Efficiency multithreshold decoders for self-orthogonal block codes for optical channels

Valery Zolotarev, Gennady Ovechkin, Dina Satybaldina, Nurlan Tashatov, Aigul Adamova, and Vitaly Mishin

***Abstract*—** This paper presents a least complexity multithreshold decoding algorithm for self-orthogonal block codes (SOC). Proposed decoding method has been expanding on the base of the optimization methods of functional of many discrete variables. This paper reviews operation principles of multithreshold decoders (MTD), compares their efficiency with other decoder's efficiency (Viterbi and turbo decoders, decoder for low-density parity check codes) and presents possibilities of the MTD for high-speed codes, suitable for use in the optical channels. The results of fast and compact implementations of SOC Encoder and MTD architectures using Xilinx's Virtex5 and Altera Stratix FPGA devices are presented and analysed.

***Keywords*—**iterative decoding, multithreshold decoder, optical communications, self-orthogonal codes.

I. Introduction

Currently various digital communication systems are applied for the exchange of information. Such systems are used for data transmission with wired and wireless communication channels, in which information can be distorted under the influence of various kinds of interference. It is unacceptable for many applications. Therefore, error-correcting coding tools are used in any digital transmission, and its using reduces the proportion of uncorrected errors to an acceptable.

Absolute necessity to use error-correcting coding has been elucidated for information channels for many decades ago. Use of codes reduces the need in signal power per 10 times. This is extremely important in many cases the use of digital radio. This is especially important on board the spacecraft, when the increase in actual physical transmitter power is technically impossible.

Currently develops optical communication systems (OCS) that provide transmission of large amounts of data at high speed around a hundred Gbit/s. Error-correcting coding is used to improve reliability of data transmission systems, the use of which allows to increase efficiency channel usage. The main requirement is to ensure for the schemes of coding and subsequent decoding OCS with a very high reliability (the probability of error of about 10^{-17}) extremely fast decoding. Therefore, OCS can be applied only with the fastest decoders.

In [1] the effects of the contributed noises for optical CDMA have been considered. These noises are phase-induced intensity noise (PIIN), shot noise and thermal noises. The system of 25 subscribers was simulated with modified quadratic congruence (MQC) codes at the C band for the upstream signal with channel spacing 50 GHz. The system shows good results in terms of the bit error rate (BER), and suppression of multiple access interference (MAI). As the code size is increased, both the complexity for the eavesdropper to detect high spectral chip pulse signal to noise ratio (SNR) and the system capacity are increase [1].

In [2] the design and application of low-density parity-check (LDPC) coding scheme for deep space communication under solar scintillation condition is studied. Simulation results reveal that compared with the convolutional codes the proposed LDPC codes could obtain 2.4 db and 2.5 db coding gain at 8.4 GHz (X-band) and 32 GHz (Ka-band), respectively. Moreover, simulation results also show that the deep space communication system with LDPC codes is much less sensitive to scintillation fading that that with convolutional codes.

A new word-length optimization method based on Monte Carlo simulation is proposed in [3]. In the proposed optimization method, and in the process of optimizing the word-length of the channel data, the statistical distribution results of variable node's posterior probability data and check node's extrinsic message are also obtained. The optimized word-length of variable node's posterior probability data and check node's extrinsic message is concluded by the statistical distribution result and the BER curves. Compared to the pure Monte Carlo simulation, the proposed method could reduce the amount of simulation work by more than 50%, and have the same word-length optimization results.

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However, for realization enough effective decoding algorithms of LDPC codes are required the considerable calculated expenses i.e. a large number of operations that leads to very significant increase in time of coding and decoding. High complexity of devices of coding and decoding in these systems complicates these algorithms for work in real time and essentially limits speed of information transfer.

Fastest decoders should only consist of a large number of the fastest microelectronic elements - large blocks of memory or long shift registers. They should not contain long chains of feedbacks, which greatly reduces the rate of advance data on such registers. Results in [4, 5] showed that the most suitable for high-speed systems according to these criteria are multithreshold decoders (MTD) for self-orthogonal block codes [6, 7, 8]. For MTD shown that they allow almost optimal (i.e., as well as then iterative exponentially complex code length methods) to decode even very long codes with linear complexity of implementation, that demonstrating good correction capability.

In present paper some new important MTD properties are discussed. The other parts of the paper are arranged in the following way. Application of the iterative optimization procedures for search the best decision of a decoder is considered in Section II. In Section III a multithreshold decoding algorithm is described. Simulation results for MTD and other binary error correction methods are shown in Section IV. In Section IV results a hardware implementations of the SOC Encoder and multithreshold decoder are presented too. Section V gives the possibilities MTD for high-speed codes, suitable for applying in the OCS. Section VI shows the main conclusions of the paper.

II. The functional global optimization principle

Development of methods for decoding error correction codes for a long time surprisingly had nothing to do with the optimization methods of functional of many discrete variables. However, decoding, i.e. search of the only one code word among an exponentially large number of possible messages, is natural to be considered from this standpoint.

The vast majority of decoding algorithms that have been developed earlier did not use a well-known variety of powerful iterative optimization procedures to search the best decision of a decoder. Such procedures could easily be applied to search code words that are at the minimum possible distance from the received message.

