Comparison of responses of nonlinear circuits and nonlinear resonant mediums

Leonid A. Rassvetalov

Abstract - Responses of the nonlinear resonant medium represented by set oscillators are investigated. Solutions of the nonlinear equations of oscillator in the form of final Volterra series in the time and frequency domains, corresponding to anharmonicity, nonlinear excitation and nonlinear attenuation are used. Due to time-and-frequency dualism nonlinear resonant medium allows to organize calculation of integral transformations of the convolution type in frequency space with the same connectivity, as multiplication in time space. Integral transformation of input signals responses' character is considered. Attention is paid to the duality both to the mediums under consideration and to classical nonlinear circuits.°

Keywords: oscillator, resonant medium, nonlinearity, Volterra series, nonlinear circuit, dualism.

1 INTRODUCTION

Due to the time-frequency dualism nonlinear resonant (NRM) medium makes possible to calculate integral transformations of the convolution type in frequency space with the same connectivity as multiplication in time space.

In this case nonlinear effects will lead not to frequency mixing resulting in generation of oscillations with combinational frequencies, but to time mixing, i.e. to generation of signals (pulses) at combinational instants of time [1], [2]. This time-frequency dualism phenomenon is illustrated by fig. 1.

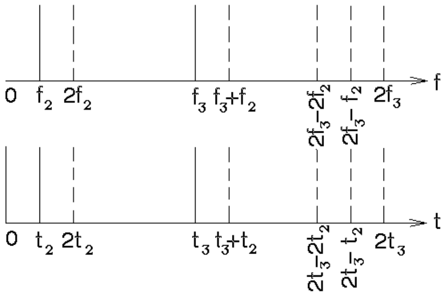

Fig. 1. Responses of nonlinear systems to multisignal excitation: above – responses of the nonlinear circuit to a series of harmonic excitations, below – responses of the nonlinear resonant medium to the excitation in the form of delta functions.

The time positions of responses are as rigidly connected to the time position of excitation pulses in nonlinear frequency space as combinational frequencies arising in a nonlinear circuit are connected to the excitation frequencies. Let us define the resonant medium as a set of high-Q oscillators, resonating in a frequency band. Such representation depicts the medium's local heterogeneity.

The term "oscillator" here covers concepts such as separate micro particles or medium collective excitations – quasi-particles – under quantum-mechanical consideration, as well as molecules or even the macroscopical particles carrying all properties of the substance - in the classical approach. In such a model the nonlinear properties of the medium can be provided both by the interaction of external excitation with a separate oscillator, and by the interaction between separate excited oscillators and thus reduced to the following types

- Anharmonicity;
- Nonlinear excitation;
- Nonlinear attenuation;
- Nonlinear interaction between oscillators

Solving one-partial problem is sufficient for the first three kinds of nonlinearity. In the latter case, it is required to resolve a problem of many particles for the description of the model. The medium's response to the external excitation will be calculated by summing the responses of separate oscillators regarding with respect to their frequency distribution density $g(\omega)$. It is appropriate to mention here that the resonant medium represented by a set of oscillators is a real frequency space and it is convenient to describe it in terms of frequency representation. Similar problems were observed in works [4]-[6].

2 PROBLEM FORMULATION

The response of such nonlinear resonant medium - echo-signal – is a result of in-phase summation of oscillations of the excited oscillators, therefore the term «phased echo» is frequently used for this signal definition Specific physical and mathematical models distinguished by both the big variety, and significant complexity are used in various type echo researches. In the applications, the theory of a spin echo [3] is discussed at length, yet the analysis is limited to small-signal approximation. The statistical analysis of the known various medium echo phenomenon physical and mathematical models, not limited by the small-signal approximation frameworks, represents significant mathematical difficulties. The volume of such calculations even more increases due to the big variety of concrete physical mechanisms of echo – signals formation.

