State Estimation Method of Sound Environment System with Multiplicative and Additive Noises

Hisako Orimoto, and Akira Ikuta

***Abstract*—** The observation data are usually contaminated by the additional external noise (i.e., background noise) of arbitrary distribution type in the real sound environment. Furthermore, the fluctuation factors appearing in the real environmental system can be roughly classified into two categories, that is, the external and internal fluctuation factors. The former like an external noise can be often considered as an additive model because it originally comes from the outside on the system and coexists independently of the internal system state, while the later like a gain fluctuation can be considered as a multiplicative model because it does not coexist with the internal state and completely disappears in the case without the existence of internal state. In this study, a new type of successive state estimation method based on the Bayes' theorem is theoretically proposed for a mixed model type stochastic system. More specifically, by considering the liner and higher order correlation information between the state and observation, a noise cancellation algorithm is theoretically proposed. Furthermore, the effectiveness of the proposed theory is experimentally confirmed by applying it to observation data in the real sound environment.

***Keywords*—** Additive External Noise, Noise Cancellation Method, Multiplicative Noise.

I. INTRODUCTION

IN the real sound environment system, the observed data contain the effects of various fluctuation factors such as noises in addition to the specific signal. Therefore, it is necessary to estimate only the specific signal based on the observation data by introducing some kinds of signal processing method [1],[2],[3]. For example, in order to estimate several evaluation quantities for the specific signal, like L_x ((100- x) percentile level), L_{eq} (averaged energy on decibel scale) and peak value, based on the observed noisy data, it is fundamental to estimate the momentarily fluctuating wave form of the specific signal. Furthermore, the fluctuation characteristics of the specific signal and the noises show complex forms of non-Gaussian type.

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H. Orimoto is with the Department of Management Information Systems, Prefectural University of Hiroshima, 734-8558 Japan (corresponding author to provide phone: +82-251-9804; e-mail: orimoto@pu-hiroshima.ac.jp).

A. Ikuta is with the Department of Management Information Systems, Prefectural University of Hiroshima, 734-8558 Japan (e-mail: ikuta@pu-hiroshima.ac.jp).

In general, the fluctuation factors can be classified into two categories of external and internal factors. The former factor can be expressed in an additive model because this factor can exist independently even when the input signal does not exist. On the other hand, the later factor can be reasonably expressed in a multiplicative model with correlation for the input signal because this factor exists in only case of existence of the input signal. In particular for the real sound environmental system contaminated by an external additive noise and multiplicative fluctuation factor of arbitrary distribution types, it is necessary to newly introduce a mixed model type stochastic model by combining the above additive and multiplicative system models.

Several noise cancellation methods such as Kalman filter [4],[5], extended Kalman filter [6], unscented Kalman filter [7], and particle filter [8] have been proposed up to now. However, these studies have been focused on mainly the suppression of additive noise, and the countermeasure methods of multiplicative noise have not been proposed.

In this study, by paying our attention to the observation mechanism in a sound environment system which is considered both of the additive external noise based on the additive property of energy variable and the internal noise expressing in a multiplicative form, a noise cancellation method is proposed. More specifically, first, in the real situation when the stochastic system is affected by a multiplicative random fluctuation factor as well as an additive noise factor of arbitrary probability distribution form, a mixed type stochastic process model is established. Next, based on Bayes' theorem in an expansion series form suitable to a signal processing with lower and higher order statistics for the observation data, a recursive estimation algorithm of the specific signal is derived. Finally, the validity of the proposed method is experimentally confirmed by applying it to the real data observed in the sound environment.

II. CANCELLATION METHOD FOR ADDITIVE AND MULTIPLICATIVE NOISES

A. Modeling for Sound Environment System

As shown in Fig. 1, in sound environment system containing an internal noise w_k with multiplicative fluctuation for the input signal x_k , and an external noise v_k of non-Gaussian type with known statistics, a noise cancellation method to estimate x_k recursively based on the successive output observation y_k at a discrete time k is proposed.

In the case of focusing on energy variables satisfying the additive property between the input signal x_k and the external noise v_k [3],[9], the output observation y_k is expressed as

$$y_k = w_k x_k + v_k . \quad (1)$$

Furthermore, a time transition model:

$$x_{k+1} = F x_k + G u_k , \quad (2)$$

is introducing for the input signal x_k. Here, u_k is a random input, and two parameters F, G can be decided on the basis of the correlation information of time series of x_k.

