Bayesian channel estimation in chaos based multicarrier CDMA system under slowly varying frequency selective channel

Meher Krishna Patel, Stevan M. Berber, *Senior Member, IEEE,* and Kevin W. Sowerby, *Senior Member, IEEE*

***Abstract*—This paper presents the theoretical analysis of the multicarrier-code division multiple access (MC-CDMA) system in slowly time varying frequency selective channel. Chaotic sequences are used as spreading codes of CDMA system with BPSK modulation scheme. Performance improvement in Bayesian estimator in the presence of chaotic sequence is investigated. Under perfect synchronization assumption, bit error rate (BER) in closed form is derived under imperfect channel estimation for downlink communication system. Simulation results show that there is significant performance improvement in MC-CDMA system as compared to CDMA system.**

***Index Terms*—Bayesian estimation, Chaotic sequence, CDMA, Frequency selective channel, Multicarrier Communication**

I. INTRODUCTION

Fading is the phenomena which makes wireless communication more difficult as compare to other communication systems e.g. optical fiber communication and wired communication etc. Therefore for many wireless systems, independent of whether time division multiple access (TDMA) or code division multiple access (CDMA) is employed, estimation of channel fading coefficient is necessary for high speed communication. Channel estimates can be updated frame by frame for slower fading rate as compare to frame rate. If channel coefficients change significantly within the frame then it is necessary to update coefficients iteratively based on symbol by symbol basis [1], [2].

Various estimation methods have been studied in last few decades and each method has its own advantages and disadvantages. Minimum mean square estimators (MMSE) [3], [4], [5] are easy to implement and perform well in flat fading environment. But these estimators require correlation computation and have poor performance for time varying channel estimation. Bayesian estimators [6], [7], [8], [9] used prior knowledge of data to generate posterior analysis. Therefore estimator performance extensively depends on prior informations. On the other hand, neural networks [10], [11], [12] do not require prior knowledge of channel statistic, but there is huge computational burden for training process. Finally, particle filters [13], [14], [15], [16] use the sequential Monte Carlo sampling method to implement recursive Bayesian filter. But these filters have very high computational load for correcting each particle, which results in higher energy consumption. Therefore hardware implementation of these filters are difficult.

Direct sequence-CDMA system has received much attention in wireless communication system due to its higher capacity, robustness against fading and anti-interference capability [17]. All users share complete available spectrum and distinguish from each other by their spreading code at the receiver. Further, frequency selective channel severely degrades the performance of the CDMA system due to it's intersymbol interference (ISI) effect. To overcome this degradation RAKE receivers in CDMA systems are widely investigated [18], [19], [20], [21], [22], [23], [24], [3], [25], [26].

On the other hand multicarrier systems are also drawn significant attention in wireless communication system [17]. In this system the transmitted data is divided into a number of slow data streams and each stream is modulated by one subcarrier. At the receiver each subcarrier data is demodulated separately. Finally the demodulated data are reassembled. Therefore transmitted data rate is reduced whereas overall received data rate is same as the single carrier system. Since the transmission data rate is reduced therefore channel can be considered as flat fading channel and ISI effect due to frequency selective channel does not occur in this case.

High data rate and bandwidth efficient digital communication technologies urge the need to develop new transceiver structures. Therefore MC-CDMA system is bandwidth efficient system and can be used for high data rate communication system. In this system spreading code is serial to parallel converted and then each chip is modulated by different carrier frequency. MC-CDMA system are widely investigated for binary spreading sequences [27], [28], [4], [29], [30]. Few papers are presented in the field of chaos based MC-CDMA system [31], [32], [33] but to our best knowledge, no paper has been published on BER performance of chaos based MC-CDMA system under complex frequency selective fading channel coefficients and noise, even though chaos based CDMA system have gained significant interest among the researchers in last decade [34], [35], [23], [12], [36], [37], [38], [39], [21], [32], [33].

For this reason the objective of this research work is to study the Bayesian channel estimator for chaos based MC-CDMA system under frequency selective channel for downlink communication. Maximum a prior (MAP) estimator equation is derived for these systems, which needs a prior knowledge of channel statistics. Further, we have also derived the ML estimation equation for considering the case where the mean and variance of the channel is unknown at the receiver. Two algorithms are derived to consider the multiplexed pilot-data case and added pilot-data case. In multiplexed pilot-data case, after demultiplexing, channel estimation can be performed directly on the extracted pilot signal. Whereas for added pilot-data case, pilot needs to be extracted by multiplying corresponding chaotic sequence, before channel estimation process. Performance difference in these two methods have been shown using simulation results. Further, BER in closed form is derived for imperfect channel estimation case. Finally, we have shown that the results in [40] is a special case of this paper.

