PID robust control using Taguchi Method

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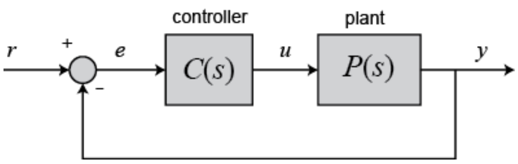
Abstract: The control of FOPD (first order plus delay) systems with high normalized time (ratio of time delay and time constant) is a very hard task, especially when the system parameters are subjected to variations. A great number of physical systems show such model behavior. In process control today, proportional integral derivative (PID) (continuous and digital) controllers still predominate and are sufficient for most needs with more than 95 per cent of control loops being of the PID type. In this work, we present a method of determination of an optimal robust PID controller based on the ISE (integral squared-error), using Taguchi method. This approach is very useful when the process parameters are prone to variations in a given range. Once the criteria of optimization posed, a statistical analysis is needed to determine if the control parameters are significant or not. The design of experiments method is certainly the most appropriate to tackle this type of problem. Indeed it allowed us to quantify the weight of the factors that affect the output and evaluate their interactions. The analysis of the results led us to define the most appropriate set up that minimizes the negative effect of noise factors.

Keywords: Robust control, Taguchi, controller, ISE, PID, noise factors

I INTRODUCTION:

We will consider the following unity feedback system:



The output of a PID controller, equal to the control input to the plant, in the time-domain is as follows:

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_p \frac{de}{dt} \quad (1)$$

The variable (e) represents the tracking error, the difference between the desired input value (r) and the actual output (y). This error signal (e) will be sent to the PID controller, and the controller computes both the derivative and the integral of this error signal. The control signal (u) to the plant is equal to the proportional gain (K_p) times the magnitude of the error plus the integral gain (K_i) times the integral of the error plus the derivative gain (K_d) times the derivative of the error.

This control signal (u) is sent to the plant, and the new output (y) is obtained. The new output (y) is then fed back and compared to the reference to find the new error signal (e). The controller takes this new error signal and computes its derivative and its integral again, ad infinitum.

The transfer function of a PID controller is found by taking the Laplace transform of Eq.(1).

$$K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \quad (2)$$

K_p = Proportional gain K_i = Integral gain K_d = Derivative gain

A proportional controller (K_p) will have the effect of reducing the rise time and will reduce but never eliminate the **steady-state error**. An integral control (K_i) will have the effect of eliminating the steady-state error for a constant or step input, but it may make the transient response slower. A derivative control (K_d) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

The effects of each of controller parameters, K_p, K_d, and K_i on a closed-loop system are summarized in the table below.

Table1: Effects of PID parameters.

CL RESP	RISE TIME	OVER SHOOT	SETTLIN G TIME	S-S ERROR
Kp	Decreas	Increase	Small Change	Decrease
Ki	Decreas	Increase	Increase	Eliminat
Kd	Small Change	Decrease	Decrease	No Change

Note that these correlations may not be exactly accurate, because K_p, K_i, and K_d are dependent on each other. In fact, changing one of these variables can change the effect of the other two.

Consequently considerable advantages could be obtained by implementation the Taguchi's approach in the process of automation control [1]. Proportional integral derivative (PID) controllers still predominate in process control today and are sufficient for most needs as more than 95 per cent of control loops are of the PID type. A vast amount of literature is available that addresses the problem of PID controllers tuning [2,3]. A complete study of PID tuning for different forms of transfer functions has been done by O'Dwyer [4]. The most important factor in all of these tuning methods is the form of the actual transfer function of a plant. If the structure of this transfer function is significantly different from the one used to derive the tuning formulas, then a substantial amount of system unpredictability should be expected. However, this problem is simplified when there is no structural mismatch between the actual and the assumed transfer functions. In this case there is no structural uncertainty, but rather the uncertainty is isolated to the actual value of the parameters used to model the plant. If the model and the actual plant have the same structure type, then the parameters of the transfer function become of primary concern. In a study made by Bialkowski [5], it was shown that 30 per cent of their control loops functioned poorly due to incorrect PID controller settings. It is, therefore, evident that a tuning refinement in the presence of model parameter uncertainties is of great importance.

