Circuit Representation of Load and Line Asymmetries in Three-Phase Power Systems

Diego Bellan, Gabrio Superti Furga, and Sergio A. Pignari

***Abstract*—**This work deals with circuit representation of load and line asymmetries in three-phase power systems. Indeed, it is well known that under the assumption of symmetrical phases, power systems are effectively analyzed through the symmetrical component transformation leading to uncoupled sequence circuits. In practical power systems, however, the assumption of symmetrical phases is not always met for several reasons including, for example, single-phase loads (e.g., high-speed railway systems) and untransposed overhead transmission lines. From a circuit viewpoint, when the symmetrical component transformation is applied, such asymmetries result in coupled sequence circuits. As a consequence, a voltage on the negative-sequence circuit arises, leading to the well-known detrimental phenomenon of voltage unbalance emission. In this paper, a rigorous circuit representation of coupling between positive- and negative-sequence circuits is derived by means of the definition of an ideal transformer with complex turns ratio. The proposed equivalent circuit is useful to get deeper insight into the fundamental mechanisms of voltage unbalance generation and identification. Moreover, the proposed circuit representation can be readily implemented into software for circuit simulation.

***Keywords*—**Asymmetrical three-phase loads and lines, power system analysis, symmetrical components transformation, voltage unbalance emission.

I. Introduction

THREE-phase power systems under steady state conditions are usually analyzed in the frequency (i.e., phasor) domain by means of the symmetrical component transformation [1]. Other similar transformations are available, operating in the frequency or in the time domain [2]-[6]. The main assumption underlying such methods is the symmetry of three-phase loads and lines, i.e., the three impedances must be equal and with the same coupling coefficient (if any). The reason is clearly due to the fact that, under such assumption, the transformations mentioned above decouple the modal circuits (e.g., the positive, negative, and zero sequence circuits in the symmetrical component transformation), allowing a much easier and direct analysis. Moreover, under the assumption of system symmetry, only the positive-sequence circuit needs to be solved, whereas the negative-sequence circuit is not excited. If the assumption of symmetrical load/line is not met, the transformations provide coupled modal circuits, leading to voltages and currents also on the negative-sequence circuit, i.e., the well-known phenomenon called voltage unbalance emission [7]-[9]. In the relevant technical literature, voltage unbalance emission is analyzed by treating the coupling of positive- and negative-sequence circuits as a two-port network, but an equivalent circuit representation is not provided. Such a circuit representation, however, would be useful to get deeper insight into the phenomenon of voltage unbalance emission allowing a clear identification and characterization of each emission source [10]-[11]. This is an important issue because asymmetry can be a feature of modern power systems for several reasons. First, single-phase loads drawing significant electrical power (e.g., high-speed railway systems) result in asymmetrical loads for a three-phase system. Second, asymmetrical geometric arrangement (e.g., non-ideal transposition) of overhead transmission lines can result in asymmetrical behavior of lines.

The paper is organized as follows. In Section II the background concerning the symmetrical component transformation is recalled. In Section III, the effects of asymmetrical loads are investigated and a circuit representation is derived by resorting to the definition of an ideal transformer with complex turns ratio, whose definition and properties are presented in the Appendix. In Section IV the methodology is applied to asymmetrical transmission lines, i.e., lines characterized by conductors with unequal mutual inductances due to asymmetrical geometric arrangement. Finally, conclusions are drawn in Section V.

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II. Background

Three-phase power systems with symmetrical loads can be conveniently analyzed by resorting to the well-known symmetrical components transformation. Indeed, even in the case of mutual coupling between the phases, the assumption of symmetrical system results in three uncoupled sequence circuits. Solving each sequence circuit is much simpler than solving the system as a whole.

The transformation matrix, in its rational form, is defined as

$$\mathbf{S} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

where

$$\alpha = e^{j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{2}$$

and $\alpha^2 = \alpha^*$. The transformation matrix is a Hermitian matrix, i.e., $\mathbf{S}^{-1} = \mathbf{S}^{T*}$.

The symmetrical components transformation when applied to phasor voltages provides

$$\begin{bmatrix} V_+ \\ V_- \\ V_0 \end{bmatrix} = \mathbf{S} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{3}$$

where V_+, V_-, and V_0 are the positive, negative, and zero sequence voltages. Of course, the same transformation applies to phasor currents.