The purpose of this publication is to elaborate a unified description of the echo phenomenon regardless of the concrete physical mechanism of its formation, suitable for the echo-processors work analysis in the structure of various radio engineering systems effected by signals and interference of any intensity. The mechanisms of nonlinearity mentioned above have peculiarities related to the amplitude behavior of responses and the responses phase - exciting pulses phase dependence. The dependence of responses shape on the shape of excitation pulses is the same for all types of nonlinearity. Phase characteristics are defined by the kind of nonlinearity.

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3 PROBLEM SOLUTION

3.1 NRM MODEL WITH ANHARMONIC OSCILLATORS

Let us present the equation of the i-th anharmonic oscillator as follows

$$D_i\, y_i(t) + F_i\,[y_i(t)] = x(t), \qquad (1)$$

where $x(t)$ - external excitation, $D_i = \frac{d^2}{dt^2} + 2\sigma_i \frac{d}{dt} + \omega_{0i}^2$ - linear operator, σ_i and ω_{0i} - loss characteristic and resonant frequency of linear approximation, correspondingly, $y_i(t)$ – response of the i-th oscillator, $F_i\left[y_i(t)\right] = \sum_{k=2}^{p} a_k y_i^k(t)$ is a polynomial of the p-th degree, a_k - constants including power constants and geometrical values.

To solve (1) let us pass to the equivalent integral relation:

Let us denote weight (pulse) function of a linear equation $Dy_i = x$ by h:

$$h_i(t,\tau) = \begin{cases} \frac{1}{\omega_{0i}} e^{-\sigma_i(t-\tau)} \sin\omega_{ei}(t-\tau), & t \ge \tau \\ 0, & t < \tau, \end{cases} \qquad (2)$$

$$\omega_{ei} = \sqrt{\omega_{0i}^2 + \sigma_i^2} \approx \omega_{0i}$$

Thus we receive

$$y_i(t) = \int_{t_0}^{T} h_i(t-\tau)\left(D_i y_i\right)_\tau d\tau = \int_{t_0}^{T} h_i(\tau)\left(D_i y_i\right)_{t-\tau} d\tau \qquad (3)$$

Let us multiply (1) by h and integrate over t

$$\int_{t_0}^{T} h_i(t-\tau)\left(D_i y_i\right)_\tau d\tau + \int_{t_0}^{T} h_i(t,\tau) F\left(y_i(\tau)\right) d\tau = \int_{t_0}^{T} h_i(\tau)\left(D_i y_i\right)_{t-\tau} d\tau, \qquad (4)$$

taking into account (3) we have

$$y_i(t) = \int_{t_0}^{T} h_i(t,\tau) x(\tau) - \int_{t_0}^{T} h_i(t,\tau) F\left[y_i(\tau)\right] d\tau \qquad (5)$$

or

$$y_i(t) = \int_{t_0}^{T} h_i(\tau) x(t-\tau) - \int_{t_0}^{T} h_i(\tau) F\left[y_i(t-\tau)\right] d\tau$$

Let us accept function $y_i(t)$ as a first approximation of the solution

$$y_{i1}(t) = \int_{t_0}^{T} h_i(t-\tau) x(\tau) d\tau, \qquad (6)$$

Let us define other approximations by a recurrence formula

$$y_{ij}(t) = y_{i1}(t) - \int_{t_0}^{T} h_j(t,\tau) F\left[y_{i(j-1)}(\tau)\right] d\tau, \qquad (7)$$

$$i = 2,3,\ldots$$

Applying (7) we get a series containing functionals of the following kind

$$V_{in}\left[x(\tau)\right] = \int_{t_0}^{T} \ldots \int_{t_0}^{T} h_{in}(t,\tau_1,\ldots,\tau_n) x(\tau_1)\ldots x(\tau_n) d\tau_1 \ldots d\tau_n, \qquad (8)$$

which are the homogeneous Volterra functionals of the n-th degree

As all the oscillators are equivalent let us omit i-oscillator belonging index hereinafter to make the record shorter. In the end of calculations it is necessary to find the sum of all oscillators responses, which will be substituted by integration with the form of absorption line g(ω). In this notation (8) will be rewritten in the following way

$$V_n\left[x(\tau)\right] = \int_{t_0}^{T} \ldots \int_{t_0}^{T} h_n(t,\tau_1,\ldots,\tau_n) x(\tau_1)\ldots x(\tau_n) d\tau_1 \ldots d\tau_n \qquad (9)$$

Function $h_n(t, t_1, \ldots, t_n)$ is called a functional kernel V_n.