Note that the Viterbi algorithm commonly applied in communication technology and used to decode short convolutional codes by the likelihood maximum, does not also belong to a class of optimization procedures, since it directly searches for the optimal decision based on a very easy-to-implement total search method.

However, some decoding algorithms, in particular, threshold decoders already have almost precisely the properties that are needed to implement the full, effective and at the same time exceptionally simple iterative decoding procedures.

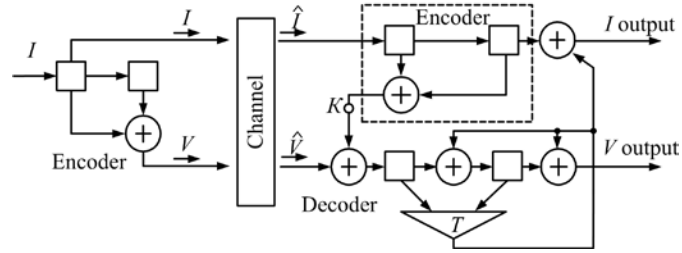

Fig. 1 a special form of convolutional coding system, explaining new interpretation of the syndrome vector

These procedures allow search global the functional extreme of a very large variable number. To confirm this, let us consider the simplest example of convolutional threshold coding/decoding system with the code rate R = 1/2 and the minimum code distance d = 3, shown in Fig. 1.

As you can see, a simple majority decoder that corrects one error in this simple example contains an exact copy of the coder. This copy forms its own estimates of the code check symbols by informational code symbols taken from the channel, perhaps, with errors. These symbols appear at the point K of the decoder and then, after summation in the half-adder with check symbols received from the channel $\hat{V}$, form a syndrome vector symbols $\overline{S}$, which depends only on the channel error vector. Then these symbols are going to the decoder threshold element T from the syndrome register.

The form of TD in the shown coding/decoding scheme allows specifying a simple way to organize proper optimization procedure, i.e. to find the best possible decoding decision. Let us turn to the fact, which has never been discussed relating to any linear code before: the decoder syndrome register contains the check symbol difference between the vector $\overline{Q} = (\hat{I}, \hat{V})$ received from the channel with distortions and the code word $\overline{A}_C$, which informational symbols coincide with the informational part of the vector $\overline{Q}$ received from the channel.

Hence, the total difference between the code word – the current hypothesis – decoder $\overline{A}_I$ decision on the transmitted code word and received noisy vector $\overline{Q}$ will be in such a modified decoder of majority type, where TD will be added with only one new vector, which should always match the difference between the received vector $\overline{Q}$ and $\overline{A}_I$ – current hypothesis of the decoder for informational symbols. This decoder will contain the current value of a total difference, and therefore, will allow measuring the full distance between the current decoder decisions contained in its information register and received vector. This distance should be reduced to minimum that will correspond to decision of the optimum decoder, which is usually achieved by the exponentially complicated total search methods.

This approach to the problem of high-performance decoding is the basis of special iterative multithreshold decoders

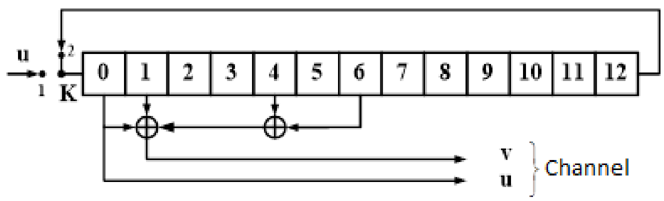

Fig. 2 encoder block SOC with R = 1/2, d = 5 and n = 26

(MTDs) being developed since 1970 [6, 7, 8], which are slightly different from classical TDs and just as simple to implement, as their prototype.

For further discussion, it is also important that, in contrast to the situation shown in Fig.1, information part of vector Q should not be directed always to the input of the internal coder in the decoder (right part of Fig.1). It is possible to send any information flow to the input of this coder. Then this syndrome vector, of course, will be the difference in check symbols between the vector received from the channel and the same code word, which informational symbols were sent to the input of the internal coder. This enhanced understanding of the message syndrome will be actively used in the subsequent consideration of the MTD properties.

Thus, changes that need to be done in the usual TD to convert it to MTD, are the next: decisions of all threshold elements about decoding symbols changes must be stored in the new additional difference register D, originally, of course, containing zeros. These decisions are then used by subsequent threshold decoder elements as an additional check for further correction of decoding symbols. This decoder already measures total distances between newer potential decisions $\overline{A}_i$ and received vector $\overline{Q}$. It changes decoding symbols so that each new decision of such MTD is always closer to vector received from the channel. This allows, in many cases, almost completely implement correcting capabilities of the codes used.

III. Multithreshold Decoding

Let's describe operating principles of MTD for self-orthogonal systematic block or convolutional code (SOC) [6, 7, 8]. For implementation of operation of encoding SOC it is possible to use the elementary diagrams constructed on the basis of shift registers. The example of the diagram of the coder block SOC, set by an ancestor polynomial $g(x) = 1+x+x^4 + x^6$, is shown in a Fig.2. This code is characterized by parameters of code length, length of information sequence, code speed and the minimum code distance of $n=26$, $k=13$, $R=1/2$, $d=5$, respectively. The similar diagram is used for encoding convolution SOC.