This paper is organized as follows. In section II chaos based MC-CDMA system is shown. Analytical performance of this system is presented in section III. MAP and ML estimation algorithms are derived in section IV. Simulation results are shown in section V. Finally some concluding remarks are given in section VI.

II. System model

Fig. 1 shows the baseband representation of the chaos based MC-CDMA system. The baseband signal s_k^g for g^{th} user at k^{th} chip instant, is given by

$$s_k^g = \gamma_i^g x_k^g \tag{1}$$

where $\gamma_i^g$is the g^{th} user symbol at i^{th} time instant, x_k^g is k^{th} chip of the g^{th}user chaotic spreading sequence or chip within an information bit (i.e. $k = 1, \cdots, 2\beta$) and 2β is the spreading factor.

After adding all the users' data, each chip of the resultant signal is modulated by different carriers. Finally all the modulated chips are added together before transmission. Therefore transmitted signal S_i is given by

$$S_i = \sqrt{2}\sum_{n=1}^{N}\sum_{j=1}^{J}\gamma_i^j x_{k,n}^j \cos(2\pi f_n t) \tag{2}$$

where N and J are the number of subcarriers and users respectively, f_n is the frequency of n^{th} subcarrier and $x_k^j = [x_{k,1}^j, \cdots, x_{k,N}^j]$ is the chaotic sequence of j^{th} user.

For L multipath channels, the received symbol r_i of g^{th} user at i^{th} time instant is given by

$$r_i = \sum_{l=0}^{L-1} h_{i,l} S_{i-\tau_l} + \xi_i \tag{3}$$

where ξ_i denotes the complex additive white Gaussian noise at i^{th} time instant with power spectral density equal to N_0 and τ_l is the delay for l^{th} path with respect to first path. Delay of first path is assumed to be zero i.e. $\tau_0 = 0$ and other delays are more than one chip durations. $h_{i,l}$ is the complex channel coefficients for l^{th} path at i^{th} time instant, which is assumed to be known at the receiver.

III. Performance analysis of MC-CDMA system

In fig. 1, since chaotic chips are spread to bit level using multicarriers therefore the small chip delay caused by the channel in equation (3) can be neglected i.e. $S_{i-\tau_l} \approx S_i$. Hence received signal can be written as

$$r_i = \sum_{l=0}^{L-1} h_{i,l} S_i + \xi_i = \alpha_i e^{j\phi_i} S_i + \xi_i \tag{4}$$

where

$$\sum_{l=0}^{L-1} h_{i,l} = \alpha_i e^{j\phi_i} \tag{5}$$

In above equation α_i and ϕ_i are amplitude and phase component of the resultant channel coefficients.

Estimated fading coefficients $(\hat{\alpha}_i e^{j\hat{\phi}_i})$ are multiplied with this received signal. The resultant signal is multiplied by subcarriers and then passes through low pass filter to remove higher frequency terms as shown in Fig. 1. Since subcarriers are orthogonal to each other therefore there is no cosine term in the low pass filter output. This output is then multiplied to g^{th} user chaotic sequence and the decision variable is given by

$$\begin{aligned} Z_i^g = \mathrm{Re}\{&\alpha_i e^{j\phi_i}\hat{\alpha}_i e^{j\hat{\phi}_i}\sum_{n=1}^{N}\sum_{j=1}^{J}\gamma_i^j x_{k,n}^j x_{k,n}^g + \\ &\sqrt{2}\hat{\alpha}_i e^{j\hat{\phi}_i}\sum_{n=1}^{N}\xi_i\cos(2\pi f_n t)x_{k,n}^g\} \end{aligned} \tag{6}$$

or

$$Z_i^g = Z_i^{ag} + Z_i^{bg} + Z_i^{cg} \tag{7}$$

where

$$\underbrace{Z_i^{ag} = \alpha_i\hat{\alpha}_i\cos(\phi_i - \hat{\phi}_i)\sum_{n=1}^{N}\gamma_i^g x_{k,n}^g x_{k,n}^g}_{Desired\ Signal} \tag{8}$$