The solution of (2) will be found by the iterative method that results in the representation of $y(t)$ in the form of Volterra finite series in case of weak nonlinearity (a_k <<1, k = 2, 3, …, N):

$$y(t) = \sum_{p=1}^{N} a_i \int_{E^p} h_p(\tau_1,\ldots,\tau_p) \prod_{r=1}^{i} x(t-\tau_r) d\tau_r, \qquad (10)$$

where E^p - p-dimensional Euclidean space, in which Volterra kernels $h_p(\tau_1, \tau_2, \ldots, \tau_p)$, representing pulse functions of nonlinear transformation of the p-th order are determined. So, for example,

$$h_2(\tau_1,\tau_2) = \begin{cases} \int_{-\infty}^{\infty} h(\tau) h(\tau_1-\tau) h(\tau_2-\tau) d\tau, & \tau_1,\ \tau_2 \ge 0, \\ 0 & \text{for all other values } \tau. \end{cases}$$

3.2 NRM MODEL WITH NONLINEAR ATTENUATION

It is possible to write the equation of an oscillator with nonlinear attenuation in the form

$$\frac{d^2y}{dt^2} + 2\sigma(y)\frac{dy}{dt} + \omega_0 y = x(t), \qquad (11)$$

where nonlinear function $\sigma(y)$ is represented by a polynomial

$$2\sigma(y) = \sum_{k=1}^{N} c_k y^k$$

We rewrite (11) in the operational form

$Dy + F(y) = x$,

where

$D = \frac{d^2}{dt^2} + 2\sigma\frac{d}{dt} + \omega_0^2,\ 2\sigma = c_1,\quad F(y) = \frac{dy}{dt}\sum_{k=2}^{N} c_k y^k$. The corresponding this equation relation takes the form (5).

After plain transformations we write the second approximation of the solution as

$$y_2(t) = y_1(t) - \sum_{k=2}^{N} c_k \int_{E^{k+1}} h(t,\tau_1,\ldots,\tau_{k+1}) \prod_{r=1}^{k+1} x(\tau_r) d\tau_r,$$

where $y_1(t) = \int_{E^1} h(t,\tau) x(\tau) d\tau$ - a linear approximation,

$$h(t,\tau_1,\ldots,\tau_{k+1}) = \int_{E^1} h(t,\tau) h(\tau,\tau_1) \ldots h(\tau,\tau_k) h'(\tau,\tau_{k+1}) d\tau,$$

the accent stands for a derivative.

3.3 NRM MODEL WITH NONLINEAR EXCITATION

The elementary mechanism of echo occurrence in such medium is linked to presence of cubic nonlinearity in the function describing the process of excitation of the oscillators system [1]. If an oscillator is excited by two δ-functions operating at $t = t_1$ and $t = t_2$ time moments with amplitudes A_1 and A_2, respectively, the response of a separate oscillator in this case can be written in the form of

$y(t,\omega)= [A_1\cos\omega(t - t_1) + A_2\text{cjs}\,\omega(t - t_2)]^3$

Having raised the sum to the third power we leave only one term describing the response at $t = 2t_1 - t_2$:

$$y(t,\omega) = ... + \frac{3}{4}A_1A_2^2\cos\omega\left[t-\left(2t_2-t_1\right)\right]$$

Having summed the responses of all oscillators we have

$$y(t) = \frac{3}{4}A_1A_2^2\int_{-\infty}^{\infty}\cos\omega\left[t-\left(2t_2-t_1\right)\right]d\omega,$$

that represents δ-function at $t = 2t_2 - t_1$ moment of time and corresponds to a two-impulse echo.