Let's describe the principles of operation of the encoder on the example provided by scheme. Check bits generates in the encoder during operation in accordance with the following algorithm:

Check bits generates in the encoder during operation in accordance with the following algorithm:

1. Before starting to encode code block key K is in state 1.

2. Information vector $\mathbf{u} = (u_0, u_1, \ldots, u_{12})$ applied one character input shift register. As a result, information symbol u_0 is located in cell 12, u_1 - the cell 11, etc.

3. Key K is transferred to state 2.

4. For j from 0 to 12 to perform cyclic shift register, and then calculates the j-th checking bit v_j:

$$v_j = \sum_{k=1}^{4} u_{(j-g_k) \bmod 13} \quad . \tag{1}$$

As a result of the algorithm generated a checking vector $v = (v_0, v_1, ..., v_{12})$, which, together with an information vector defines the code word $c = (u, v)$, which is transmitted through the channel.

Let's describe the principle multithreshold decoding of SOK. In a situation, where the decoder after transmission of a binary symmetric channel (BSC) rather than a distorted codeword noises message $y = (u', v')$ of length n. First calculated syndrome $\boldsymbol{s} = \mathbf{H}\boldsymbol{y}$ (here H - check matrix code) of the received message, and for each information symbol u_j, $1 \le j \le k$, stands set $\{s_p\}$ syndrome elements with numbers $\{p\}$, called checks relative to the character u_j and containing error e_j in this symbol.

First, as in the usual threshold decoder is calculated syndrome $s = Hy$ (here H - check matrix CSOC) of the received message, and each information symbol u_j, $1 \le j \le k$, find the set of elements $\{s_p\}$ syndrome with numbers $\{p\}$ called checks against symbol u_j and containing, as an error term e_j in this symbol.

In addition to the threshold decoder in MTD injected binary vector d of length k, called the difference, initially filled with zeros. The basic step is to decode that for arbitrarily chosen symbol u_j computed likelihood function L_j, independent of its related inspections and j-th element of vector d:

$$L_j = \sum_{p \in \Theta_j} S_p + d_j \tag{2}$$

where d_j - a symbol of the difference vector, related to decoded symbol u_j (0 or 1); S_p - p-th element of the syndrome vector, which is part of a number of checks regarding decoded symbol u_j; Θ_j - a set of of numbers of checks, controlling the j-th information symbol. The example of MTD implementation for encoder from Fig. 1 is given in Fig. 3.

Let us note again that, according to the MTD work rule, its difference register D initially contains only zeros. Thus, at the first iteration of error correction, MTD works just like a conventional TD. Only at subsequent decoding attempts, MTD starts to really take into account the contents of the corresponding cells in register D, as a result of which it keeps the properties to improve TD decisions at all changes of the message informational symbols.

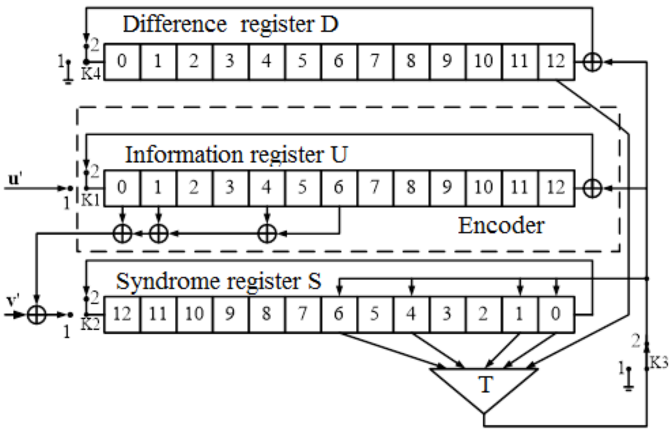

Fig. 3 Multithreshold decoder for SOC

MTD can be easily modified as the normal threshold decoder for adding checks in (2) with certain coefficients. Output bits define reliability for the decision made when dealing with multiple levels quantized soft modem solutions. Using of soft decisions of the demodulator can achieve 1.4 ... 1.7 dB better results than using hard decisions of the demodulator. The expression (2) to calculate the likelihood function takes the form L_j

$$L_i = \sum_{p \in \Theta_j} (2s_p - 1) w_p + (2d_j - 1) w_j \qquad (3)$$

where $\{w_p\}$– factors reflecting the reliability checks $\{s_p\}$; w_j – factor reflecting the reliability of the received symbol u_j. Logarithm of the likelihood ratio can be used as the grade received from the channel symbols

$$w_j = \ln \frac{P(u_j = 1 | r_j)}{P(u_j = 0 | r_j)} \qquad (3)$$

where r_j – symbol received from the channel corresponding to the transmitted information symbols u_j. Values $w_{j.}$ can be quantized into several levels to simplify the calculations.

As far as the main ideas of multithreshold algorithm's basis have been considered, let us turn to a strictly formal justification of their capabilities.

Suppose we are given binary linear systematic block or convolutional code with the code rate R = k/n, where k – is a number of information symbols, n – the code length.