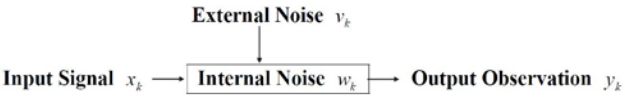


Fig. 1 sound environment system contaminated by internal and external noises

B. Derivation of Noise Cancellation Method

To derive an estimation algorithm for the specific signal x_k, we place our basis on the Bayes' theorem for the conditional probability distribution [1],[2],[3].

$$P(x_k \mid Y_k) = \frac{P(x_k, y_k \mid Y_{k-1})}{P(y_k \mid Y_{k-1})}, \quad (3)$$

where $Y_k (= \{y_1, y_2, ..., y_k\})$ is a set of observation data up to time k. By expanding the conditional joint probability distribution $P(x_k, y_k \mid Y_{k-1})$ in a statistical orthogonal expansion series [10] on the basis of the well-known standard probability distributions describing the dominant part of the real fluctuation, the following expression is derived.

$$P(x_k \mid Y_k) = \frac{P_0(x_k \mid Y_{k-1}) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \varphi_m^{(1)}(x_k) \varphi_n^{(2)}(y_k)}{\sum_{n=0}^{\infty} A_{0n} \varphi_n^{(2)}(y_k)} \quad (4)$$

with

$$A_{mn} = < \varphi_m^{(1)}(x_k) \varphi_n^{(2)}(y_k) \mid Y_{k-1} > . \quad (5)$$

The above two functions $\varphi_m^{(1)}(x_k)$ and $\varphi_n^{(2)}(y_k)$ are orthonormal polynomials of degrees m and n with weighting functions $P_0(x_k \mid Y_{k-1})$ and $P_0(y_k \mid Y_{k-1})$. Based on (4), the estimate of the polynomial function $f_M(x_k)$ of x_k with M th order can be derived as follows.

$$\hat{f}_M(x_k) = < f_M(x_k) \mid Y_k > = \frac{\sum_{m=0}^{M} \sum_{n=0}^{\infty} C_m^M A_{mn} \varphi_n^{(2)}(y_k)}{\sum_{n=0}^{\infty} A_{0n} \varphi_n^{(2)}(y_k)}, \quad (6)$$

where C_m^M is an appropriate constant satisfying the following equality:

$$f_M(x_k) = \sum_{m=0}^{M} C_m^M \varphi_m^{(1)}(x_k) . \quad (7)$$

As examples of standard probability functions for the specific signal and the observation, Gaussian distribution is adopted:

$$P_0(x_k \mid Y_{k-1}) = N(x_k; x_k^*, \Gamma_{x_k}),$$
$$P_0(y_k \mid Y_{k-1}) = N(y_k; y_k^*, \Omega_k) \quad (8)$$

with

$$N(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{(x_k - \mu)^2}{2\sigma^2}\},$$
$$x_k^* = < x_k \mid Y_{k-1} >, \quad \Gamma_{x_k} = < (x_k - x_k^*)^2 \mid Y_{k-1} >,$$
$$y_k^* = < y_k \mid Y_{k-1} >, \quad \Omega_k = < (y_k - y_k^*)^2 \mid Y_{k-1} > . \quad (9)$$

The orthonormal polynomials with two weighting probability distributions in (8) are then specified as

$$\varphi_m^{(1)}(x_k) = \frac{1}{\sqrt{m!}} H_m\left(\frac{x_k - x_k^*}{\sqrt{\Gamma_{x_k}}}\right),$$
$$\varphi_n^{(2)}(y_k) = \frac{1}{\sqrt{n!}} H_n\left(\frac{y_k - y_k^*}{\sqrt{\Omega_k}}\right), \quad (10)$$

where $H_m(\bullet)$ denotes the Hermite polynomial with m th order [10],[11]. Therefore, by selecting

$$f_1(x_k) = x_k, \quad f_2(x_k) = (x_k - \hat{x}_k)^2, \quad (11)$$

in (7), the estimates for mean and variance (i.e., conditional mean and variance) of x_k which are the first and second order statistics, can be expressed as follows:

$$\hat{x}_k = < x_k \mid Y_k > = \frac{\sum_{n=0}^{\infty} \{A_{0n} C_0^1 + A_{1n} C_1^1\} \varphi_n^{(2)}(y_k)}{\sum_{n=0}^{\infty} A_{0n} \varphi_n^{(2)}(y_k)}, \quad (12)$$

$$P_{x_k} = < (x_k - \hat{x}_k)^2 \mid Y_k >$$
$$= \frac{\sum_{n=0}^{\infty} \{A_{0n} C_0^2 + A_{1n} C_1^2 + A_{2n} C_2^2\} \varphi_n^{(2)}(y_k)}{\sum_{n=0}^{\infty} A_{0n} \varphi_n^{(2)}(y_k)} \quad (13)$$

with

$$C_0^1 = x_k^*, \quad C_1^1 = \sqrt{\Gamma_{x_k}}, \quad C_0^2 = \Gamma_{x_k} + \left(x_k^* - \hat{x}_k\right)^2,$$
$$C_1^2 = 2\sqrt{\Gamma_{x_k}}\left(x_k^* - \hat{x}_k\right), \quad C_2^2 = \sqrt{2}\Gamma_{x_k} . \quad (14)$$

Using (1), two variables y_k^* and Ω_k in (9) can be calculated as follows:

$$y_k^* = < w_k > x_k^* + < v_k >, \quad (15)$$

$$\Omega_k = < (w_k - < w_k >)^2 > (\Gamma_{x_k} + x_k^{*2}) \quad (16)$$

Furthermore, using (1), each expansion coefficient A_{mn} defined by (5) can be given in functional forms of x_k^*, Γ_{x_k} and statistics of noises. Some of the expressions are shown as follows:

$$A_{00} = 1, \quad A_{01} = A_{02} = A_{10} = A_{20} = 0,$$

$$A_{11} = \sqrt{\frac{\Gamma_{x_k}}{\Omega_k}} < w_k >, \quad A_{21} = A_{12} = 0, \quad A_{22} = \frac{\Gamma_{x_k}}{\Omega_k} < w_k^2 > . \quad (17)$$

In addition, from (2), the following relationships are obtained.

$$x_{k+1}^* = F \hat{x}_k + G < u_k >, \quad (18)$$

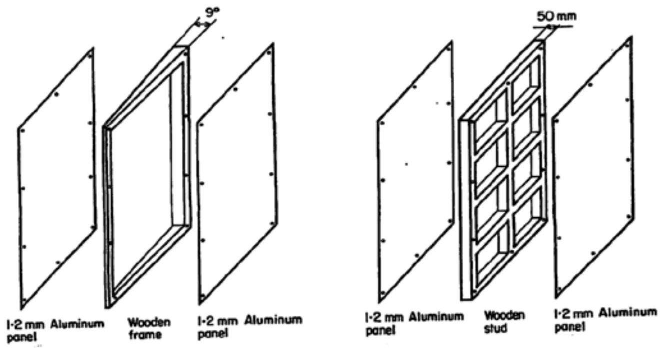
$$\Gamma_{x_{k+1}} = F^2 P_{x_k} + G^2 < (u_k - < u_k >)^2 > . \quad (19)$$

By combining (12), (13) with (18), (19), the estimation of the specific signal can be performed in a recursive way.

III. APPLICATION TO SOUND ENVIRONMENT

A. Estimation of Input Signal for Idealized Sound Environment

In order to confirm the effectiveness of the proposed method, the proposed method was applied to architectural acoustic data in idealized sound environment. The following three kinds of insulating wall structures have been installed between adjoining reverberation rooms (i.e., the transmission and reception rooms): (i) a single wall which consists of an aluminum panel; (ii) a non-parallel double wall which consists of two aluminum panels fixed at an angle of 9 degrees with an air gap between them (cf. Fig.2(a)); (iii) a double wall with a sound bridge, which consists of two aluminum panels connected by a latticed wood sound bridge (the distance between the two aluminum panels is 50 mm; cf. Fig.2(b)).



(a) (b)

Fig.2 sketches of two types of actual acoustical structures discussed in this paper (in addition to a single wall); (a) non-parallel double wall (b) double wall with sound bridge

Each aluminum panel has a surface density of 3.22 kg/m^2 and a thickness of 1.2 mm. The area of air-gap, apart from the bridged part, is 1.46 m^2.