A separable Volterra kernel submits to similar transformation:

$$h_n\left(t,\tau_1,...,\tau_n\right) = \prod_{r=1}^{n}h_{1r}\left(t-\tau_r\right)$$

Volterra functional of the n-th order with such kernel can be written in the following simple form

$$V_n = \int_{E^n}h_n\left(t,\tau_1,...,\tau_n\right)\prod_{r=1}^{n}x\left(\tau_r\right)d\tau_r =$$

$$\left[\int_{E^1}h_{1n}\left(t-\tau\right)x\left(\tau\right)d\tau\right]^n$$

3.4 Spectral representation of NRM

Let us consider reaction of a nonlinear circuit to the sum of signals with arbitrary spectral densities and compare it with reaction of NRM, expressed in the terms of input action spectrum S ($j\omega$) and multivariate Fourier-transform $K_n(\omega_1, ,\omega_n)$ of Volterra kernels $h_n(\tau_1,\ldots,\tau_n)$ [7].

Functions $K_n(\omega_1, \ ,\omega_n)$ are n-dimensional NRM gains.

$$K_n\left(\omega_1,...,\omega_n\right) =$$

$$\int_0^{\infty}...\int_0^{\infty}h_n\left(\tau_1,...,\tau_n\right)e^{-j\left(\omega_1\tau_1+...+\omega_n\tau_n\right)}d\tau_1,...d\tau_n. \qquad (12)$$

Then the time form of NMR reaction can be presented as

$$y(t) = \sum_{p=1}^{n}\frac{1}{2^p}\int_0^{\infty}...\int_0^{\infty}K_p\left(\omega_1,...,\omega_p\right)\prod_{k=1}^{p}S\left(\omega_k\right)e^{j\omega_0 t}d\omega_k$$

The response $y(t)$ all oscillators system on a signal $x(t)$ we shall write down in the reduced form of the record found in[2].

$$y(t) = \sum_{i=1}^{N}y_i(t)\sum_{p=1}^{\infty}a_{2p-1}\frac{C_{2p-1}^{p-1}}{2^{2p-1}}\left(\frac{2\pi}{T}\right)^{2p-2}\times$$

$$\int_{-\infty}^{\infty}g\left(\omega\right)\dot{K}_{2p-1}\left(\omega\right)\dot{S}\left(\omega\right)\left|\dot{S}\left(\omega\right)\right|^{2p-2}d\omega, \qquad (13)$$

where p – number of "plus" signs in the gain arguments, a_i – power constants, function $g(\omega)$ has the meaning of the oscillator frequency distribution density.

The minimal order of nonlinearity in the equation (1) providing echo phenomena equals to three. This is the cubic medium and it can be represented by the radio engineering equivalent shown in Fig. 2.

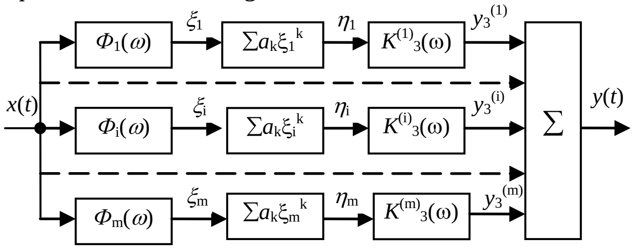

Fig. 2. Simplified equivalent of the cubic medium.

The first term of expression (13) describes linear transmission of the signal x(t) through the filter with the gain g (jω); the second one corresponds to the nonlinear transformation of the third order and rather adequately characterizes the processes occurring in the nonlinear resonant medium:

$$y\left(t \mid p=2\right) = 2\frac{3}{8}\frac{a_3}{2\pi}\times$$

$$\times\text{Re}\left\{\int_{-\infty}^{\infty}g\left(\omega\right)S\left(j\omega\right)\left|S\left(j\omega\right)\right|^2e^{j\omega t}d\omega\right\}. \qquad (14)$$

The spectral density of the output signal corresponding to this transformation

$$S_y\left(j\omega \mid p=2\right) = \frac{3a_3}{8}\left[S\left(j\omega\right)\left|S\left(j\omega\right)\right|^2\right]g\left(\omega\right), \qquad (15)$$

defines possible responses of medium.