After transmission over BSC without memory an optimum decoder that minimizes the average probability of decoding errors among many possible 2k of equiprobable code words $\{\overline{A}\}$ choose a vector $\tilde{A}$, for which the Hamming distance $r = |\overline{Q} \oplus \tilde{A}|$ (where $\overline{Q}$ – received message, $\overline{Q} = \overline{A} \oplus \overline{E}$; $\overline{A}$ – transmitted code word; $\overline{E}$ – channel noise vector, □– modulo 2; |x| – Hamming weight of the vector x) would be minimal for the whole set of code words $\{\overline{A}\}$.

For proving convenience of the following propositions, let us represent any binary code vector $\overline{X}$ of length n by a pair of vectors $\overline{X}_I$ and $\overline{X}_V$ of length k and (n–k) respectively, related to information and check parts of the vector:

$$\overline{X} = (\overline{X}_I, \overline{X}_V).$$

Then, assuming that the check matrix of the code has a systematic form $H = (P^T : I)$, we have the following lemma.

Lemma 1. For each code vector $\overline{A}$ and the received message $\overline{Q}$ the following relation is true

$$\overline{A} \oplus \overline{Q} = (\overline{D}, H(\overline{Q}_I \oplus \overline{D}, \overline{Q}_V)), \qquad (4)$$

where vector $\overline{D}$ of the length *k* is defined by the relation

$$\overline{A}_I = \overline{Q}_I \oplus \overline{D}. \qquad (5)$$

Proof. Due to the code linearity

$$\overline{S} = H(\overline{Q}_I \oplus \overline{D}, \overline{Q}_V) = H(\overline{A}_I, \overline{A}_V \oplus \overline{A}_V \oplus \overline{Q}_V) =$$
$$= H\overline{A} \oplus H(\overline{0}_I, \overline{A}_V \oplus \overline{Q}_V),$$

where $\overline{0}_I$ – is a zero information word.

As $H\overline{A} = 0$, whereas $\overline{A}$ – code word, and $H(\overline{0}_I, \overline{A}_V \oplus \overline{Q}_V) = \overline{A}_V \oplus \overline{Q}_V$, as $\overline{A}_V \oplus \overline{Q}_V$ is multiplied only by identity submatrix *I* of the matrix *H*, then vector $\overline{S}$ is

$$\overline{S} = \overline{A}_V \oplus \overline{Q}_V. \qquad (6)$$

After substitutions in the right part (4) taking into account (5), we find that

$$(\overline{D}, \overline{S}) = (\overline{D}, \overline{A}_V \oplus \overline{Q}_V) = (\overline{D} \oplus \overline{Q}_I \oplus \overline{Q}_I, \overline{A}_V \oplus \overline{Q}_V) = \overline{A} \oplus \overline{Q}.$$

Thus, the syndrome vector $\overline{S}$, is actually (as it was shown in Fig.1), a difference in the check symbols between partially distorted message came out from the channel and the above-defined code word.

The lemma is proved.

The essence of the lemma amounts to the fact that the difference $\overline{B} = \overline{Q} \oplus \overline{A}$ for any code word $\overline{A}$ and received vector $\overline{Q}$ is defined by a pair of vectors $(\overline{D}, \overline{S})$. By definition, in $\overline{D} = 0$ the vector $\overline{S}$ is a usual syndrome of the received message $\overline{Q}$: $\overline{S} = H\overline{Q}$. For simplicity with $\overline{D} \neq 0$ we will call $\overline{S}$ a syndrome too, as this generalization seems natural and is not resulting in any contradictions.

Searching all possible vectors $\overline{A}$, we can find a vector $\tilde{A}$, which minimizes $|\overline{B}|$ and is OD decision. Unfortunately, total search decoding algorithms are too complex. Therefore, let us consider a decoding algorithm, which is very close to the known threshold error correction method and therefore is very easy to implement.

1. Let the decoder at the first preparatory stage perform calculation and memorizing of a syndrome vector of the

received message $\overline{S}$. After that, the decoding procedure is begun.

2. Choose an information symbol i_j and the usual sum of the syndrome component s_{j_k} is calculated for it, containing as additives the error e_j in decoding symbol i_j (i.e. we need to find the sum of checks $s_{j_k} \in \{S_j\}$, where $\{S_j\}$ – is a set of checks related to the component e_j, corresponding to a symbol i_j) and symbol d_j (vector $\overline{D}$ component), which is also related to decoded symbol i_j:

$$L_j = \sum_{s_{j_k} \in \{S_j\}} s_{j_k} + d_j. \qquad (7)$$

Let us assume that initially $\overline{D} = 0$, as before decoding, decoder memory has only received vector $\overline{Q}$, because the decoder does not have any other more preferable hypotheses of the received message.

Let us choose threshold T as equal to a half of all addends in (7). For SOC, this number is $T = d/2 = (J+1)/2$.

3. Let finally all $J = d–1$ checks and i_j and d_j are inverted at $L_j > T$ and remain unchanged at $L_j \leq T$.

4. The decoded information symbol is chosen, and decoder will return to step 2, unless the decision to terminate a decoding procedure is made.