Symmetrical three-phase loads can be described in terms of an impedance matrix with the following structure

$$\mathbf{Z} = \begin{bmatrix} Z & Z_m & Z_m \\ Z_m & Z & Z_m \\ Z_m & Z_m & Z \end{bmatrix} \tag{4}$$

By defining the column vectors

$$\mathbf{V}_s = \begin{bmatrix} V_+ \\ V_- \\ V_0 \end{bmatrix},\ \mathbf{I}_s = \begin{bmatrix} I_+ \\ I_- \\ I_0 \end{bmatrix},\ \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix},\ \mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{5}$$

the transformed current/voltage relationship for a symmetrical load can be written

$$\mathbf{V}_s = \mathbf{S}\mathbf{Z}\mathbf{S}^{-1}\mathbf{I}_s = \mathbf{Z}_s\mathbf{I}_s \tag{6}$$

where

$$\mathbf{Z}_s = \begin{bmatrix} Z_+ & 0 & 0 \\ 0 & Z_- & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \tag{7}$$

and

$$Z_+ = Z_- = Z - Z_m \tag{8a}$$
$$Z_0 = Z + 2Z_m \tag{8b}$$

The diagonal form of the sequence impedance matrix (7) leads to the above-mentioned uncoupled sequence circuits when the transformation is applied to the whole three-phase system.

III. Asymmetrical Loads

Let us consider an asymmetrical three-phase load consisting in three uncoupled impedances Z_a, Z_b, and Z_c taking values possibly different with respect to the average value Z. The assumption of uncoupled impedances is reasonable since load asymmetry results from different single-phase loads applied to each phase of the power system. We assume therefore

$$Z_a = Z + \delta Z_a \tag{9a}$$

$$Z_b = Z + \delta Z_b \tag{9b}$$

$$Z_c = Z + \delta Z_c \tag{9c}$$

where the impedance deviations δZ_a, δZ_b, and δZ_c are such that $\delta Z_a + \delta Z_b + \delta Z_c = 0$ since Z is the impedance average value.

The asymmetrical impedance matrix can be written

$$\tilde{\mathbf{Z}} = \begin{bmatrix} Z+\delta Z_a & 0 & 0 \\ 0 & Z+\delta Z_b & 0 \\ 0 & 0 & Z+\delta Z_c \end{bmatrix} =$$

$$= \begin{bmatrix} Z & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix} + \delta Z_a \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$+ \delta Z_b \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \delta Z_c \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \mathbf{Z} + \mathbf{\Delta}_a + \mathbf{\Delta}_b + \mathbf{\Delta}_c \tag{10}$$

By applying the transformation (6) to (10), the average impedance matrix $\mathbf{Z}$ remains unchanged (i.e., $\mathbf{Z}_s = \mathbf{Z}$) since its structure is the same as (4) with out-of-diagonal elements equal to zero. Therefore, only the transformed version of matrices $\mathbf{\Delta}_a$, $\mathbf{\Delta}_b$, and $\mathbf{\Delta}_c$ must be analyzed. After some algebra, the following expressions can be obtained:

$$\mathbf{\Delta}_{sa} = \mathbf{S}\mathbf{\Delta}_a\mathbf{S}^{-1} = \frac{\delta Z_a}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{11}$$

$$\mathbf{\Delta}_{sb} = \mathbf{S}\mathbf{\Delta}_b\mathbf{S}^{-1} = \frac{\delta Z_b}{3} \begin{bmatrix} 1 & \alpha^2 & \alpha \\ \alpha & 1 & \alpha^2 \\ \alpha^2 & \alpha & 1 \end{bmatrix} \tag{12}$$

$$\mathbf{\Delta}_{sc} = \mathbf{S}\mathbf{\Delta}_c\mathbf{S}^{-1} = \frac{\delta Z_c}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha^2 & 1 & \alpha \\ \alpha & \alpha^2 & 1 \end{bmatrix} \tag{13}$$

In this work, the analysis is performed for power systems with three wires. Therefore, the zero-sequence circuit corresponding to the zero-sequence voltage and current in (6) is not defined. Thus, by taking into account (11)-(13), the positive- and negative-sequence components of the sequence impedance matrix $\tilde{\mathbf{Z}}_s$ provide the following relationship between positive- and negative-sequence variables:

$$\begin{bmatrix} V_+ \\ V_- \end{bmatrix} = \left(Z \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{\delta Z_a}{3} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \right.$$

$$\left. + \frac{\delta Z_b}{3} \begin{bmatrix} 1 & \alpha^2 \\ \alpha & 1 \end{bmatrix} + \frac{\delta Z_c}{3} \begin{bmatrix} 1 & \alpha \\ \alpha^2 & 1 \end{bmatrix} \right) \begin{bmatrix} I_+ \\ I_- \end{bmatrix} \tag{14}$$