Figure 3 shows the block diagram of the experimental arrangement in the two reverberation rooms. Road traffic noise recorded on a data recorder in advance was used as an input sound source, and white noise generated by the noise generator was emitted as a background noise. The transmitted noise intensity is measured in the presence of strong background noise. In the experiments, the signal to noise ratio on a dB scale is defined in advance as usual by the difference between the two observed values measured in the two cases of employing only the signal and only the noise. Of course, this value of the signal to noise level is measured at the microphone position. The signal to noise ratio at this microphone is positively set in advance to be about 0 dB, as one of the typical worst cases in the experiments, for the purpose of confirming the effectiveness of the proposed estimation method: that is, the adopted sound insulation system has a random output whose averaged signal level is almost equal to that of the background noise at the microphone.

Input and output data were measured with interval of 1 s as two kinds of data (i.e., learning data and prediction data). The scatter diagram between the input and output signals of the learning data is shown in Fig. 4. It is reasonable to express the relationship between x_k and y_k as the following system model.

$$y_k = (a + \varepsilon_k) x_k + b = a x_k + \varepsilon_k x_k + b \, , \tag{20}$$

where two parameters a and b are regression coefficients in a linear regression model between the input x and the output y :

$$y = ax + b \, . \tag{21}$$

Two parameters a and b in (20) were estimated from the learning data by applying the least squares method. Furthermore, ε_k in (20) denotes the fluctuation of the inclination a in the linear regression, and $\varepsilon_k x_k$ expresses the fluctuation around the linear regression model in (21). For random variables with the non-negative fluctuation region $[0, \infty]$ like energy, power, intensity and amplitude, the fluctuation width around the regression curve is not principally independent of the mean value (i.e., regression function). In general, the fluctuation around the mean value becomes larger (or smaller) as the mean value becomes larger (or smaller). Since the scatter diagram between the input and output data for the sound insulation system in Fig. 3 show the tendency that the fluctuation around the regression function corresponding to mean value becomes larger as the input and output data become larger as shown in Fig. 4, the system model in (20) is introduced as the input and output relationship. The term $\varepsilon_k x_k$ in (20) denotes the fluctuation around the linear regression $y_k = a x_k + b$, and the values of $\varepsilon_k x_k$ depend on the input x_k. Furthermore, since the output data are usually measured under existence of an external noise (i.e., background noise) V_k , the relationship can be derived from (20).

$$y_k = w_k x_k + v_k \, , \tag{22}$$

where w_k ($= a_k + \varepsilon_k$) is a multiplicative noise and v_k ($= b_k + V_k$) denotes an additive noise in (1).

For comparison, estimation results in four cases of (i) regarding $\varepsilon_k x_k$ as external additive noise ε_k independent of the input signal, by introducing a system model: $y_k = a_k x_k + \varepsilon_k + b_k + V_k$ instead of (22), where the statistics of ε_k are obtained from the data of ε_k after evaluating from $\varepsilon_k = y_k - a_k x_k - b_k$ by use of the learning data, (ii) ignoring ε_k by introducing a system model: $y_k = a_k x_k + b_k + V_k$ instead of (22), (iii) applying Kalman filter (KF) to the system model in (i), (iv) applying the extended Kalman filter (EKF) to the system model in (22) were obtained. The estimation results in the cases of (i) and (ii) are shown as "Compared Method 1" and "Compared Method 2" respectively. The experimental results for the estimation results using double wall with sound bridge are shown in Figs. 5-8. Though the estimation algorithm derived in this study is expressed in an infinite expansion series, only the finite expansion series can be applied to the real estimation problem. Since many state estimation methods like KF consider the statistics until the second order like mean and variance of external noise, the proposed method considering the statistics until the second order was also applied to the real estimation in order to compare the proposed method with the previous methods in the same condition. More specifically, the estimation algorithms considering (12) with $n \le 1$, and (13) with $n \le 2$ were adopted.

The RMS errors of the estimations for three kinds of insulating wall structures are shown in Table 1. The proposed method shows more accurate estimation than the results by the compared methods, and the validity of the proposed method has been confirmed.