We find NRM response to excitation in the framework of the second approximation in the form of sum of arbitrary form signals [2]:

$$S\left(j\omega\right) = \sum_{i=1}^{N}S_i\left(j\omega\right)\exp\left(-j\omega t_{1i}\right), \qquad (16)$$

where S_i ($j\omega$) - spectrum of the i-th signal, t_{1i} - its time shift relative to the first signal which time position (in a general sense) is given to be a zero time: $t_{1i} = 0$. …

Substituting (16) in (15), we receive:

$$S_{out}^{(3)}\left(j\omega\right) = \frac{3}{8}a_3 g\left(j\omega\right)\left\{\sum_{i=1}^{N}\left|S_i\right|^2S_i\exp\left(-j\omega t_{1i}\right) + \right.$$

$$2\sum_{i=1}^{N-1}\sum_{n=i+1}^{N}S_n\left|S_i\right|^2\exp\left(-j\omega t_{1n}\right) +$$

$$\sum_{i=1}^{N-1}\sum_{n=i+1}^{N}S_i^*S_n^2\exp\left[-j\omega\left(2t_{1n}-t_{1i}\right)\right] + \qquad (17)$$

$$\left.2\sum_{i=1}^{N-2}\sum_{n=i+1}^{N-1}\sum_{m=n+1}^{N}S_i^*S_nS_m\exp\left[-j\omega\left(t_{1m}+t_{1n}-t_{1i}\right)\right]\right\},$$

where only the terms, satisfying the principle of causality remain.

We present a nonlinear circuit as a inercialless nonlinear converter (NNC) connected in series with a linear filter selecting some (first) spectral band (fig. 3).

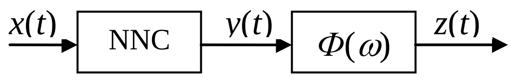

Fig. 3. Representation of a nonlinear circuit

Characteristic of a NNC exited by the input signal $x(t)=\sum_{i=1}^{N}x_i(t)$ is a polynomial $y(t)=\sum_{k=1}^{N}b_k x^k(t)$

For every x_i (t) the spectral density is defined $S_i(\omega)=\int_{-\infty}^{\infty}x_i(t)\exp(-j\omega t)\,dt$. In this case

$$y(t)=\sum_{k=1}^{N}b_k\left[\sum_{i=1}^{n}x_i(t)\right]^k . \qquad (18)$$

To avoid excessively intricate calculation we limit ourselves to the analysis of a polynomial of the third degree, letting $N = 3$ in (18). Then

$$y=b_1\left[\sum_{i=1}^{n}x_i^2+2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}x_i x_j\right]+$$
$$b_3\left[\sum_{i=1}^{n}x_i^3+3\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}x_i x_j^3+3\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}x_i^2 x_j\right]+$$
$$b_3\left[+6\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=j+1}^{n}x_i x_j x_k\right]$$

The oscillation spectrum y (t) can be written in the following form:

$$S_y(\omega)=b_1\sum_{i=1}^{n}S_i(\omega)+$$
$$b_2\left[\sum_{i=1}^{n}S_i(\omega)*S_i(\omega)+2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}S_i(\omega)*S_j(\omega)\right]+$$
$$b_3[\sum_{i=1}^{n}S_i(\omega)*S_i(\omega)+3\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}S_i(\omega)*S_j(\omega)*S_j(\omega)+$$
$$3\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}S_i(\omega)*S_i(\omega)*S_j(\omega)+$$
$$6\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=j+1}^{n}S_i(\omega)*S_j(\omega)*S_k(\omega)], \qquad (19)$$

where $S_i(x)*S_j(x)=\int S_i(\xi)S_j(x-\xi)\,d\xi$.