From a circuit point of view, the four additive impedance terms in (14)

$$\mathbf{z}_s = Z \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15a}$$

$$\boldsymbol{\delta}_{sa} = \frac{\delta Z_a}{3} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{15b}$$

$$\boldsymbol{\delta}_{sb} = \frac{\delta Z_b}{3} \begin{bmatrix} 1 & \alpha^2 \\ \alpha & 1 \end{bmatrix} \tag{15c}$$

$$\boldsymbol{\delta}_{sc} = \frac{\delta Z_c}{3} \begin{bmatrix} 1 & \alpha \\ \alpha^2 & 1 \end{bmatrix} \tag{15d}$$

correspond to a series connection of four two-port networks (see Fig. 1) whose explicit circuit representations can be derived as follows.

The two-port network corresponding to (15a) is a pair of uncoupled impedances *Z*.

The circuit representation of (15b) consists in an impedance $\delta Z_a/3$ connected in parallel with the two ports (see Fig. 2). An ideal transformer with unit turns ratio can be added for homogeneity with the following equivalent networks.

The two-port networks (15c) and (15d) are similar each other. Notice that since the two matrices in (15c) and (15d) are not symmetrical, the equivalent circuit cannot be given in terms of reciprocal components only. A simple representation, however, can be derived by resorting to the definition of an ideal transformer with complex turns ratio (see Appendix).

In fact, for the two-port network (15c) it can be easily shown that the equivalent circuit (see Fig. 3) consists in an ideal transformer with complex turns ratio given by $\alpha^2 = \alpha^*$, and an impedance $\delta Z_b/3$ connected in parallel to one of the two ports (in fact, by reporting the impedance through the ideal transformer its value does not change since the scaling coefficient is $|\alpha^2| = 1$).

For the two-port network (15d) the circuit representation is similar to (15c), where the ideal transformer has complex turns ratio α instead of α^2, and the parallel impedance is $\delta Z_c/3$ (see Fig. 4).

The complete circuit representation of coupling between positive- and negative-sequence circuits is shown in Fig. 5.

A. *Special Case: Asymmetrical Reactive Load*

The general results derived above, of course, hold also in the special case where asymmetry is only in the reactive part of the load impedances. In this case, however, a simpler equivalent circuit can be derived by means of an alternative approach.

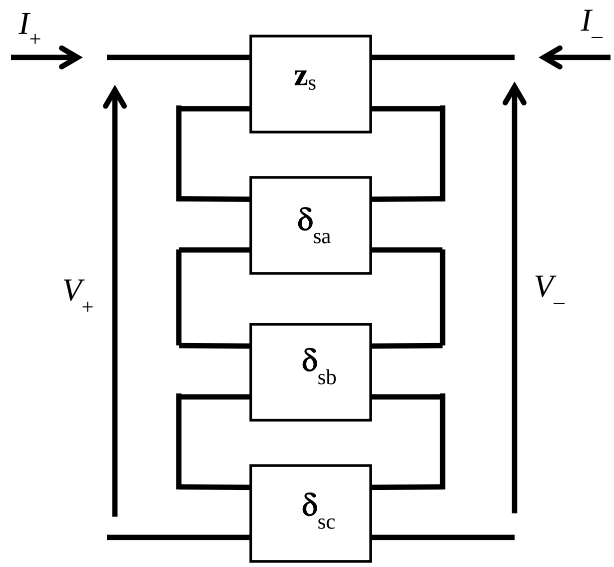

Fig. 1. Series connection of two-port networks representing nominal impedances and coupling due to impedance asymmetries.

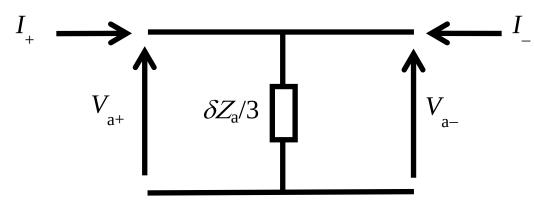

Fig. 2. Two-port network representing asymmetry in Z_a.

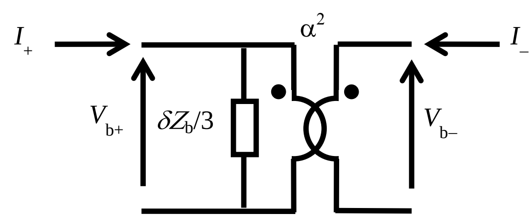

Fig. 3. Two-port network representing asymmetry in Z_b.