$$\underbrace{Z_i^{bg} = \alpha_i\hat{\alpha}_i\cos(\phi_i - \hat{\phi}_i)\sum_{n=1}^{N}\sum_{j\neq g, j=1}^{J}\gamma_i^j x_{k,n}^j x_{k,n}^g}_{Interuser\ Interference} \tag{9}$$

$$\begin{aligned} Z_i^{cg} = &\sqrt{2}\hat{\alpha}_i\sum_{n=1}^{N}\cos(2\pi f_n t)x_{k,n}^g \times \\ &\underbrace{\{\mathrm{Re}(\xi_i)\cos\hat{\phi}_i + \mathrm{Im}(\xi_i)\sin\hat{\phi}_i\}}_{Noise} \end{aligned} \tag{10}$$

The probability of error $\left(P^{(g)}(a_i)\right)$ for the i^{th} bit of g^{th} user is given by [35]

$$\begin{aligned} P^{(g)}(a_i) &= \tfrac{1}{2}Pr\left(Z_i^{(g)} < 0 \;\middle|\; \gamma_i^{(g)} = 1\right) \\ &+ \tfrac{1}{2}Pr\left(Z_i^{(g)} \geq 0 \;\middle|\; \gamma_i^{(g)} = -1\right) \\ &= \tfrac{1}{2}\mathrm{erfc}\left(E\left[Z_i^{(g)} \;\middle|\; \gamma_i^{(g)} = 1\right] / \sqrt{2\mathrm{var}\left[Z_i^{(g)} \;\middle|\; \gamma_i^{(g)} = 1\right]}\right) \end{aligned} \tag{11}$$

where Pr and erfc are the probability operations and complementary error function respectively.

Solving above equation (see Appendix A for derivation) and omitting the subscript i that is related to bit under investigation, we have final equation of $\left(P^{(g)}(a)\right)$ as

Therefore final BER equation is given by

$$\begin{aligned} &P^{(g)}(a) = \\ &\tfrac{1}{2}erfc\left[\left\{\tfrac{2\Psi}{N} + \tfrac{2(J-1)}{N} + \left(\alpha^2\cos^2(\phi - \hat{\phi})\tfrac{E_b}{N_0}\right)^{-1}\right\}^{-1/2}\right] \end{aligned} \tag{12}$$