For the first decoding attempt the proposed procedure while all $d_j = 0$, is similar to the usual algorithm for TD. Let us below refer to the decoder that implements the proposed algorithm, a multi-threshold decoder (MTD). When performing the basic 2…4 steps of decoder procedure, all k information symbols of the message can be searched in any order, and that is the essence of multithreshold method, multiple times, up to 5, 20 or more times. Of course, some decoder decisions can be incorrect for some symbols, and some of these errors can be corrected at the next iterations-attempts to decode the same symbols.

In this case, the following theorem is true.

Theorem 2. The fundamental theorem of multithreshold decoding (FTMTD) [6].

If at any *j*-th step of decoding MTD changes the information symbol i_j, then:

a) MTD finds a new code word $\overline{A}_2$, closer to the received message $\overline{Q}$, than the code word $\overline{A}_1$, which corresponded to value i_j prior to *j*-th decoding step:

$$|\overline{B}_1| = |\overline{A}_1 \oplus \overline{Q}| > |\overline{A}_2 \oplus \overline{Q}| = |\overline{B}_2|;$$

b) After completion of the *j*-th step it is possible to decode any subsequent symbol i_k, $k \neq j$, so that its change will result in further approaching to the received message.

Proof. According to Lemma 1, prior to the decoding of symbol i_j it is true that

$$|\overline{B}_1| = |\overline{A}_1 \oplus \overline{Q}| > |\overline{A}_2 \oplus \overline{Q}| = |\overline{B}_2|;$$

Where

$$\overline{A}_1 = (\overline{A}_{1I}, \overline{A}_{1V}), \overline{A}_{1I} = \overline{Q}_I \oplus \overline{D}_1$$

The weight of vector $\overline{B}_1$ before this step, equal to $|\overline{B}_1| = |\overline{D}_1| + |\overline{S}_1|$, can be represented as an ordinary sum of weights $W_1 = L_{1j}+X$, where L_{1j} is defined by (7) and is equal to the sum of checks and symbol d_j at the threshold element, and X – is the weight of the other components $\overline{S}_1$ and $\overline{D}_1$, not included in L_{1j}.

Consider code vector $\overline{A}_2$, differing from $\overline{A}_1$ only in one information symbol i_j, and the respective difference $\overline{B}_2 = \overline{A}_2 \oplus \overline{Q}$. Since $\overline{B}_1$ and $\overline{B}_2$ differ only in the components coming to the threshold element, then $|\overline{B}_2| = L_{2j} + X$, where $L_{1j} + L_{2j} = J+1$, because due to the code linearity each check and d_j are exactly equal to 1 in one of vectors $\overline{B}_i$.

Since MTD changes i_j, if $L_{1j} > T$, it is essential for that to have $L_2 < L_1$ and, consequently, $|\overline{B}_1| > |\overline{B}_2|$, which proves item a) of the theorem.

Further, it is obvious that if the symbol i_j was not changed, it is possible to decode any other symbol i_k, $k \neq j$, as the conditions of the lemma are hold. In case of change i_j in accordance with the rules of MTD functioning after decoding of i_j equations $\overline{A}_{2I} = \overline{Q}_I \oplus \overline{D}_2$ and $\overline{S}_2 = H(\overline{Q}_I \oplus \overline{D}_2, \overline{Q}_V)$ hold. Here $\overline{D}_2$ differs from $\overline{D}_1$ in symbol d_j, and changes through feedback from the threshold element of checks related to i_j, those components of $\overline{S}_1$ are inverted, in which $\overline{S}_2$ differs from $\overline{S}_1$. Hence, we find that after changing i_j for the previously defined vectors $\overline{D}_2$, $\overline{A}_2$ and $\overline{S}_2$ there is the following equation

$$\overline{A}_1 = (\overline{A}_{1I}, \overline{A}_{1V}), \overline{A}_{1I} = \overline{Q}_I \oplus \overline{D}_1$$

similar to the one, occurring prior to the change of i_j (by Lemma 1). Thereby for subsequent decoding steps and changes of symbols i_k, $k \neq j$, further strictly monotonic approximation to the message received from the channel $\overline{Q}$ will be implemented.

The main MTD theorem is proved.

This theorem implies that the MTD for each change of decoding symbols is getting closer to the received vector $\overline{Q}$, thus finding new current and more likelihood vectors $\overline{A}_i$. MTD views and compares not an exponentially great amount of code words but only pairs of ones differing only in a single information symbol with one of the compared words being in the decoder. In case when the second code word turns out to be closer to vector $\overline{Q}$, than the one in MTD, the decoder will switch over to that word to perform further comparison with the new intermediate vector $\overline{A}_i$. It is clear that in principle it is possible to carry out a large number of decoding attempts for all code symbols. In that way convergence to the optimum decoder decision-vector $\tilde{A}$ will be realized. It is crucial that MTD complexity remains the same as for customary TD: a linear, i.e. theoretically the lowest possible.

Therefore, the main theorem of multithreshold decoding states that the simplest of known threshold procedures for each change of decoded symbols provides a strict convergence to the optimal decision, i.e., strict increase of the likelihood of each new MTD decision. In this case, complexity of the procedure for the message of the length k is not proportional to 2k, but simply to k. There is no known similar proved property of strictly monotonic convergence to OD decision for any other decoding algorithms of low complexity.