To date, there is no research about robustness of PID controllers under noise condition resulting from parameter uncertainty. The quality of PID controllers tuning is the result of a number of parameters. Some of these are controllable while others are noise factors [2,3,6].

The object of this paper is the search of robustness of the PID controllers using the design of experiments method [7]. For that purpose we first need to select the factors of the PID controllers and identify the noise factors that cause undesirable variation upon the quality characteristic.

2 METHOD AND RESULTS:

Conventional methods of PID controllers tuning:

The Ziegler and Nichols [6] tuning method is based on the calculation of the ultimate period (Pu) and ultimate gain (Ku) of the system. The pair Pu, Ku can be calculated by letting the process loop gain increase until the system begins sustained oscillations [2,3].

An alternative method for tuning is the relay feedback method [2]. In this method the system is forced to oscillate by introducing a non-linear feedback of the relay type in order to generate a limit cycle oscillation in the system. The amplitude of the system oscillations can be controlled since it is proportional to the relay amplitude. In the PID tuning mode, when the steady state is reached, the system oscillates at a frequency with period and amplitude close to the ultimate period (Pu) and ultimate gain (Ku) of the open loop system. We can also use the auto-tune rules in the case where a controller based on minimizing a criterion needed in servo tuning ISE case [8].

Quality characteristic of PID controllers:

Clarifying things it is assumed that the model of process and actual plant have the same transfer function define by equation (1) [2,3] where, G_s is the DC-gain of process model, τ is the time delay and T is the time constant.

$$G(p) = \frac{G_s e^{-\tau s}}{(1+Ts)} \qquad (3)$$

The PID controller may be implemented in continuous or discrete time, in a number of controller structures. The ideal continuous PID controller time is expressed in Laplace form as follows [1,2]:

$$C(p) = K_p + \frac{K_I}{s} + K_d s \qquad (4)$$

This form is known as parallel form. This controller may be in the following one (filtered form) which is physically realisable, provided N is taken in the range 10-20, without any change in performance. (Equation 3) .

$$C(p) = K_p + \frac{K_I}{s} + \frac{K_d s}{1+\frac{K_d s}{N}} \qquad (5)$$

The performance index used is the integral-squared-error (ISE) and is mathematically defined by [2,3,6]

$$ISE = \int_0^\infty (y_{sp} - y(t))^2\, dt \qquad (6)$$

where y(t) and ysp is the current output and the desirable output (set point) of process model, respectively.

This particular performance criterion is widely used for controllers tuning because its minimisation is related to minimisation of error magnitude and duration [6]. Thus, in this study, the ISE will be taken as the quality characteristic to be observed.

Control factors of PID controllers and noise:

In the PID controllers, the KP, KI and KD parameters are the control factors since they can be changed by the conventional auto-tuner to minimize the quality characteristic ISE [2,6].

$$K_p = \frac{1.048}{G_s}(\frac{T}{\tau})^{0.869}$$

$$T_i = \frac{K_p}{A_i} = \frac{G_s}{0.74 - 0.13\frac{\tau}{T}}$$

$$T_d = \frac{A_d}{K_p} = 0.348T(\frac{\tau}{T})^{0.914} \quad (7)$$

With :

$$0.1 < NTD = \frac{\tau}{T} \leq 1 \quad (8)$$

The experience reveals that non-linear behaviour of the control factors of a PID can be determined only if more than two levels are used [9]. Therefore, in our investigation we will assign three levels for each control factor. Also literature reveals that between KP, KI and KD there is no significant interaction (there is no dependence), which affects the quality characteristic ISE [2,6]. The design of experiments method will allow us to verify this statement as this method is well suited to assess the interactions of factors that affect the output [10].In this study, after the auto-tuning method [8] ,the initial optimum set of values is determined for the control factors under ideal conditions, i.e. w ithout considering any noise factor, is KP=2, KI=2 and KD=0.5. The three levels of three control factors are identified for study as presented in Table 2.

