State space MPC Using State Observers

P. Chaupa, J. Novák, P. Januška

***Abstract*—**The article is focused on state observers and their usage in model predictive control (MPC). The observers are used to track and reconstruct states of a model of a controlled system. Linear time-invariant (LTI) state space models are used in the article because this type of models is often used in different MPC techniques If the states of the controlled system are immeasurable a state observer (filter) is used to calculate current states in each control step. The paper is especially focused to finite impulse filters (FIR) as these filters do not require knowledge of initial state - contrary to infinite impulse response (IIR) filters. Different observers are tested and compared with proposed filters based on quadratic and linear programming. Filters were used in a very noisy environment to evaluate filter robustness. Then MPC using promising filters was applied to a three tank model Amira DTS 200.

Keywords—Model predictive control, state observers, FIR filters, noise.

I. Introduction

The model predictive control (MPC) is very popular and successful technique for control of technological processes. The control algorithm is based on model of the controlled system [1]. The model is used to predict future output courses of the controlled system on the basis of current state of the system and future course of the system inputs. There are many types of models which are implemented in MPC. The model can be divided to categories according to various criteria: linear vs. nonlinear; time invariant vs. time varying; state space vs. input-output, etc.

This paper is focused on linear time invariant (LTI) state space models of the controlled plant. This model category can be used even for nonlinear or time-varying systems when linearizing the system in some working point. Then the LTI model provides good representation of the controlled system in a neighborhood of the working point. State space models can also represent biased control systems by adding a uncontrollable state to the model. This state represents a bias which is added to the output of the original system. Time-varying processes can be model by linear systems with on-line identification [2], [3].

Usage of state space representation of the model is very useful and popular. One of its main advantages is general approach to computation of all signals in the control circuits regardless of the type of the model – the same approach is valid for multi-input multi-output systems, single-input single-output systems, systems with measurable disturbance, systems without measurable disturbances, etc.

In case of state space MPC is used, computation of future courses of control signals is based on knowledge of current state of the controlled plant. The current states are used as parameters of a criterion which is to be minimized by future course of the control signals. Thus, knowledge of the current states is crucial for computation of the control signal. In general, the states are not measurable and must be computed using input-output data of the controlled system. A control circuit block, which is used to compute current state, is referred to as state observer or state filter. The term “observer” is more common in the control systems area while the term “filter” is more used in the signal processing. The aim of the state observer is to reconstruct current state on the basis of previous inputs and outputs. The observers can be divided into two classes: IIR (Infinite Impulse Response) and FIR (Finite Impulse Response). The IIR observers require knowledge of the system states at the beginning of the observer horizon while FIR observers don’t require initial state. Well-known IIR state observers are Kalman filter for stochastic systems [4] and Luenberger observer for deterministic systems [5]. The IIR filters in their recursive form are popular in the area of control systems but their convergence is often not guaranteed by design. In this case the convergence has to be verified for each application.

Non-recursive FIR filters are popular in signal processing [6], [7], [8]. They are characterized by guarantee of stability, robustness to temporary changes of system parameters, etc. The FIR filters are using, as well as predictive control itself, the receding horizon principle. The current state estimates are computed from previous inputs and outputs of the system on a finite horizon. This horizon is moved forward each sample step. Moreover, linearity, filter error minimization and independence of the system state at the beginning of the horizon are incorporated into filter by design [9].

This paper continues in the work presented in previous paper which is focused mainly to comparison of individual observers [10]. Several observers with a good performance [11] are compared with proposed observers based on H_1, H_2 and H_∞ norms.

The paper is organized as follows. Section 2 presents the observers which were used in MPC. General principles of MPC are shortly recalled in Section 3. Section 4 presents controlled system which was used to test the observers – Amira DTS200 Three Tank System [12]. Several control

P. Chalupa is with the Tomas Bata University in Zlin, Faculty of Applied Informatics, nam. T. G. Masaryka 5555, 760 01 Zlin, Czech Republic (phone: +420 57603 5204, e-mail: chalupar@fai.utb.cz).

J. Novak is with the Tomas Bata University in Zlin, Faculty of Applied Informatics, nam. T. G. Masaryka 5555, 760 01 Zlin, Czech Republic (e-mail: jnovak@fai.utb.cz).