Although we have just proved a theorem for the MTD algorithm on convergence of its decisions to OD decision, we should not forget that this is an iterative application of the simplest threshold function to the decoded symbols. At high noise levels in the channel with random errors, at first, and subsequent iterations of decoding it is possible to have incorrect decision of the threshold element decoder in any individual decoded symbols. On the other hand, at all changes of decoded symbols, according to the proven results, this decoder only strictly improves its decisions by likelihood. However, this means that after incorrect decisions on decoded symbols in the next steps, MTD can correct its own errors made in previous iterations. In a high noise level, a part of initially incorrect decisions of MTD in the first iterations of the error correction can be significant. Even so, relative to the whole message received from the channel, each new decision of MTD, as it follows from the fundamental theorem, is always strictly more probable.

Finally, let's note that the theorem does not implies that transition from one code word $\bar{A}$ to another will continue for as long as $|\bar{B}| = |\bar{Q} \oplus \bar{A}|$ will not be minimal, i.e. $\bar{A}$ becomes a decision of OD $\tilde{A}$. Thus, the MTD is not the optimal decoder. All of the following chapters of this book will be devoted to search for such codes and decoders for which the decoding process, even for high noise levels will actually almost always last as long as it reaches the vector $\tilde{A}$, decision of the total search optimum decoder.

Let us further assume that MTD has reached the optimum decoder decision, i.e. there are symbols of vector $\tilde{A}$ in the MTD information register. Then it is true that

Corollary. MTD does not change the decision of an optimum decoder.

Proof. If the MTD changes a single information symbol in vector $\tilde{A}$, then it would mean that there is another code vector $\tilde{A}^*$, which is closer to $\bar{Q}$, what $\tilde{A}$, that is impossible, because, by definition, the closest to $\bar{Q}$ word is a vector $\tilde{A}$.

The corollary is proved.

Thus, the stability of MTD decision corresponding to the optimum decision is shown: having reached that, MTD is going to stay there. It is very important as the algorithm implies an opportunity of multiple changes of the decoding symbols.

It might also be noted that during the proving of the main MTD theorem the uniqueness of the decoded symbol ij was not used in any meaningful way. It follows that the decoding procedure can be applied to any group of information symbols.

IV. Software and Hardware Implementations of the Multithreshold Decoder

Let's compare characteristics of MTD and other binary error correction methods in channels the additive white gaussian noise (AWGN) and a binary phase modulation (FM2) for the binary codes with a code rate R = ½ (Fig. 4). Theoretically, decoder can work with these parameters channel and codes when the signal/noise ratio equal of 0.2 dB (curve “C = ½” in Fig. 4).

Convolutional codes have found most widespread practical use in actual communication systems. Viterbi algorithm [9] and various concatenated codes is often used to decode them. These methods emerged and developed in the 70s - 80s of the last century. Turbo [10, 11] and low density codes [12] are actively developed in recent times by foreign experts, the effectiveness of which is very high. For example, methods for decoding turbo codes recommended standard CDMA2000, provide characteristics represented by a curve «3) TCC CDMA 2000 (n=3600)».

Small probability of error decoding can be achieved by low-density codes of length a million bits when working less than 0.1 dB capacity a Gaussian channel (LDPC (n=1000000)). Efficiency of decoders low density codes of shorter length is shown in Figure by curved lines «5) 802.16e LDPC(n=2304)» and «6) DVB-S2 LDPC(n=64800)». Unfortunately, all of these methods when working in a big noise still have a very large implementation complexity, making it difficult to practical use in high-speed data transmission and storage. Efficiency of MTD is presented for code with length of 20,000 bits, a code distance d = 9 and code rate R = 1/2 in Fig. 4 «Curve 7) MTD (n = 20000, d = 9)». MTD perform only a quick simple addition and comparison of integers, which makes them very attractive for use in existing and newly developed high-speed digital data transmission systems.

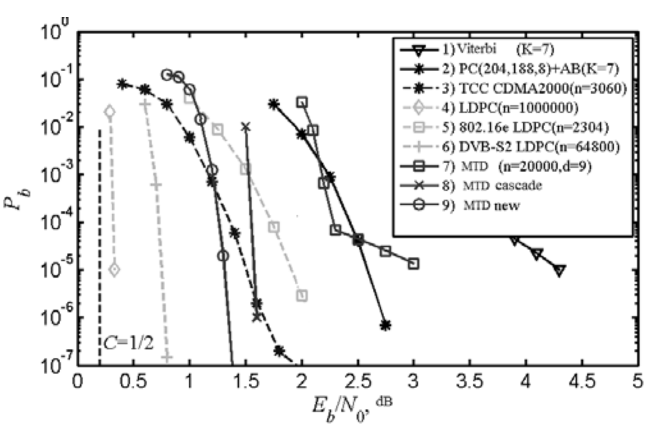

Fig. 4 performance of error-correcting codes with R = 1/2 over AWGN channel and FM2

Developers are constantly looking for ways to increase its efficiency, despite the good correction capability provided by the original MTD algorithm.

One way to approach the area of effective MTD work up your bandwidth is code selection, the least prone to error propagation (EP) decoding [6-8]. This property is reflected in the fact that after the decoder at work makes a mistake, the error probability in the following symbols increases significantly. In [6] proposed an approach to assess the susceptibility of code and build EP codes with minimum EP. Codes are necessary to obtain the best performance, in which there are multiple branches of information and verification. Example encoder such code is shown in Fig. 5 containing two information and two checking branches.