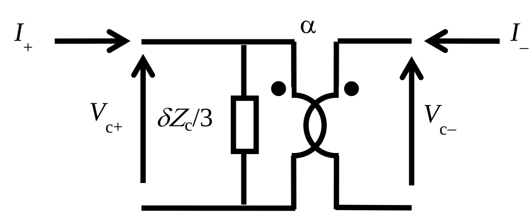

Fig. 4. Two-port network representing asymmetry in Z_c.

In this case the asymmetrical part of the load impedance matrix can be written

$$\mathbf{Z} = \begin{bmatrix} jX_a & 0 & 0 \\ 0 & jX_b & 0 \\ 0 & 0 & jX_c \end{bmatrix} \tag{16}$$

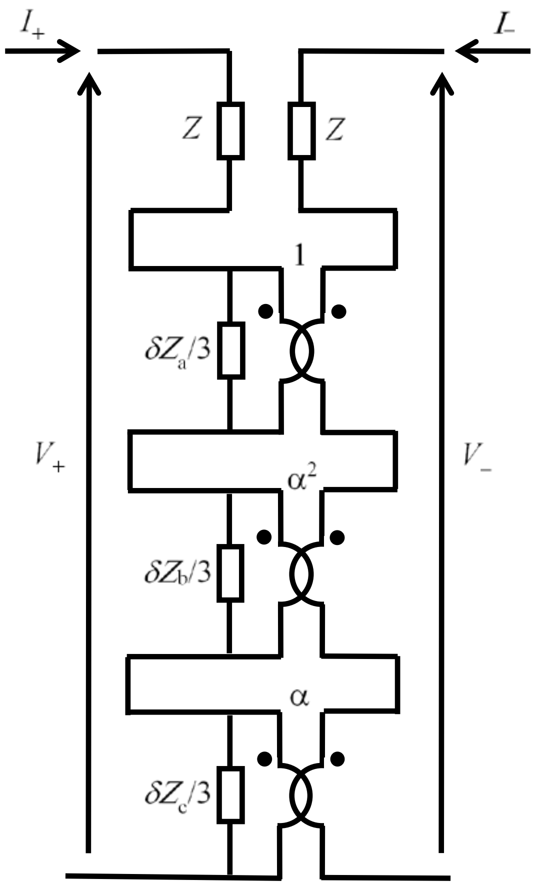

Fig. 5. Complete two-port network representation of coupling between positive- and negative-sequence circuits due to asymmetries in load impedances.

By applying the symmetrical component transformation we obtain:

$$\mathbf{Z}_s = j\begin{bmatrix} X & X_- & X_+ \\ X_+ & X & X_- \\ X_- & X_+ & X \end{bmatrix} = j\mathbf{X}_s \tag{17}$$

where X is the average reactance

$$X = \frac{X_a + X_b + X_c}{3} \tag{18}$$

and

$$X_+ = \frac{X_a + \alpha X_b + \alpha^2 X_c}{3} \tag{19}$$

$$X_- = \frac{X_a + \alpha^2 X_b + \alpha X_c}{3} \tag{20}$$

The matrix $\mathbf{X}_s$ can be written as the sum of its symmetric and antisymmetric parts:

$$\mathbf{X}_s = \begin{bmatrix} X & \frac{X_- + X_+}{2} & \frac{X_- + X_+}{2} \\ \frac{X_- + X_+}{2} & X & \frac{X_- + X_+}{2} \\ \frac{X_- + X_+}{2} & \frac{X_- + X_+}{2} & X \end{bmatrix} + \begin{bmatrix} 0 & \frac{X_- - X_+}{2} & \frac{X_+ - X_-}{2} \\ \frac{X_+ - X_-}{2} & 0 & \frac{X_- - X_+}{2} \\ \frac{X_- - X_+}{2} & \frac{X_+ - X_-}{2} & 0 \end{bmatrix} \tag{21}$$

