed algorithm was proved using the Lyapunov method.

We have treated a numerical simulation example in order to test the performances and the efficiencies of the proposed robust self-tuning control explicit scheme; where the self-tuning control explicit scheme on the basis of the RLS parametric estimation algorithm with forgetting factor was used to compare different simulation results. The obtained simulation results are satisfactory.

Acknowledgment

The authors acknowledge the ministry of higher education and scientific research in Tunisia for funding this work.

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