When using code such a structure can achieve significant reduction in the breeding of errors by reducing the number of common errors involved in decoding the various bits of information [8].

When using code such a structure can achieve significant reduction in the breeding of errors by reducing the number of common errors involved in decoding the various bits of information [6, 7, 8]. In [13], the authors show that only by a proper choice of code and optimization of its structure without complicating decoding scheme can get additional energy gain of the order of 1 .. 1.5 dB.

The next area is the work under these extremely efficient and extremely simple algorithms associated with the development of concatenated coding schemes. Cascading should only be with very simple codes to the overall complexity of the scheme has not increased. Focuses on this approach concatenated codes used in the MTD, with parity codes, Hamming codes and short self-orthogonal codes [14, 15]. Analytical calculations and computer simulation results show, that application of such schemes allows you to bring the effective area of MTD bandwidth channel 1 .. 2 dB and reduce the probability of error decoding for 2 .. 5 orders of magnitude without a significant complication of the decoding scheme. This scheme allows to provide efficacy comparable with the efficiency of the best methods of error correction. The complexity of the cascade scheme decoder is very small. As a result of this concatenated MTD easy to implement as a conventional MTD decoder for speeds of 500 Mbit / s or even higher.

Additional improvement in decoding performance self-orthogonal codes possible with non-significant complication decoding algorithm [16]. Features presented one such decoder in Fig. 4 with curve "9) MPDnew". It illustrates the very high energy efficiency of the proposed algorithm at a distance of only 1.1 dB of channel capacity. The absolute majority of other error correction algorithms are in such high noise extremely difficult. Provided characteristics comparable or even better than many well-known characteristics of turbo decoders and LDPC codes.

It turned out that MTD has another, no less effective way to speed up calculations. In fact, MTD as a conventional TD, mostly only counts the sums of checks. These are sums of a

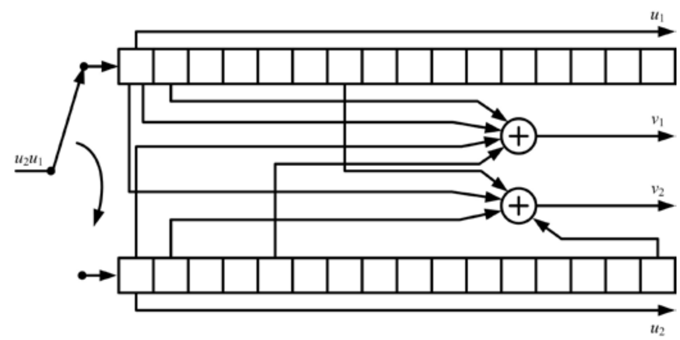

Fig. 5 encoder block SOC with two information and two checking branches

small number of short integers. Therefore, we can formulate conditions for decoder elements, parameters of codes used, modem, and threshold values in threshold elements that allow even at high noise levels to implement functions of summation and comparison in a threshold element by simple and rapid means.

This problem has also been solved completely and unambiguously. Currently, for many types of codes it is possible to implement such threshold elements that will instantly give a decision at each shift step of registers of a convolutional MTD. This is the second level of parallelization. In some cases, it is necessary to adapt code polynomials of the used codes to requirements of a particularly simple parallelization and acceleration of threshold elements.

As a result of implementation of the second-level parallelization of operations, MTD decoder turns into a device that somewhat does not perform any calculations for external observer. In other words, at each data shift step, threshold elements in MTD shift registers instantly create decisions on forthcoming changes of decoded symbols.

But this means that such MTD somewhat does not made any calculations and all restrictions on performance of this decoder are connected only with a marginal speed of data movement through decoder shift registers and the number of parallel working registers in a decoder. Single-bit modulo 2 adders, adders of small integers and standard shift registers are the fastest elements of digital technology. Therefore, simple estimates show that performance of such hardware MTD with the described approach is about three decimal orders of magnitude higher than that of other algorithms with a high noise level, and may range widely. This method of multifold MTD acceleration is patented in [17].

Fig. 6 shows the latest achievements in the field of high-speed MTD-type decoders based Xilinx's Virtex5 and Altera Stratix FPGA devices for the code rate R $\approx$1/2. Curve 1 refers to the development of convolutional MTD based on FPGA Xilinx at 100 Mb/s speed, which can be easily implemented at speeds up to 480 Mbit/s. It is much better than Viterbi algorithm (VA, curve 3) and only little differs from capabilities of a standard and highly efficient concatenated decoding scheme with VA and decoder of a Reed – Solomon code (VA+RC, curve 4), but significantly easier for them.

Curve *2* corresponds to MTD with 40 decoding iterations, constructed based on Altera FPGA for speeds of the order of

1.6 Gb/s [18]. Let's emphasize its most important feature: this conventional basic decoder scheme of this type, which is not even belong to concantenated structures, is much more effective in terms of noise level at which it operates than the previous schemes in Fig.6. Such high efficiency of this decoder is due to new developments in the search for actual codes with low error propagation at their correction, which, in turn, allowed move to a larger number of decoding iterations. It can be argued that such a scheme is certainly among the best nonconcantenated error correction procedures known in the theory and technique of coding. Of course, the success of MTD algorithm in such high noise levels in accordance with the fundamental code properties is only possible with a substantial increase in their length and decoding delay.