This work was supported by the European Regional Development Fund under the project CEBIA-Tech No. CZ.1.05/2.1.00/03.0089.

setting is compared in Section 5 and results are summarized in Conclusion.

II. Observers

The observers studied in this paper are designed for discrete LTI state space system without direct feed-through. The system is described by the following equation:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{1}$$

where x_k is the state vector, u_k is the vector of inputs, y_k is the vector of outputs of the system. Symbols w_k and v_k represents disturbances which are assumed to be a white noise. Finally, the symbol k represents current time sample.

Most of FIR observers, which will be described in following sections, use matrix equation expressing relation of recent inputs, recent outputs and current states. If the system matrix A is nonsingular, it can be derived from (1):

$$\begin{aligned} x_{k-1} &= A^{-1}x_k - A^{-1}Bu_{k-1} - A^{-1}Gw_{k-1} \\ y_{k-1} &= Cx_{k-1} + v_{k-1} = CA^{-1}x_k - CA^{-1}Bu_{k-1} - CA^{-1}Gw_{k-1} + v_{k-1} \end{aligned} \tag{2}$$

Recursive application of equation (2) over horizon of N previous samples leads to the following equation:

$$Y_{k-1} = \bar{C}_N x_k + \bar{B}_N U_{k-1} + \bar{G}_N W_{k-1} + V_{k-1} \tag{3}$$

where

$$\begin{aligned} Y_{k-1} &= \begin{bmatrix} y_{k-N}^T & y_{k-N+1}^T & \cdots & y_{k-1}^T \end{bmatrix}^T \\ U_{k-1} &= \begin{bmatrix} u_{k-N}^T & u_{k-N+1}^T & \cdots & u_{k-1}^T \end{bmatrix}^T \\ W_{k-1} &= \begin{bmatrix} w_{k-N}^T & w_{k-N+1}^T & \cdots & w_{k-1}^T \end{bmatrix}^T \\ V_{k-1} &= \begin{bmatrix} v_{k-N}^T & v_{k-N+1}^T & \cdots & v_{k-1}^T \end{bmatrix}^T \end{aligned} \tag{4}$$

and matrices $\bar{C}_N$, $\bar{B}_N$ and $\bar{G}_N$ are defined as:

$$\bar{C}_N = \begin{bmatrix} CA^{-N} \\ CA^{-N+1} \\ CA^{-N+2} \\ \vdots \\ CA^{-1} \end{bmatrix} \quad \bar{B}_N = -\begin{bmatrix} CA^{-1}B & CA^{-2}B & \cdots & CA^{-N}B \\ 0 & CA^{-1}B & \cdots & CA^{-N+1}B \\ 0 & 0 & \cdots & CA^{-N+2}B \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}B \end{bmatrix}$$
$$\bar{G}_N = -\begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-N}G \\ 0 & CA^{-1}G & \cdots & CA^{-N+1}G \\ 0 & 0 & \cdots & CA^{-N+2}G \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}G \end{bmatrix} \tag{5}$$

The aim of the observer is to compute current state x_k. This is usually done by minimizing a criterion. If some of the states are measurable and some have to be determined by observer, this notation can be easily used. State (and subsequently matrices A, B, and C) can be reordered to the following form:

$$x_k = \begin{bmatrix} x_{k,imm} \\ x_{k,m} \end{bmatrix} \tag{6}$$

where $x_{k,imm}$ are immeasurable states and $x_{k,m}$ are measurable states. The matrix $\bar{C}_N$ is divided into two parts corresponding to the two parts of x_k. The term $\bar{C}_N$ in the equation (3) is superseded by:

$$\bar{C}_N x_k = \begin{bmatrix} \bar{C}_{N,imm} & \bar{C}_{N,m} \end{bmatrix} \begin{bmatrix} x_{k,imm} \\ x_{k,m} \end{bmatrix} = \bar{C}_{N,imm} x_{k,imm} + \bar{C}_{N,m} x_{k,m} \tag{7}$$

The second term $\bar{C}_{N,m} x_{k,m}$ is known and thus only $x_{k,imm}$ states are to be determined.

The following subsections deal with observers which were successfully tested in [10] and therefore are used in MPC in the framework of this paper.