Table 2: PID parameters

Factors	Range	Level 1	Level 2	Level 3
Kp	1.6-2.4	1.6	2	2.4
Ki	1.6-2.4	1.6	2	2.4
Kd	0.5-0..5	0.4	0.5	0.6

Noise factors are those parameters of process model (equation (1)) that cannot be controlled or are too expensive to control or cannot be identified. In equation (1), the Gs (DC-gain), τ (delay time) and τ/T (normalized time delay, ration of process time delay to time constant, dimensionless factor) are generally known to be the most uncertain factors; even small uncertainties in their values (uncontrollability) lead to poor control. In this study, the first noise factor NF1 (DC-gain, Gs) could be higher (up to 1.1) than the original estimate of 1. The second noise factor NF2 (time delay, τ) could be higher (up to 0.55s) than the original estimate of 0.5 s and the third noise factor NF3 (Time constant could be higher (up to 1.1) than the original estimate of 1. So, the process parameters are subject to a 10 % change; Table 3.

Table 3: Noise factors

Factors	Range	Level 1	Level 2
NF1 (Gs)	1-1.1	1	1.1
NF2 (τ)	0.5-0.55	0.5	0.55
NF3 (T)	1-1.1	1	11

As we can see the quality characteristic ISE (output) depends on three controllable factors (KP, KI and KD) and three noise factors (NF1, NF2 and NF3) . The purpose of this work is to determine the importance of these factors and their reactions in order to find the optimum set up which will lead us to the robust design by obtaining the desired ISE, which is its minimum. As stated earlier in order to achieve this we shall apply the method of experiments method .However we cannot run this analysis considering these six factors at once because of inheriting constraints. In deed on one hand we must assign three levels to Kp, KI and KD as explained earlier (constraint relative to the number of required levels). On the other hand we only have two levels available for the NF's. Therefore the only option left is to run the experiments for the controllable factors with their three levels for each combination of the noise factors. Hence the orthogonal arrays which we work with are presented in table 3. Thus, ISE is evaluated for the nine runs for each of four different combinations of the three noise factors (two levels at each noise factor). The ISE response was computed for each combination of control and noise matrix experiments using the MATLAB SIMULINK program by simulating the control system equations. For each combination of control factor levels, the mean and the standard deviation (std) were also evaluated.The choice of this set up of orthogonal arrays was done according to Byrne and Taguchi[11], Montgomery and Ross[12] and Roy [13]. The results of these simulations are presented in Table 4. For the ith combination of the noise factors the output is represented by Ri .

From Table 4 one can guess that the best results are obtained in trials number 4 and 7. We also can see how the noise factors depending on their levels affect the results. We then apply the design of experiments analysis separately to the four results in order to determine the weight of each factor and evaluate their interactions [14]. This will lead us to the optimum set-up.

Table 4: Simulations Results .

NF1			1	1	2	2
NF2			1	2	1	2
NF3			2	1	1	2
Kp	Ki	K_d				
1	1	1	0.7486	0.7805	0.7079	0.7901
1	2	2	0.7741	0.8111	0.7340	0.8279
1	3	3	0.8086	0.8525	0.7699	0.8752
2	1	3	0.7014	0.7570	0.7063	0.7645
2	2	1	0.7160	0.7808	0.7040	0.7998
2	3	2	0.7404	0.8112	0.7305	0.8347
3	1	2	0.6788	0.7804	0.7296	0.7962
3	2	3	0.6962	0.8290	0.8200	0.8533
3	3	1	0.7311	0.8648	0.7765	0.9006

We first report the polynomials which represent the mathematical models of the results that we obtained with the design of experiments method:

R1 = + 0.71 - 0.046 Kp + 0.025 Ki - 0.001 Kd - 0.005 KpKi - 0.0005 KpKd - 0.012KiKd + …

R2 = + 0.79 + 0.012 Kp + 0.026 Ki + 0.012 Kd + 0.020 KpKi - 0.017 KpKd + 0.013 KiKd +…