The major advantage offered by MTD algorithms along with high efficiency is the possibility of extremely high performance hardware implementation. Since these algorithms allow their full parallelization, it allows decoding MTD in rate that matches the speed of shift registers (for the high-speed circuit design elements) in the selected element base. Currently no known other types of algorithms that would at least partially possessed similar properties. These features high speed MTD work you can always save at virtually any modifications and improvements MTD methods known to date.

V. MTD Application to Improve the Reliability of Data Transmission in the OCL

Let's consider the possibilities MTD used in conjunction with high-speed codes, for example, suitable for applying in the OCS. Fig. 7 shows the characteristics of various MTD for convolutional codes with a code rate R = 4/5 in a Gaussian channel. Curve 1J shows the possibilities of the use of fairly complex MTD decoder of Japanese specialists [4,5]. These schemes present the feedback data transfer, markedly reducing speed of the circuit that these networks would be more high speed. Curve 2Dec given for MTD decoder using code, the lower bound optimum decoding which corresponds to the curve 2Opt.

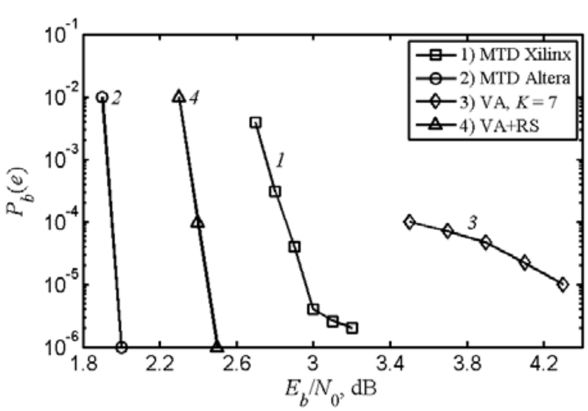

Fig. 6 characteristics of standard VA and best MTD decoders in a Gaussian channel

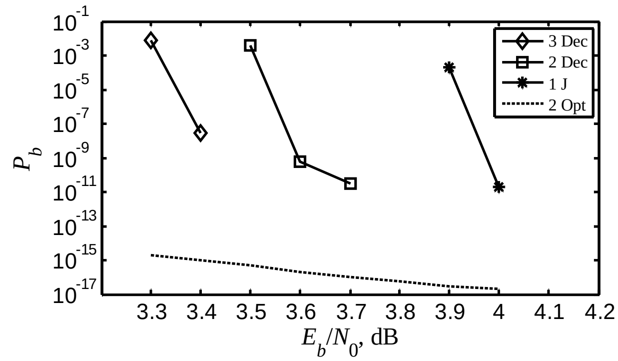

Fig. 7 efficiency MTD for codes with a code rate R = 4/5 over Gaussian channel

Simulation results shown that increasing of the ratio Eb/N0 signal/noise values we seen decrease of error probability Pb (e) for MTD with such codes. MTD characteristics reached a level of optimum decoder at E_b/N_0=3,7 dB or less. The results of these experiments suggest that the code with MTD will reach the optimum decoder level. It is determined by the lower boundary 2Opt with a noise level of 3.7 dB. Achieving a level of optimum decoding can be taken for granted, at least when E_b/N_0=3,8 dB.

The curve 3Dec corresponds to the option MTD application when not required to achieve conventional coding methods noncascade very small error probability at the output of the decoding apparatus. Curves 2Dec and 3Dec match MTD work at significantly greater level of noise than the decoder, the characteristics of which are given in the curve 1J. MTD represented by curve 3Dec, requires about 0.5 million code symbols delay decision in 25 iterations of decoding convolutional code.

It is understood that the ability to operate at high noise level allows the use of all the algorithms with different modifications in MTD and various types of circuits cascaded. All results of the three options MTD shown in Fig. 5, in the case of cascade schemes error correction will be, of course, improved. However, even when the characteristics of the second code are initially noncascade better than in concatenated coding schemes obtained in [5]. Improvement of parameters third code will be particularly noticeable at staging because it operates at a higher noise level than the two previous. But when cascading all types always have to take additional measures in order to not greatly reduce the processing speed, as this violates the principle of instant correction of errors in the MTD as it moves through shift registers decoder.

VI. Conclusions

It is shown that a fundamentally new level of performance and processing speed compared with absolutely all known methods of error correction can be achieved by using different types of MTD algorithms. MTD algorithms allow us to solve the problem to ensure high reliability of data transmission without any additional modification of these algorithms. Their use is equally simple and effective at the hardware and software implementation.

MTD methods are truly unique algorithms capable of providing efficient decoding at high noise level. They perform a very small number of transactions and the highest levels of reliability of storing digital information and its processing speed in very large-scale databases, optical disks, etc. In all these cases very limited resources are used, such as simple microprocessors or the cheapest FPGA, which determines the ease and efficiency of the new methods of error-correcting the coding.

Great deal of additional information on multithreshold decoders can be found on websites [19].

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