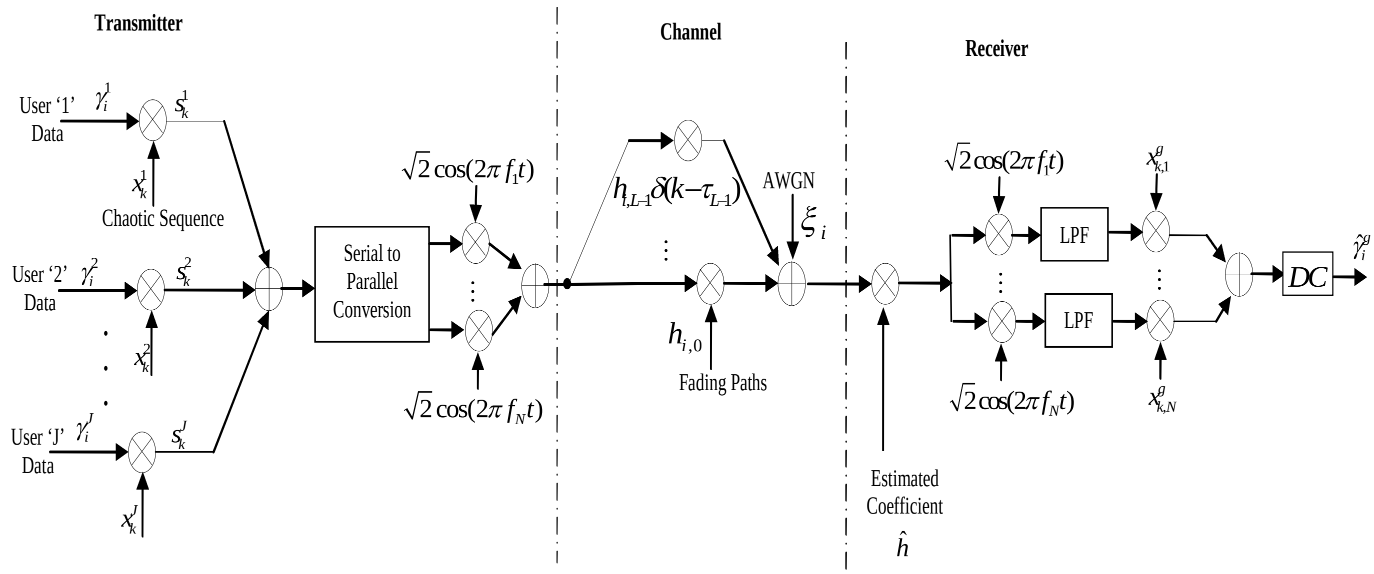

Fig. 1. Block Diagram of Proposed System

where $\Psi = \text{var}\{(x^g_{k,n})^2\}/P_c^2$

Putting $N = 2\beta$ then above equation becomes

$$P^{(g)}(a) = \frac{1}{2}erfc\left[\left\{\frac{\Psi}{\beta} + \frac{(J-1)}{\beta} + \left(\alpha^2\cos^2(\phi-\hat{\phi})\frac{E_b}{N_0}\right)^{-1}\right\}^{-1/2}\right] \tag{13}$$

For perfect flat fading channel estimation case i.e. $\alpha_i e^{j\phi_i} = \hat{\alpha}_i e^{j\hat{\phi}_i}$, BER equation derived in [40] is the special case of equation (13). Comparing equation (13) with results in [40], we can see that performance of MC-CDMA system is same as DS-CDMA system for flat fading channel. But effect of ISI is removed by MC-CDMA system in frequency selective fading channel.

IV. MAP AND ML ESTIMATOR

In this section we have derived Bayesian estimator equations for two cases, i.e. for multiplexed pilot-data and added pilot-data case.

Fig. 2 shows the baseband representation of Bayesian channel estimator. In this figure channel estimation is performed after multiplying the chaotic signal to received pilot symbols. The wireless channel is assumed to be quasi-static fading channel i.e. path gains are constant over a symbol duration. At i^{th} time instant, the received pilot signal at any user can be described as:

$$y_i = \left(\sum_{j=1}^{N} \mathbf{s}_{j,i}^T \mathbf{C}_{j,i}\right)\mathbf{h}_i + \xi_i \tag{14}$$

where $\mathbf{s}_i = [s_i, s_{i-1}, \cdots, s_{i-L+1}]^T$ is the transmitted pilot symbols, $\mathbf{h}_i = \left[h_{0,i}, h_{1,i}, \cdots, h_{L-1,i}\right]^T$ is the quasi-static time varying channel for j^{th} user and ξ_i is the zero mean White Gaussian noise with variance of σ_w^2. L and N represent the total number of paths and users respectively. $\mathbf{C}_i = diag[\mathbf{c}_i, \mathbf{c}_{i-1}, \cdots, \mathbf{c}_{i-L+1}]$ is the diagonal matrix with elements $\mathbf{c_i}$ of length 2β known as spreading factor. $\mathbf{c}$ is the spreading sequence for pilot symbols. Subscript j denotes that the symbol is related to j^{th} user. Channel coefficients are assumed to be Gaussian distributed [41] i.e. $\mathbf{h} \sim N(\mathbf{m_h}, \sigma^2_{\mathbf{h}})$ where $\mathbf{m_h}$ and $\sigma^2_{\mathbf{h}}$ are the mean and variance of the channel respectively.

A. MAP and ML estimation with multiplexed pilot and user data

If pilot is multiplexed with user data then it can be extracted using demultiplexer at the receiver and can be processed by channel estimator. Here we have to assume that fading and nosie have same effect on pilot and user data symbols. In this case, the condition distribution function $p\left(y_i|\mathbf{h}_i\right)$ for j^{th} user is defined as

$$p\left(y_i|\mathbf{h}_i\right) = \frac{1}{\sqrt{2\pi\sigma_w^2}}\exp\left(-\frac{\left(y_i - \mathbf{s}_{j,i}^T\mathbf{C}_{j,i}\mathbf{h}_i\right)^2}{2\sigma_w^2}\right) \tag{15}$$