The third term’s structure coincides (19) with the structure of expression (17) to the insignificant constant factors provided that operations of convolution were substituted for operations of multiplication in (17)

The distinction is complex conjugation of one of any product factors in the expression (51 indicating substitution of correlation for convolution of time functions. But this circumstance is absolutely insignificant for NC description as spectra of signals in these circuits’ clusters about frequencies $+\omega_i$ and $-\omega_i$.

Let signals x_i (t) further be narrow-band in radio engineering sense and allowed to be presented in the form of x_i (t) = A_i (t) cos[$\omega_i t$ + φ_i (t)], where A_i (t) and $\varphi_i(t)$ - slowly varying envelope and a phase. Then, considering band limitedness of signals in (19), we get for positive frequencies:

$$S_y(\omega)=b_1\sum_{i=1}^{n}S_i(\omega-\omega_i)+$$
$$+b_2\left\{\sum_{i=1}^{n}\left[\frac{1}{2}S_{ii}(\omega)+\frac{1}{2}S_{ii}(\omega-2\omega_i)\right]\right.$$
$$\left.+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[S_{ij}(\omega-\omega_i-\omega_j)+S_{ij}(\omega-|\omega_i-\omega_j|)\right]\right\}$$
$$+b_3\left\{\frac{1}{4}\sum_{i=1}^{n}\left[S_{iii}(\omega-\omega_i-\omega_j)+3S_{iii}(\omega-\omega_i)\right]\right.$$
$$+\frac{3}{4}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[2S_{iij}(\omega-\omega_i)\right.$$
$$\left.+S_{iij}(\omega-\omega_j-2\omega_i)+S_{iij}(\omega-|\omega_j-2\omega_i|)\right]$$
$$+\frac{3}{4}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left[2S_{ijj}(\omega-\omega_i)\right.$$
$$+S_{ijj}(\omega-\omega_i+2\omega_j)+S_{ijk}(\omega-|\omega_i+\omega_j-\omega_k|)$$
$$\left.\left.+S_{ijk}(\omega-|\omega_i-\omega_j-\omega_k|)+S_{ijk}(\omega-|\omega_i-\omega_j+\omega_k|)\right]\right\}$$

Here $S_{ijk}(\omega$ - $\omega_l)$ - convolution of (6) spectra envelopes $A_i(t)$, $A_j(t)$, $A_k(t)$, transferred to frequency ω_l. The result of three signals S_1 + S_2 + S_3 sum transformation in a nonlinear circuit of the third order is presented in Fig.4,a. Fig. 4,b displays the response of the cubic medium to the sum of three signals having the same form-time dependence as the spectral functions in fig. 4,a.

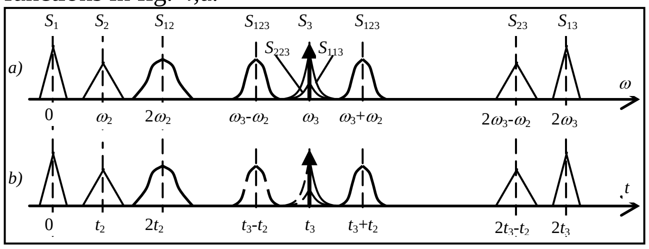

Fig. 4. Responses a) of a nonlinear circuit and b) NRM to the sum of three signals of the arbitrary form.

As the third signal δ-functions: $S_3 = \delta(\omega - w_3)$ in fig.4,a and $S_3 = \delta(t - t_3)$ in fig. 4,b are used. For clearness only basic responses of nonlinear systems, i.e. responses of two-pulse type S_{12}, S_{23} and S_{13}, and responses of three-impulse type S_{123}, S_{113} and S_{223} are shown in the figure.

Positions of the responses on the ω or t- axes are bilaterally symmetric with reference to the axes of symmetry passing through the second signals.

Responses on the frequency axis correspond to convolutions of the third signal spectrum with auto convolutions of the first and second signals spectra, and on a time base - to convolution of the third signal with auto correlative functions of the first and second signals correspondingly.