A. *Dual IIR Observer*

A standard Kalman filter [11] of the system (1) can be written as

$$\hat{x}_{k+1} = A\hat{x}_k + AP_kC^T \; CP_kC^T + R_v \;^{-1} \; y_k - C\hat{x}_k \tag{8}$$

with Riccati equation

$$P_{k+1} = A \; I + P_kC^TR_v^{-1}C \;^{-1} P_kA^T + GQ_wG^T \tag{9}$$

where I is identity matrix and Q_w is a diagonal covariance matrix of noise signal w_k in equation (1) and (2). The structure of the observer is similar to computation of control signal in LQ control design. Due to this duality, the observer is referred to as "dual IIR observer".

B. *2.2 FIR H_2 Observer*

The FIR H_2 observer is proposed for systems whose noise input matrix G is unknown or zero. Then the model of the system has the form

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{10}$$

Recursive application of equation (2) over horizon of N previous samples leads to the following equation:

$$Y_{k-1} = \bar{C}_N x_k + \bar{B}_N U_{k-1} + V_{k-1} \tag{11}$$

where vectors and matrices are defined by (4) and (5).

The objective is to minimize sum of squares of differences between measured output and estimated output:

$$\min_{\hat{x}_k} J_k \qquad J_k = \left(Y_{k-1} - \hat{Y}_{k-1}\right)^{\mathrm{T}} \left(Y_{k-1} - \hat{Y}_{k-1}\right)$$
$$\hat{Y}_{k-1} = \bar{C}_N \hat{x}_k + \bar{B}_N U_{k-1} \tag{12}$$

This optimization problem can be solved directly by matrix inversion where gain matrix is obtained. It is also possible to consider (12) as a quadratic programming problem. This approach allows application of known restrictions of the states directly to their computation (e.g. the state known to be non-negative) [13], [14].

III. Model Predictive Control

The basic idea of MPC is to use a model of a controlled process to predict N future outputs of the process. A trajectory of future manipulated variables is given by solving an optimization problem incorporating a suitable cost function and constraints. Only the first element of the obtained control sequence is applied. The whole procedure is repeated in following sampling period. The computation of a control law of MPC is based on minimization of the following criterion

$$J(k) = \lambda_e \sum_{j=1}^{N} e(k+j)^2 + \lambda_{du} \sum_{j=1}^{N_u} \Delta \mathbf{u}(k+j)^2 \tag{13}$$

where $e(k+j)$ is a predicted control error, $\Delta\boldsymbol{u}(k+j)$ is a vector of future control increments (for the system with two inputs it has two elements), N is length of the prediction horizon, N_u is length of the control horizon and λ is a weighting factor of control increments. A usage of model without integral behaviour and subsequent application of quadratic criterion (13) in the MPC control scheme (i.e. $\boldsymbol{u}$ instead of $\Delta\boldsymbol{u}$) could lead to non-zero steady state control error.

The optimization problem (13) van be solved either directly by matrix inversion or can be easily reformulated to quadratic programming problem. The latter bring an advantage of coping with constrains directly inside optimization algorithm.

IV. DTS200 System

The Amira DTS200 system consists of three interconnected cylindrical tanks, two pumps, six valves, pipes, water reservoir in the bottom, measurement of liquid levels and other elements. The pumps pump water from the bottom reservoir to the top of the left and right tanks. Valve positions are controlled and measured by electrical signals, which allow precise positioning.

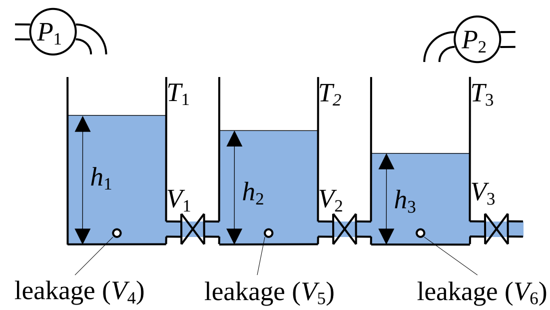

Fig. 1. Scheme of three tank system Amira DTS200

A simplified scheme of the system is shown in Fig. 1. The pump P_1 controls the inflow to tank T_1 while the pump P_2 controls the liquid inflow to tank T_3. There is no pump connected to the middle tank T_2. The characteristic of the flow between tank T_1 and tank T_2 can be affected by valve V_1, flow between tanks T_2 and T_3 can be affected by the valve V_2 and the outflow of the tank T_3 can be affected by valve V_3. The system also provides the capability of simulating leakage from individual tanks by opening the valves V_4, V_5 and V_6.