Since mean $\mathbf{m_h}$ and variance $\sigma^{\mathbf{2}}_{\mathbf{h}}$ of Gaussian distributed channel is known at receiver, therefore MAP estimation algorithm is given by [6]

$$\nabla_{\mathbf{h}}\left(-\frac{\left(y_i - \mathbf{s}_{j,i}^T\mathbf{C}_{j,i}\mathbf{h}_i\right)^2}{2\sigma_w^2} - \frac{(\mathbf{h}_i-\mathbf{m_h})\sigma_{\mathbf{h}}^{-\mathbf{2}}(\mathbf{h}_i-\mathbf{m_h})^T}{2} + \text{constants}\Big|_{\mathbf{h}=\hat{\mathbf{h}}}\right) = 0 \tag{16}$$

Above derivative reduces to following equation (see appendix B for derivation)

$$\hat{\mathbf{h}}_{MAP,i} = \mathbf{m_h} + \frac{1}{\sigma_w^2}\left(\sigma_{\mathbf{h}}^{-\mathbf{2}} + \frac{1}{\sigma_w^2}\mathbf{C}_{j,i}^T\mathbf{s}_{j,i}\mathbf{s}_{j,i}^T\mathbf{C}_{j,i}\right)^{-1} \times \mathbf{C}_{j,i}^T\mathbf{s}_j\left(y_i - \mathbf{s}_{j,i}^T\mathbf{C}_{j,i}\mathbf{m_h}\right) \tag{17}$$

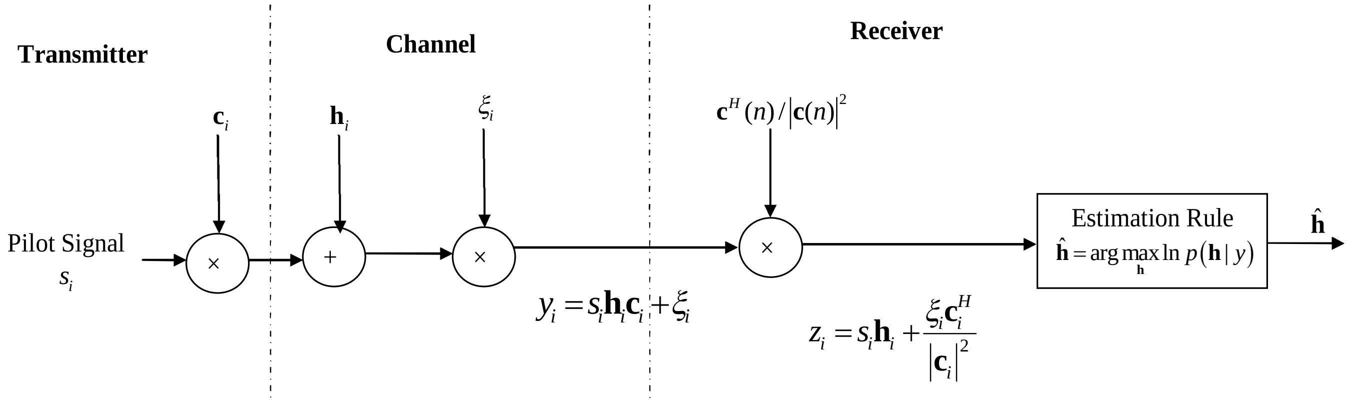

Fig. 2. Block Diagram of Bayesian Estimator

If we do not have a prior knowledge of the channel statistic, then we remove the second term in equation (16) and resultant algorithm is known as ML estimation [6] i.e.

$$\nabla_{\mathbf{h}} \left(- \frac{\left(y_i - \mathbf{s}_{j,i}^T \mathbf{C}_{j,i} \mathbf{h}_i\right)^2}{2\sigma_w^2} + \text{constants} \Bigg|_{\mathbf{h}=\hat{\mathbf{h}}} \right) = 0 \quad (18)$$

After solving derivative, we have following ML estimation equation

$$\hat{\mathbf{h}}_{ML,i} = \left(\mathbf{s}_{j,i}^T \mathbf{C}_{j,i}\right)^{-1} y_i \quad (19)$$

B. MAP and ML algorithm for added pilot and user data

For multiplexed pilot and user data, we have to assume same fading effects on both the signals. If we add pilot symbols to user symbols then fading have same effect on both the signals. Therefore same fading and noise effect assumptions can be removed. Further, in this case pilot has to be extracted from data for the channel estimation process. Since the chaotic sequences are orthogonal to each other therefore pilot symbols can be extracted by multiplying the received signal with chaotic sequence of pilot symbols. Multiplying received signal i.e. equation (14) with chaotic signal of j^{th} user we have

$$z_i = y_i \frac{\mathbf{C}_{j,i}^H}{\mathbf{C}_{j,i} \mathbf{C}_{j,i}^H} \quad (20)$$