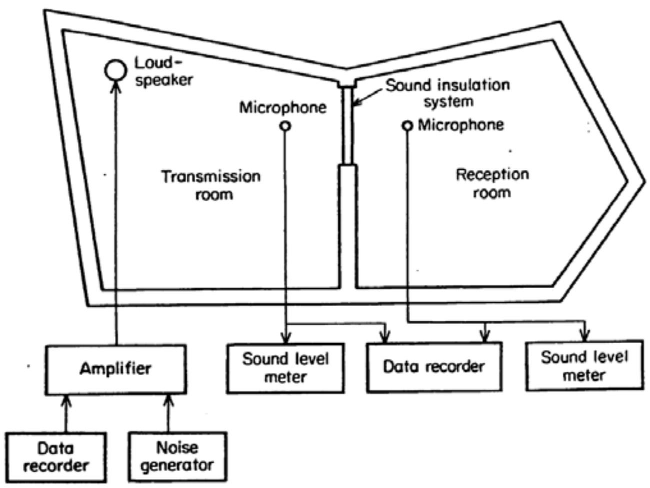


Fig. 3 block diagram of experimental arrangement

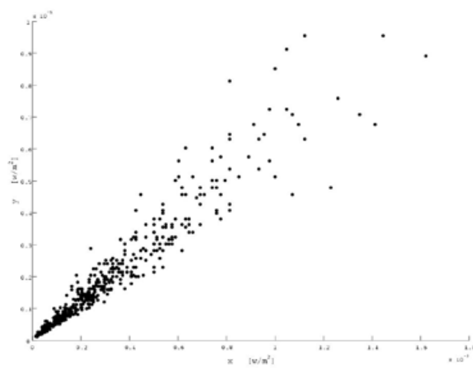


Fig. 4 scatter diagram between the input and output data for idealized sound environment

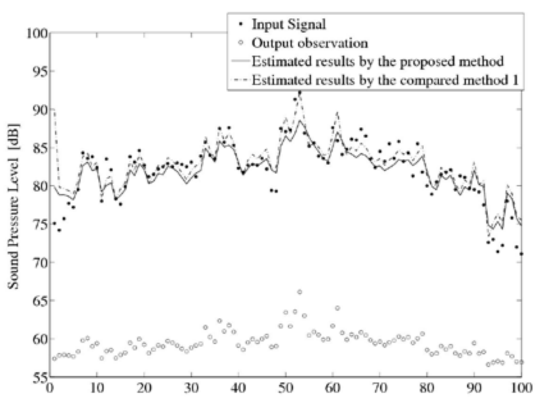


Fig. 5 comparison between two estimated results by the proposed method and the compared method 1 for idealized sound environment

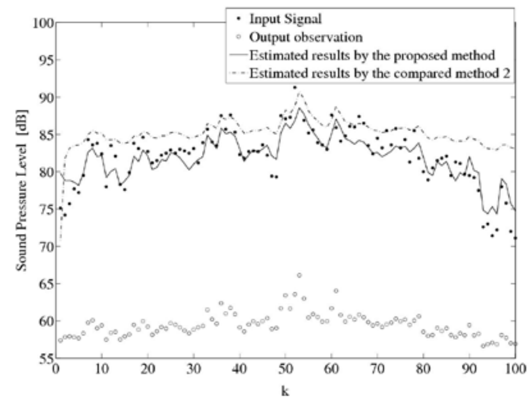


Fig. 6 comparison between two estimated results by the proposed method and the compared method 2 for idealized sound environment

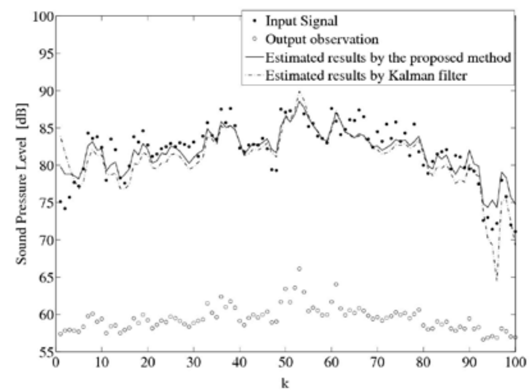


Fig. 7 comparison between two estimated results by the proposed method and the KF for idealized sound environment

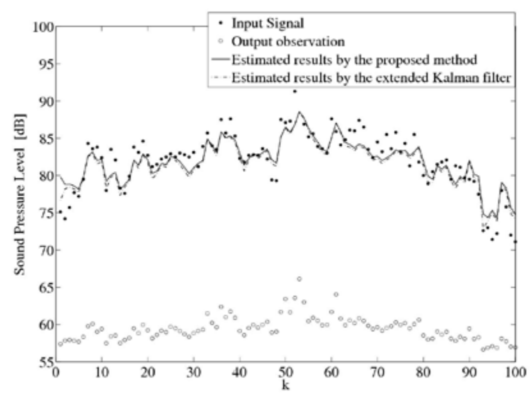


Fig. 8 comparison between two estimated results by the proposed method and the EKF for idealized sound environment