Pumps are controlled by analogue signals in range from -10V to 10V. Heights of water level are measured by pressure sensors. Each valve is operated by two digital signals which control motor of particular valve. First signal orders to start closing of the valve while the second signal is used for opening of the valve. If none of the signals is activated the valve remains in its current position. Each valve also provides three output signals: analogue voltage signal corresponds to the current position of the valve and two informative logical signals which states that the valve is fully opened or fully closed respectively.

The overall number of inputs to the modeled plant DTS200 is 14:

- 2 analogues signals controlling the pumps,
- 12 digital signals (2 for each of the 6 valves) for opening / closing of the valves.

The plant provides 21 measurable outputs which can be used as a control feedback or for measurements of plant characteristics:

- 3 analogue signals representing level heights in the three tanks,
- 6 analogues signals representing position of the valves,
- 12 logical signals (2 for each of the 6 valves) stating that corresponding valve is fully opened / closed.

For the MPC control purposes, the plant was configured as a single input single output (SISO) system. The pump P_1 served as an actuator for controlling water level in the third tank (T_3). Control voltage of the pump was transformed to the range 0-100%. Valves V_1 and V_2 were fully opened during all experiments, valve V_3 was partially opened and valves V_4, V_5 and V_6 were fully closed. Pump P_2 was not used during experiments.

A. Static Characteristic

Static characteristic of the system is depicted in Fig. 2. Control signal outside the range presented in Fig. 2 lead to saturation. If control signal is smaller than approx. 22% the tank T_3 remains empty. If the control signal is higher than approx. 42%, the tank T_1 is entirely filled by water after some time and thus the water level in tank T_3 cannot be above approx. 390 mm.

The gain of the system increases as the output increases and represent one of nonlinearities of the DTS200 system. The gain changes from 16 mm/% to 26mm/%.

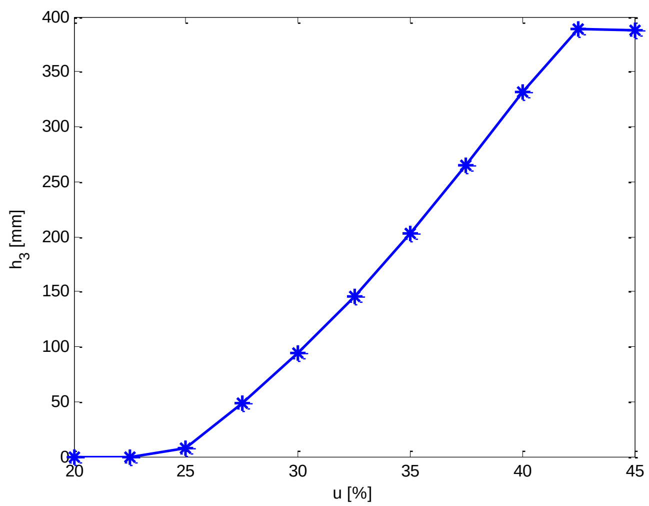

Fig. 2. Static characteristic of the controlled system

B. Identification

As the MPC and state observers presented in this paper cope with linear model of a system, an LTI (linear time invariant) model had to be created. Based on first principle analysis the system is at least of 3rd order because each cylindrical tank represents one state. But the characteristics of the system can be represented by lower order model as well.

The system was modeled by first and second order model and comparison of responses to pseudo-random signal are presented in Fig. 3 and Fig. 4. It can be seen that accuracy of the second order model is significantly better then accuracy of the first order system. Increasing of the model order further did not led to markedly better results and therefore second order model was used.