In this case MAP and ML estimation equations are given by (see appendix C for derivation)

$$\hat{\mathbf{h}}_{MAP,i} = \mathbf{m}_{\mathbf{h}} + \tfrac{1}{\sigma_w^2} \left(\sigma_{\mathbf{h}}^{-\mathbf{2}} + \tfrac{1}{\sigma_w^2} \mathbf{s}_{j,i} \mathbf{s}_{j,i}^T \right)^{-1} \times \mathbf{s}_j \left(z_i - \mathbf{s}_{j,i}^T \mathbf{m}_{\mathbf{h}} \right) \quad (21)$$

and

$$\hat{\mathbf{h}}_{ML,i} = \left(\mathbf{s}_{j,i} \mathbf{s}_{j,i}^T\right)^{-1} \mathbf{s}_{j,i} z_i \quad (22)$$

V. Simulation Results

In the simulation we compare the performance of estimators for three cases. In first case, the pilot is multiplexed with data without multiplying with chaotic sequences at the transmitter. We represent this case as 'Without Chaotic Multiplication' in the simulation results. Similarly other two cases i.e. multiplexed pilot-data and added pilot-data with chaotic sequences multiplication at transmitter, are denoted by 'Before Chaotic Multiplication' and 'After Chaotic Multiplication' respectively in the results.

First order Markov process is used to model the fading process of channel [42], which is described as

$$h_{i+1,l} = \alpha_l h_{i,l} + v_{i,l} \; l = 0, \cdots, L-1 \quad (23)$$

where $v_{i,l}$ is the complex Gaussian process for l^{th} path at time i. In the simulation, variance of Gaussian process is set to 1. α_l is the correlation coefficient that depends on maximum Doppler frequency f_d and defined as

$$\alpha_l = J_0(2\pi f_d T_s) \quad (24)$$

where $J_0(\cdot)$ is the Bessel function of first kind and zeroth order, and T_s is the signaling rate. $f_d T_s$ is set to 0.1 in the simulation.

The value of the spreading factor 2β is 50. Following Chebyshev polynomial function i is used to generate the chaotic sequence [43].

$$x_j = 1 - 2(x_{j-1})^2 \quad (25)$$

where x_j denotes the j^{th} chip value chaotic sequence.

Fig. 3 and Fig. 4 show the channel tracking performance of the three estimators at $0dB$ and $20dB$ SNR conditions respectively. Chaotic sequences spread the data over entire bandwidth during transmission and despreading takes place during reception. Further noise is spread by the chaotic sequence multiplication at the receiver. Due to this spreading of noise, performance of the estimator in the presence of chaotic sequence is better than the without chaotic spreading case, as shown in Fig. 3 and Fig. 4.

From these figures it is clear that performance of all the estimators are improved with increase in the SNR. Since chaotic sequences are directly used for channel estimation