Table 1 root mean squared error of the estimation in [dB] for idealize sound environment

	Proposed Method	Compared Method 1	Compared Method 2	KF	EKF
Single wall	1.11	1.25	1.88	1.18	1.25
Non-parallel double wall	1.26	1.61	3.11	1.56	1.73
Double wall with a sound bridge	1.84	2.28	4.30	2.45	1.89

B. Estimation of Input Signal for Real Sound Environment

The proposed method was applied to input and output sound data measured indoors and outdoors for a house shown in Fig. 9, as an example of complex sound environment systems. After generating a music sound inside the house, the sound energy data measured indoors and outdoors were regarded as the input signal x_k and the output observation y_k. The input and output data were measured simultaneously with a sampling interval of 1 s as 500 learning data points and 200 prediction data points.

By applying the proposed method to the prediction data, the fluctuation wave of x_k was estimated on the basis of the recursive observation of y_k.

The estimation results of the proposed method are shown in Figs. 10-13. These figures are compared with (i) - (iv) of Section III.A. The RMS errors of the estimation are shown in Table 2. The proposed method shows more accurate estimation than the results by the compared methods, and the practical usefulness of the proposed method has been confirmed.

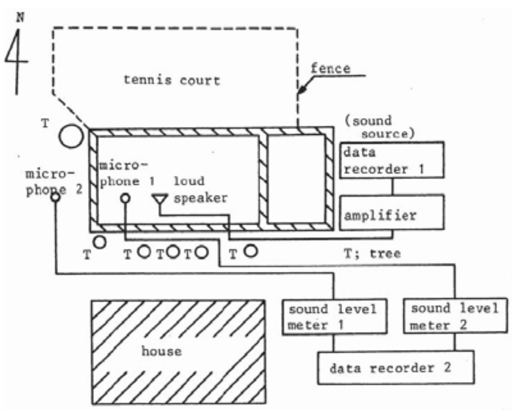


Fig. 9 a schematic drawing of the experimental set up in sound insulation system

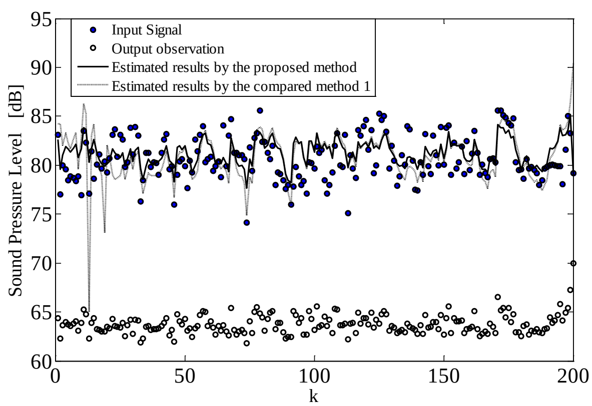


Fig. 10 comparison between two estimated results by the proposed method and the compared method 1 for real sound environment

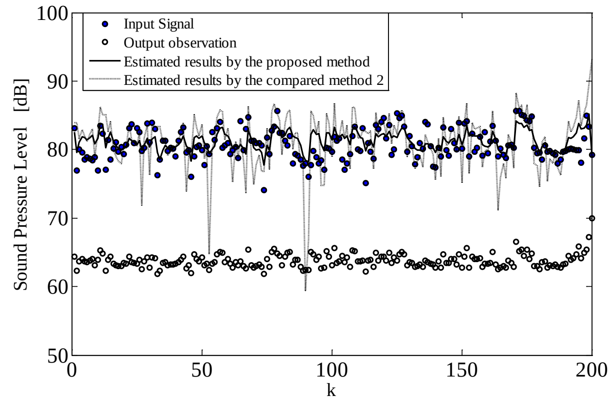


Fig. 11 comparison between two estimated results by the proposed method and the compared method 2 for real sound environment