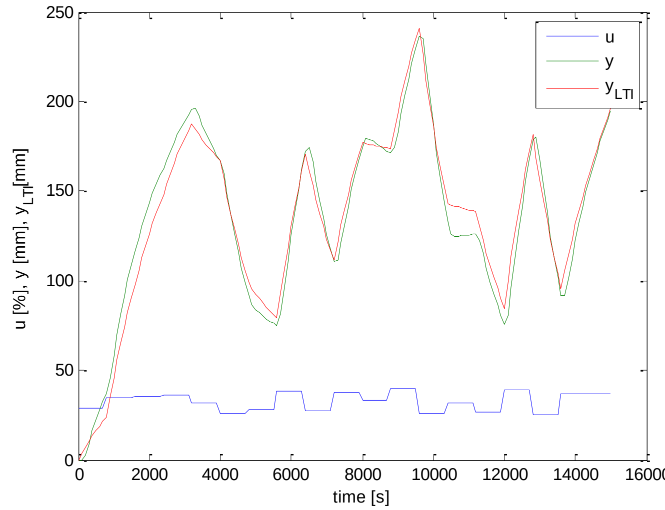

Fig. 3. Output of first order LTI model

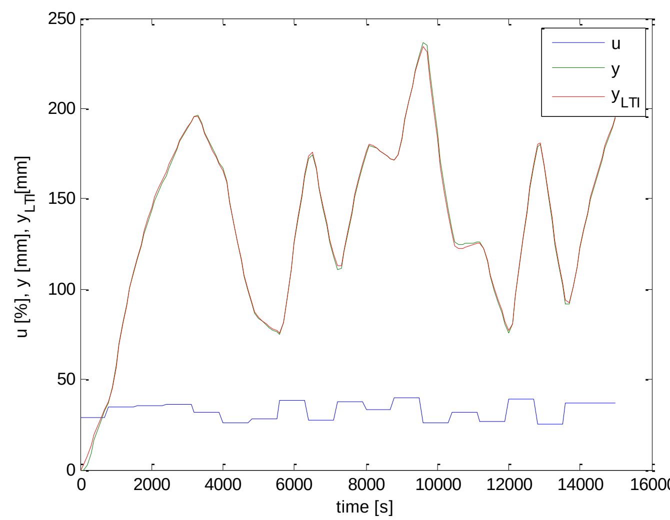

Fig. 4. Output of second order LTI model

V. Model Predictive Control of the DTS200

The DTS 200 was controlled by a MPC controller described in Section 3 using two types of state observer: dual IIR and FIR_H2; as described in Section 2.

The control scheme is presented in Fig. 5

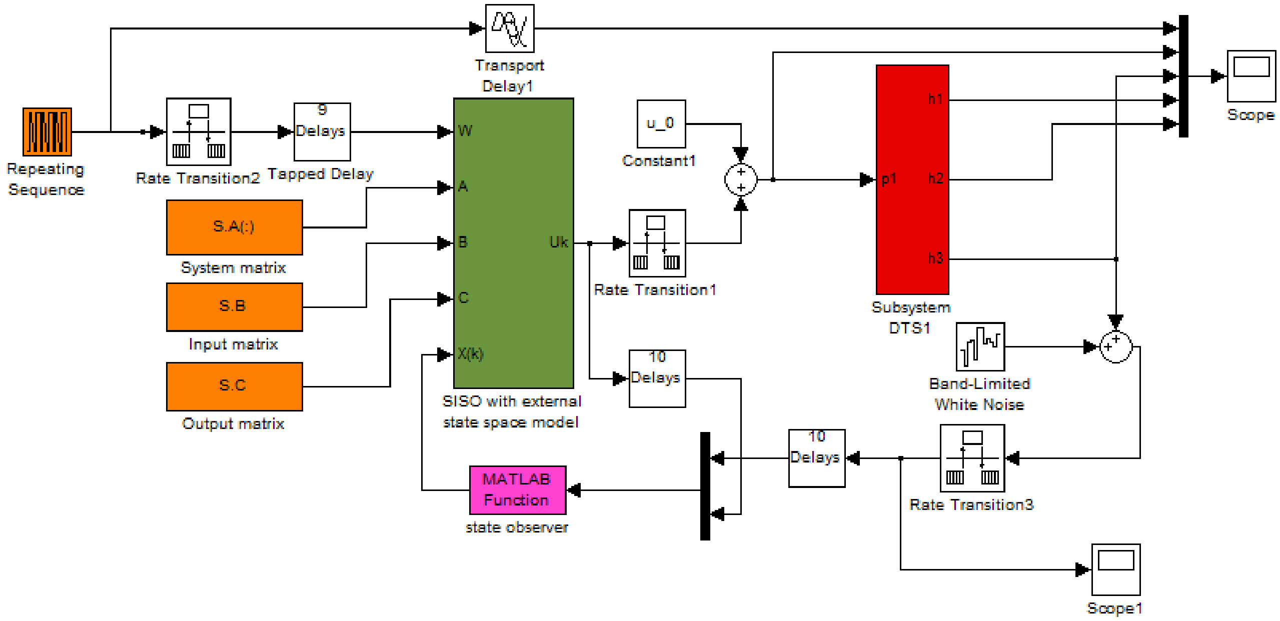

Fig. 5. Control Scheme

The scheme presented in Fig. 5 is used to test whether individual observers are able to cope with a noise which was added to a not-noisy output of the system. This can represent sensor noise or noise which affects communication line between sensor and controller.