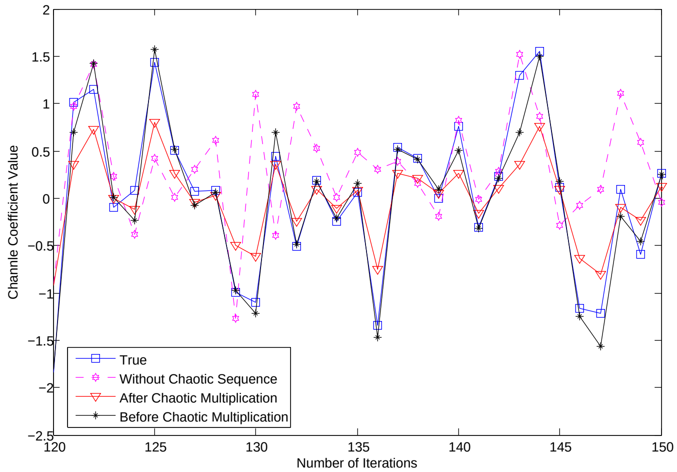

Fig. 3. MAP channel estimators performance, SNR = 0dB

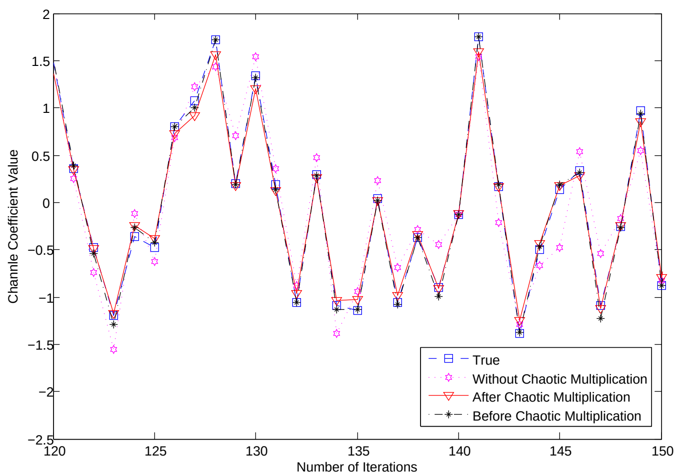

Fig. 4. MAP channel estimators performance, SNR = 20dB

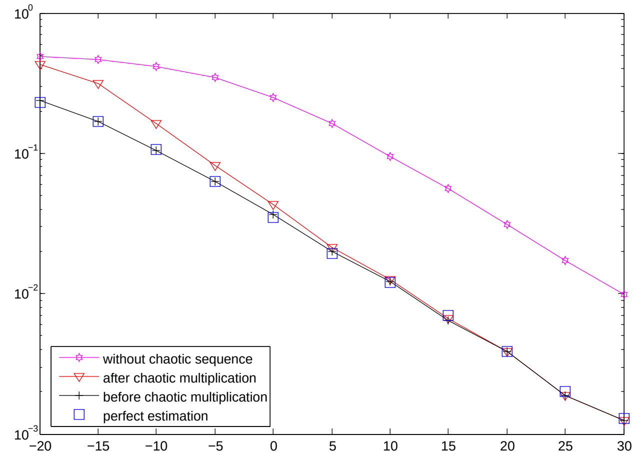

Fig. 5. BER performance of chaos based CDMA system using MAP, $2\beta = 50$

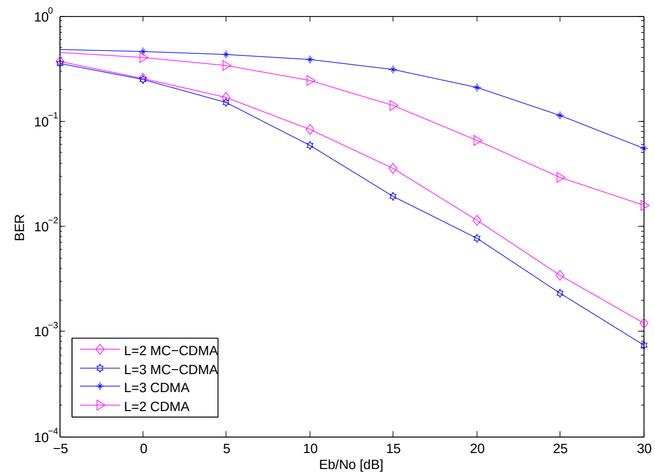

Fig. 6. Simulated BER comparison of different multipath components in one user and pilot system with added pilot-data case, $2\beta = 50$

as well as noise spreading in multiplexed pilot-data case therefore its performance is better than the added pilot-data case, for lower SNR conditions. For higher SNR conditions performance of both the chaotic estimators are practically same as shown in Fig. 4.

In Fig. 5, the BER performances are shown. $20dB$ performance improvement can be seen at $SNR = 10dB$ with chaotic spreading sequence over without spreading case. Performance improvement can be seen in 'Before Multiplication Case' over 'After Multiplication case' at lower SNR conditions. Further, performance of 'Before multiplication case' is same as the the perfect estimation case.

Different simulation results have been shown to compare the performance of the CDMA and MC-CDMA systems in fig. 6. In multipath channel simulations, second channel is delayed by one chip with respect to first channel. Similarly third channel is delayed by one chip with respect to second channel. It can be seen that performance of CDMA system degrades with increase in number of channels. Performance of this system deteriorates because of ISI effect due to multipath channels. However, the multipath frequency selective channels are converted into flat fading channels due to slow data transmission rate in MC-CDMA. Further, these multipath flat fading channels are added together, which is providing a virtual equal gain combining (EGC) effect within channels. Therefore the performance of MC-CDMA system has improved in multipath environment with increase in number of channels.