After simple algebra we obtain:

$$\mathbf{Z}_s = \begin{bmatrix} jX & j\frac{\delta X_a}{2} & j\frac{\delta X_a}{2} \\ j\frac{\delta X_a}{2} & jX & j\frac{\delta X_a}{2} \\ j\frac{\delta X_a}{2} & j\frac{\delta X_a}{2} & jX \end{bmatrix} + \begin{bmatrix} 0 & \frac{\delta X_b - \delta X_c}{2\sqrt{3}} & -\frac{\delta X_b - \delta X_c}{2\sqrt{3}} \\ -\frac{\delta X_b - \delta X_c}{2\sqrt{3}} & 0 & \frac{\delta X_b - \delta X_c}{2\sqrt{3}} \\ \frac{\delta X_b - \delta X_c}{2\sqrt{3}} & -\frac{\delta X_b - \delta X_c}{2\sqrt{3}} & 0 \end{bmatrix} \tag{22}$$

and finally, by taking into account only positive- and negative-sequence circuits:

$$\begin{bmatrix} V_+ \\ V_- \end{bmatrix} = \left(\begin{bmatrix} jX & j\frac{\delta X_a}{2} \\ j\frac{\delta X_a}{2} & jX \end{bmatrix} + \begin{bmatrix} 0 & \frac{\delta X_b - \delta X_c}{2\sqrt{3}} \\ -\frac{\delta X_b - \delta X_c}{2\sqrt{3}} & 0 \end{bmatrix} \right) \begin{bmatrix} I_+ \\ I_- \end{bmatrix} \tag{23}$$

The circuit representation of (23) consists in the series connection of only two components. The first component is a mutual inductor whose circuit representation can be given in terms of a real ideal transformer as in Fig. 6, where the turns ratio k is given by

$$k = \frac{\delta X_a}{2X} \tag{24}$$

and the leakage coefficient σ is given by

$$\sigma = 1 - k^2 = 1 - \frac{\delta X_a^2}{4X^2} \tag{25}$$

The second component in (23) corresponds to a gyrator with resistive parameter r given by

$$r = \frac{\delta X_c - \delta X_b}{2\sqrt{3}} \tag{26}$$

The complete circuit representation of coupling due to an asymmetric reactive load is shown in Fig. 7.

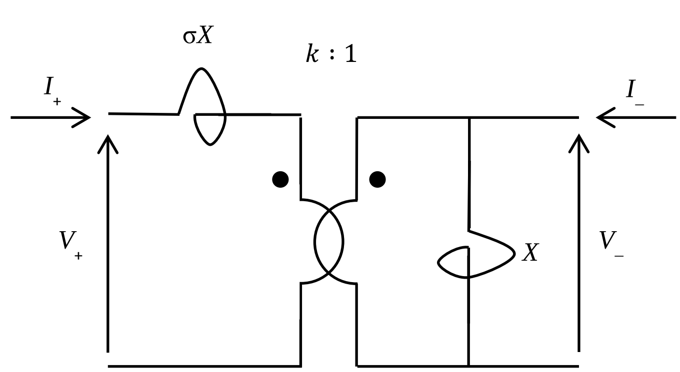

Fig. 6. Circuit representation of the inductive coupling component between positive and negative sequence circuits.

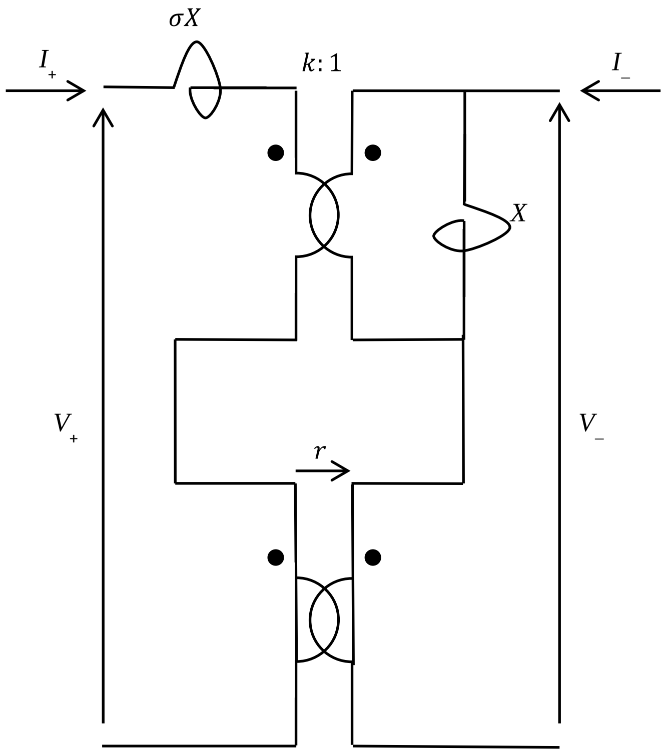

Fig. 7. Complete circuit representation for the coupling between positive and negative sequence circuits due to an asymmetrical reactive load.