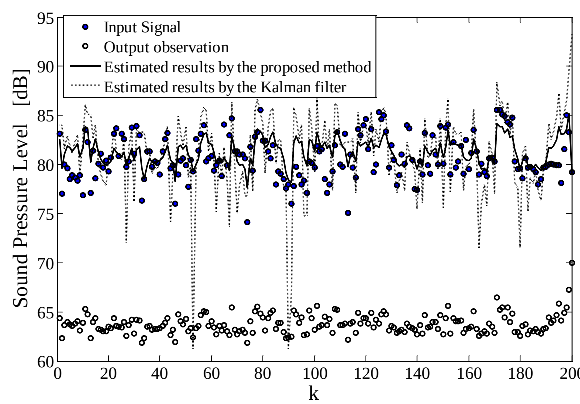


Fig. 12 comparison between two estimated results by the proposed method and the KF for real sound environment

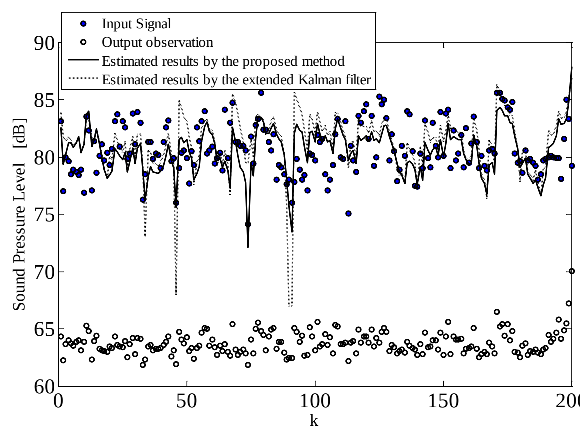


Fig. 13 comparison between two estimated results by the proposed method and the EKF for real sound environment

Table 2 root mean squared error of the estimation in [dB] for real sound environment

Proposed Method	Compared Method 1	Compared Method 2	KF	EKF
2.09	2.58	3.67	3.72	2.60

IV. CONCLUSIONS

In this study, sound environment system involving both of external noise satisfying the additive property of energy variable and multiplicative noise expressing internal fluctuation have been considered. A noise cancellation method for sound environment system with the external and multiplicative noises has been derived on the basis of Bayes' theorem in an expansion series form suitable to the signal processing by utilizing the higher order correlation information. More specifically, by expressing the internal fluctuation factor as multiplicative noise, and by considering Bayes' theorem in an orthogonal expansion expression of the conditional probability distribution reflecting the correlation information between the input and output signals in the expansion coefficients, an estimation algorithm has been derived. Furthermore, by applying the proposed method to real data in a sound environment system, the effectiveness of the theory has been confirmed experimentally.

The proposed method is quite different from the traditional standard approach. However, it is still at its early stage of study, and there are a number of practical problems to be explored in the future, starting from the result of the basic study in this paper. Some of the problems are the following.

(i) The proposed method should be applied to real estimation and prediction problems for many other sound environment systems, and its practical usefulness should be verified in each real situation.

(ii) By considering the higher order statistics as many as possible, the accuracy of estimation and prediction should be improved.

(iii) An optimal number of expansion terms in the proposed estimation algorithm of expansion expression type should be found.

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Hisako Orimoto received the B.Sc. degree from Hiroshima Prefectural Women's University in 2003, M.Sc. degree from Prefectural University of Hiroshima in 2008, and D.Eng. degree from Hiroshima University in 2009.

After a System Engineer of System Development Division, NTT DATA Chugoku Co. and a Research Associate of Hiroshima National College of Maritime Technology, she is now an Assistant Professor of Department of Management Information Systems, Prefectural University of Hiroshima in Hiroshima, Japan. Her research interests include the Digital Signal Information Processing and Data Processing**.**

Akira Ikuta received the B.E. and M.E. degrees from Kyushu Institute of Technology in 1976 and 1978, the D.Eng. degree from Hiroshima University in 1981, respectively.

After a Research Associate of Electrical Engineering at Hiroshima University and an Assistant Professor of Mechanical Systems Engineering of Kinki University, he is now a Professor of Department of Management Information Systems, Prefectural University of Hiroshima in Hiroshima, Japan. His research interests include the Digital Signal Information Processing and Wave Motion Environment (Sound, Vibration and Electromagnetic Wave).

Prof. Ikuta is an Editorial Board of Advances in Acoustics and Vibration.