The following figures present time courses obtained using both observers with short and long horizon. The real output of the system without noise is also presented but this signal is just recorded and stays hidden to the controller and observer.

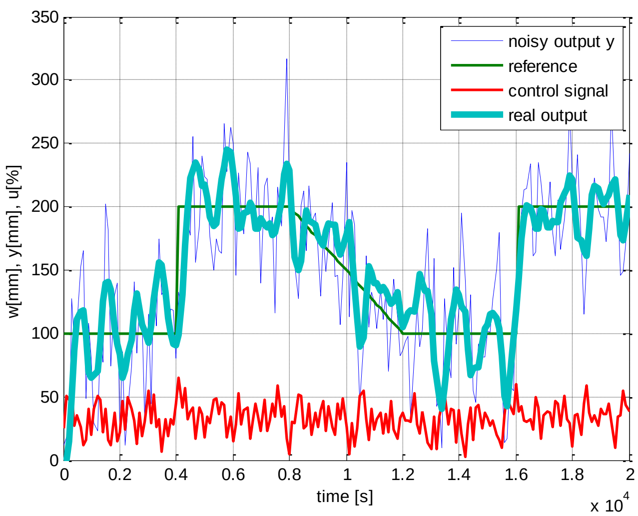

Fig. 6. Dual IIR observer, short horizon (N=2)

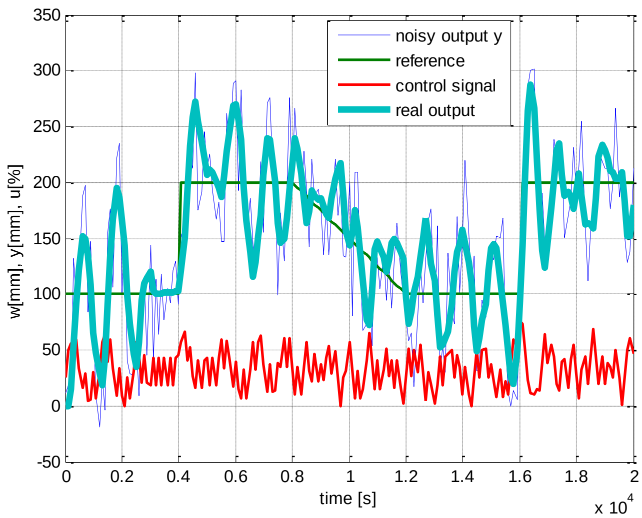

Fig. 7. FIR H2 observer, short horizon (N=2)

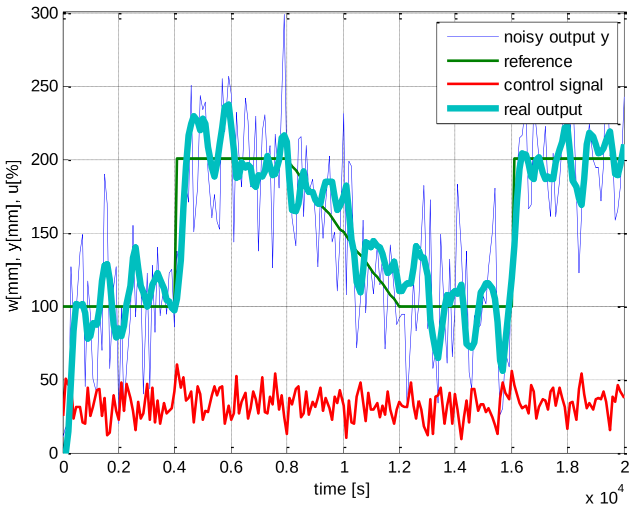

Fig. 8. Dual IIR observer, long horizon (N=10)