R3 = +0.74 + 0.033 Kp + 0.015 Ki + 0.034 Kd + 0.032 KpKi - 0.014 KpKd + 0.028 KiKd+ …

R4 = + 0.81+ 0.017 KP + 0.033 Ki + 0.011Kd + 0.022 KpKi -0.020 KpKd + 0.015 KiKd + …

There are two main observations that need to be made: first ,the levels of the noise factors affect randomly the weights of the parameters. As an example the weight of Kp varies from 0.046 to 0.017 depending on the noise level. From this we can conclude that any analysis with the design of experiments method based only on the contrallable parameters could lead to wrong results as we can only determine the local minima of the ISE with these data. Taking into account the effect of the noise factors is thus mandatory. The second observation concers the weights of the interactions (KpKi ,KpKd,KiKd) which can be quite important This contradicts Astrom et al [2] and Ziegler et al [6] . In order to find the set up that gives the global minima of the ISE we need to consider the plots of R1, R2, R3 and R4 as functions of different interactions (Fig.1a,1b,1c,1d) as well as the contour plots . They are given in Fig.1 and Fig.2 .In deed these plots allow us to assess the effects of the different factors and their interactions. We can deduct from these figures that the optimum levels of the control factors so that the effect of noise is reduced and the ISE minimised are: level 2 for the factor kp ,level 1 for ki and level 2 for Kd.