A significant particular case is when only positive-sequence voltage is imposed to an asymmetrical load. In this case, in a three-wire system, from (23) with $V_{-} = 0$ it can be readily obtained

$$I_{+} = -\frac{\mathrm{j}X}{X^2-\left(\frac{\delta X_a}{2}\right)^2-\left(\frac{\delta X_b-\delta X_c}{2\sqrt{3}}\right)^2} V_{+} \tag{27a}$$

$$I_{-} = j\frac{\frac{\delta X_a}{2}+j\frac{\delta X_b-\delta X_c}{2\sqrt{3}}}{X^2-\left(\frac{\delta X_a}{2}\right)^2-\left(\frac{\delta X_b-\delta X_c}{2\sqrt{3}}\right)^2} V_{+} \tag{27b}$$

Eq. (27b) shows that load asymmetry results in a negative-sequence current even if a symmetrical voltage is imposed.

Moreover, from (27a)-(27b) the squared value of the total rms three-phase current can be evaluated

$$I_T^2 = |I_a|^2 + |I_b|^2 + |I_c|^2 = |I_+|^2 + |I_-|^2 =$$

$$= \frac{X^2+\left(\frac{\delta X_a}{2}\right)^2+\left(\frac{\delta X_b-\delta X_c}{2\sqrt{3}}\right)^2}{\left[X^2-\left(\frac{\delta X_a}{2}\right)^2-\left(\frac{\delta X_b-\delta X_c}{2\sqrt{3}}\right)^2\right]^2}|V_+|^2 \tag{28}$$

From (28) it can be observed that load asymmetry results always in an increased value of the total rms three-phase current.

IV. Asymmetrical Lines

Power systems lines are usually transposed along their length such that each of the three conductor can be considered geometrically equivalent to the others on average basis. When the length of the line does not allow transposition, however, the mutual inductance between each conductor pair cannot be considered the same for each conductor pair [7]-[8]. A typical example is given by a line with a horizontal arrangement of the three conductors. In this case, the mutual inductance between the outer conductors is lower than the two pairs including the inner conductor. This arrangement results in an asymmetric impedance matrix in the out-of-diagonal coefficients 1-3. The impedance matrix in this case can be written

$$\widetilde{\boldsymbol{Z}} = \boldsymbol{Z} + j\delta X_m \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{29}$$

where Z is the symmetric part of the impedance matrix, and δX_m is the deviation of the mutual reactance 1-3 due to geometric asymmetry of conductor arrangement. By applying the symmetrical component transformation to (29), and by taking only positive and negative sequence variables as in Section III, for the asymmetric part (responsible for circuit coupling) we obtain

$$\begin{bmatrix} V_+ \\ V_- \end{bmatrix} = -j\frac{2}{3}\delta X_m \begin{bmatrix} \frac{1}{2} & \alpha \\ \alpha^2 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} I_+ \\ I_- \end{bmatrix} \tag{30}$$

whose circuit representation can be given in terms of an ideal transformer with complex turns ratio as in Fig. 8.

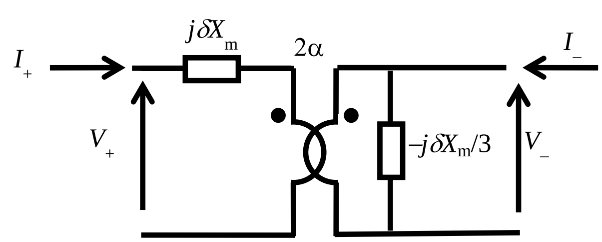

Fig. 8. Circuit representation of coupling due to line asymmetry.

Appendix

An ideal transformer with complex turns ratio k is defined by (passive sign convention at the two ports):

$$V_1 = kV_2 \tag{A1}$$

$$I_2 = -k^* I_1 \tag{A2}$$

Notice the complex conjugate of k in (A2). Indeed, such definition leads to conservation of complex power:

$$S_1 = V_1 I_1^* = kV_2 \left(\frac{-I_2}{k^*}\right)^* = -V_2 I_2^* = -S_2 \qquad \text{(A3)}$$

Moreover, the scaling coefficient for reporting impedances from the secondary to the primary side is $|k|^2$. In fact, an impedance Z_2 on the secondary side results in a primary side equivalent impedance

$$Z_1 = \frac{V_1}{I_1} = \frac{kV_2}{-\frac{I_2}{k^*}} = kk^* \frac{V_2}{-I_2} = |k|^2 Z_2 \qquad \text{(A4)}$$

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