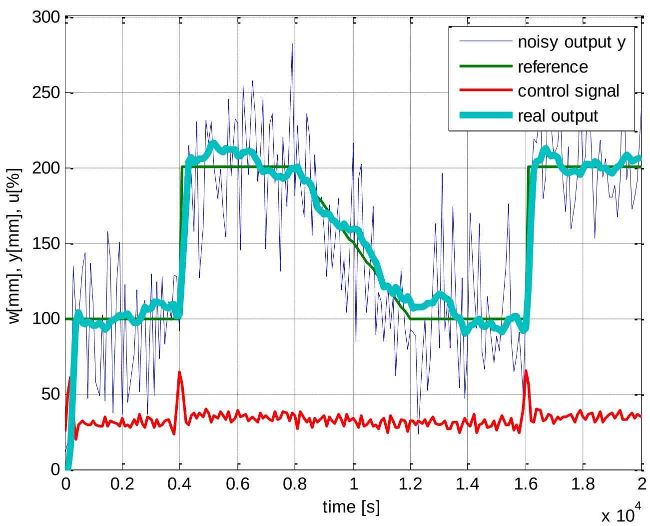

Fig. 9. FIR H2 observer, long horizon (N=10)

Values of quadratic criterion (13) calculated for whole control range are summarized in Table 1.

Table 1. Comparison of observers

Observer	J
Dual IIR, short horizon (Fig. 6)	158 278
FIR H2, short horizon (Fig. 7)	318 724
Dual IIR, long horizon (Fig. 8)	111 085
FIR H2, long horizon (Fig. 9)	51 665

VI. Conclusion

The paper was focused on comparison of FIR state observers with more commonly used IIR observers. Presented courses confirmed that the FIR filters can be used as state observers in MPC. The control quality was better for longer horizons for both FIR and IIR state observers – as expected. When using short horizon, the control performance was better for Dual IIR observer, while for longer horizons, the FIR H2 observer reached better results.

Acknowledgment

This work was supported by the European Regional Development Fund under the project CEBIA-Tech No. CZ.1.05/2.1.00/03.0089.

References

[1] E. Camacho, C. Bordons, *Model Predictive Control, 2nd ed.* Springer, London, 2007.

[2] V. Bobál, J. Böhm, J. Fessl, J. Macháček, Digital Self-tuning Controllers: Algorithms, Implementation and Applications. Springer, London, 2005.

[3] M. Kubalcik, V. Bobal, Adaptive Predictive Control of Three-Tank-System. In: *International Journal of Mathematical Models and Methods in Applied Sciences*, vol. 7 (2), 2013, pp. 157-165.

[4] R. E. Kalman, R. S. Bucy, A new approach to linear filtering and prediction problems. *In: Transactions of the ASME--Journal of Basic Engineering*, vol. 82, 1960, pp. 35–45.

[5] D. G. Luenberger, Introduction to Dynamic Systems, Theory, Models, and Applications. John Wiley & Sons, New York, 1979

[6] S. Udomhunsakul, Noise Reduction Using Adaptive Singular Value Decomposition, In: *International Journal of Circuits, Systems and Signal Processing*, vol. 7 (2), 2013, pp. 91-100.

[7] T. Veerakumar, S. Esakkirajan, I. Vennila, High Density Impulse Noise Removal Using Modified Switching Bilateral Filter, In: *International Journal of Circuits, Systems and Signal Processing*, vol. 6 (3), 2012, pp. 189-196.

[8] M. A. Pakzad, B. Moaveni, Delay-Dependent State Estimation for Time Delay Systems, In: *WSEAS Transactions on Systems and Control*, vol. 8 (1), 2013

[9] I. Y. Song, D. Y. Kim, V. Shin, M. Jeon, Receding horizon filtering for discrete-time linear systems with state and observation delays. *In: IET Radar, Sonar*, vol. 6 (4), 2012, pp. 263-271.

[10] P. Chalupa, P. Januška, J. Novák, State Observers for Model Predictive Control. *In: Nostradamus 2013: Prediction, Modeling and Analysis of Complex Systems*, 2013, pp. 295-303.

[11] W. Kwon, S. Han, *Receding horizon control: model predictive control for state models.* Springer, London, 2005.

[12] *DTS200: Laboratory Setup Three - Tank - System*, Amira, Duisburg, 2002.

[13] J. M. Maciejowski, *Predictive control: with constraints.* 1st ed. Prentice Hall, Harlow, 2002.

[14] G. C. Goodwin, J. de Dona, M. Seron, *Constrained control and estimation: an optimisation approach*. Springer, London, 2005.