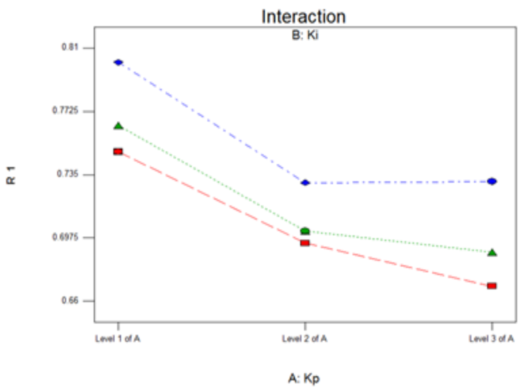


Figure 1-a Interaction plot of R1

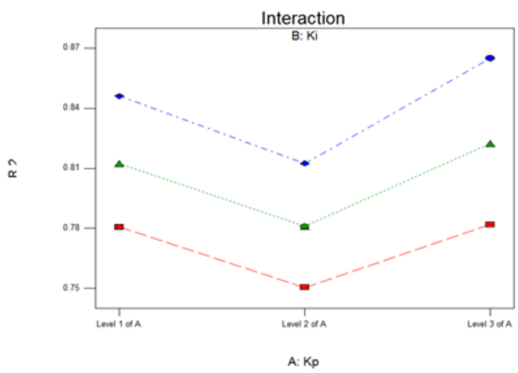


Figure 1-c Interaction plot of R2

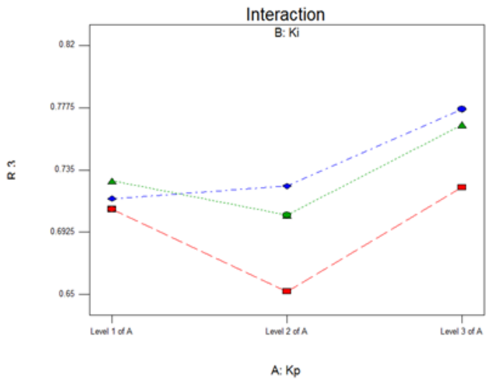


Figure 1-b Interaction plot of R3

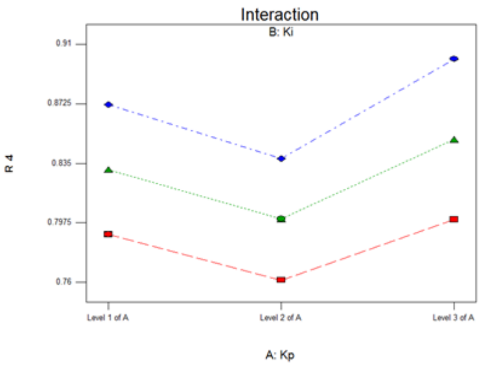


Figure 1-d Interaction plot of R4

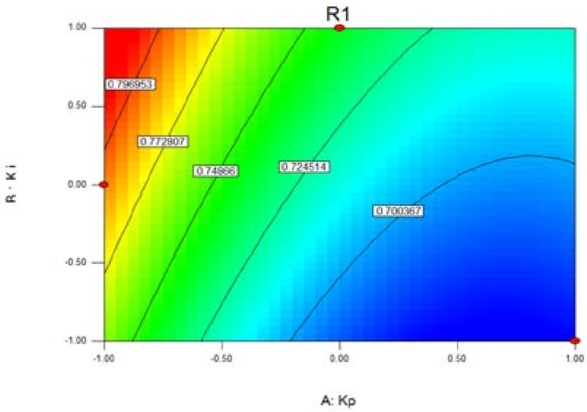


Figure 2-a Contour plot of R1

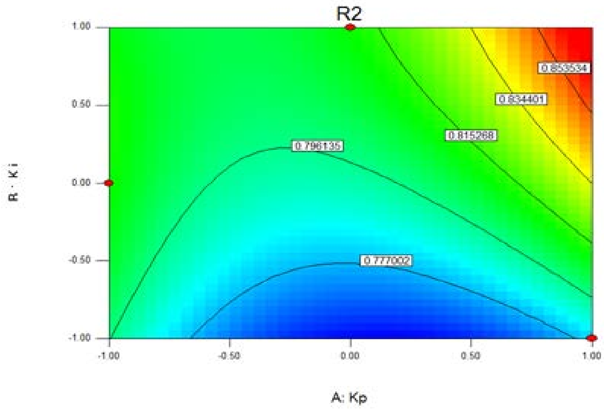


Figure 2-b Contour plot of R2

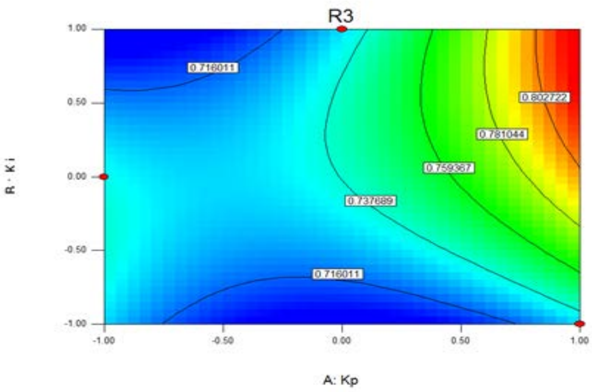


Figure 2-c Contour plot of R3

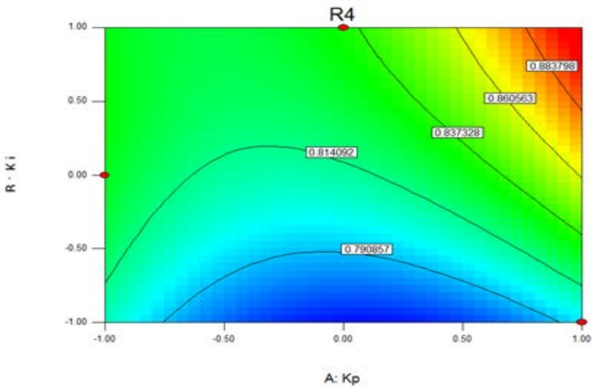


Figure 2-d Contour plot of R4

3 CONCLUSION:

The use of the design of experiments method that takes in consideration the noise factors proved to be appropriate as it allowed us to find the best set up of the tree PID control factors that reduce the negative effect of the associated random noise factors. This was achieved by analysing the effect of these factors and their interactions. The contour plots were very useful as they permit to make projections which allow a global insight of the problem. In doing so we were able to determine the best factors levels that give the best combination of the controllable factors. In addition the analysis showed that the interactions of control factors are quite important as they can be of the same magnitude. This contradicts the assertions given in reference [2] and [6].

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