It is easy to see, that according to a principle of causality no signals drawn in fig. 4,b by hatch line can arise in NRM. This is the unique and quite natural difference in responses of NC and NRM. It is also necessary to note, that there are no differences like these in the mathematical aspect: substitution

(3) in (2) results in the expression similar to (6) provided that $b_1 = b_2 = 0$.

Having analyzed the origin of the responses presented in Fig.3 we can note the following

1. NC response having the spectrum S_{123} (ω), results from the product of $S_1(t)$ S_2 (t) and heterodyning with frequency ω_3. Filter $\Phi(\omega)$ fig. 3 plays the role of integrator, NNC and the band filter connected in series can be considered as heterodyne correlator.
2. In NRM echo-signals are generated at nonlinearity of the order not lower than three unlike nonlinear circuits where responses of S_1 (t) S_2 (t) type can occur already at square-law nonlinearity

So there are no responses of the S_1 (ω) S_2 (ω) type in NRM. NRM with nonlinearity of the third order is usually called cubic. In such mediums responses of the $S_1^*(\omega)S_2^2(\omega)$ type exited by the interaction of two signals $S_1(t)\otimes S_2(t)*S_2(t)$ in time domain) can appear as well as responses of the $S_1^*(\omega)S_2(\omega)S_3(\omega)$ type resulted from the interaction of three signals $S_1(t)\otimes S_2(t)*S_3(t)$ in time domain).

The last response at $S_3(t) = \delta(t - t_3)$ can be considered as cross-correlation function (CCF)

$$R_{12}(t) = S_1(t)\otimes S_2(t) = S_1(-t)*S_2(t) = S_1^*(t)*S_2(t),$$

subjected to "heterodyning" with time t_3, i.e. to the time delay equaled to $t_3 - (t_2 - t_1) = t_3 - t_2$.

In fig. 3 filter $\Phi(\omega)$ operates as frequency selection, separating useful result of correlation of signals $S_1(t)$ and $S_2(t)$ from other responses.

CCF of signals $S_1(t)$ and $S_2(t)$ in NRM can be selected from other responses by means of time strobing. All common properties of NC and NRM responses mentioned above are consequence of the time-frequency symmetry, or dualism [8] which can be illustrated by the following table:

Table 1

№	Operation	Dual operation
1	Product	Convolution
2	Band trapping	Time trapping
3	Frequency transformation	Time delay
4	Weight processing in the frequency domain	Weight processing in the time domain

Thus, it is shown, that NC and NRM are dual systems.

If responses of one system to an input signal are known, responses of the other system can be received, making $\omega \leftrightarrow t$ replacement or replacing operations of multiplication by operation of convolution with simultaneous complex conjugation of the first factor.

Table 2 illustrates the rule of NRM response record at known NC responses.

Table 2

Nonlinear circuit	Nonlinear resonant medium	Re-mark
$S_{123}(\omega) = S_1(\omega)* S_2(\omega)*S_3(\omega)$	$S_{123}(\omega) = S_1^*(\omega)S_2(\omega)S_3(\omega)$	1
	$S_{123}(t) = S_1^*(t)*S_2(t)*S_3(t)$	2
$S_{123}(t) = S_1(t)\cdot S_2(t)\cdot S_3(t)$	$S_{123}(t) = S_1^*(t)*S_2(t)*S_3(t)$	3
	$S_{123}(\omega) = S_1^*(\omega)S_2(\omega)S_3(\omega)$	4

In this table:
1 – Operation change; 2 – Argument change
3 – Operation change; 4 – Argument change

4 Conclusion

Thus, it is shown, that nonlinear circuits of the arbitrary order are dual to nonlinear resonant media of the third order and higher. This property NRM can be used in devices of signals processing for operations of integral transformations. It is especially perspective for the devices using a photon echo phenomenon (photon echo-processors). Photon processors have very high speed and work with bidimentional signals.

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