Fig. 7 shows the performance degradation with increase in number of users in the system. Obviously, when the number of users increases the BER increases due to the increase of IUI. Further, from equation (13) it is clear that the performance degradation with increased number of users can be improved by using high values of spreading factor. Fig. 8 shows this performance improvement with increase in spreading factor value.

VI. Conclusion

In this paper, performance of chaos based MC-CDMA system is investigated and compared with the performance of classical CDMA system. Performance analysis is evaluated under frequency selective fading channel. Under perfect synchronization, analytical expression for BER in close form is

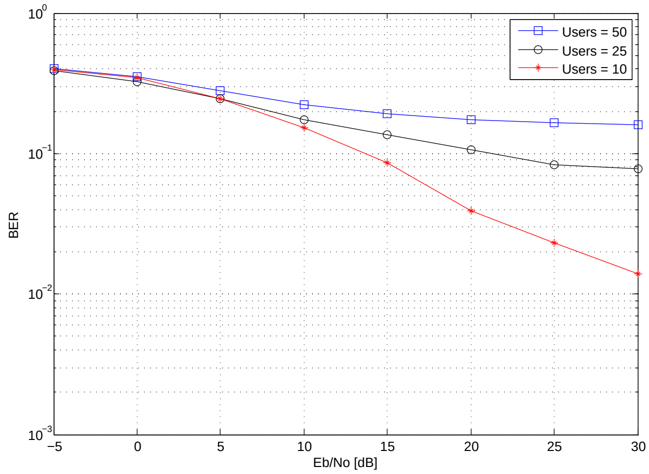

Fig. 7. Simulated BER comparison of different user in single path communication for $\beta = 50$

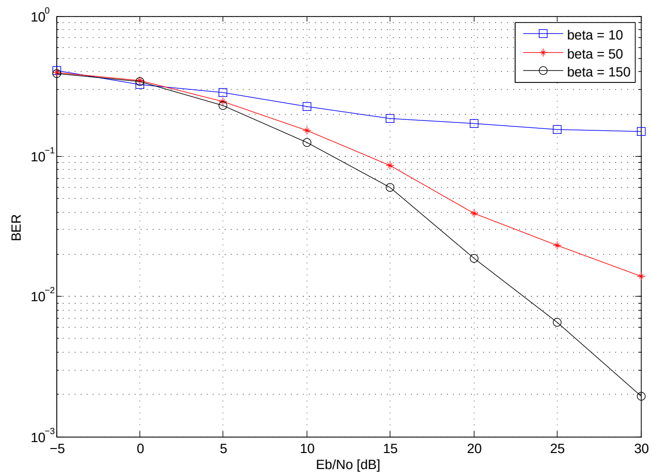

Fig. 8. Simulated BER comparison of different values of spreading factor (beta) in single path communication for 10 users

derived for imperfect channel estimation. Simulation results show that the performance of Bayesian channel estimator improves in the presence of spreading sequences. Further, analytical and simulation results show that the performance of MC-CDMA and CDMA system is equivalent for flat fading channel, but there is a significant performance improvement in MC-CDMA system under frequency selective channel.

Appendix A
Derivation of Equation (12)

Since the chaotic spreading sequences and AWGN noise are zero mean uncorrelated processes. Hence

$$E\left[Z_i^g \mid \gamma_i^g = 1\right] = \alpha_i\hat{\alpha}_i \cos(\phi_i - \hat{\phi}_i)NE\left[\left(x_k^g\right)^2\right] \\ = \alpha_i\hat{\alpha}_i \cos(\phi_i - \hat{\phi}_i)NP_c \quad (26)$$

All the three terms in (7) are uncorrelated to each other, hence var $\left[Z_i^g \mid \gamma_i^g = 1\right]$ is given by

$$\text{var}[Z_i^g \mid \gamma_i^g = 1\,] = \text{var}[Z_i^{ag}] + \text{var}[Z_i^{bg}] + \text{var}[Z_i^{cg}] \quad (27)$$

Variance of the first term Z_i^{ag} can be written as

$$\text{var}[Z_i^{ag}] = \{\alpha_i\hat{\alpha}_i \cos(\phi_i - \hat{\phi}_i)\}^2 N\left[\text{var}(x_k^g)^2\right] \quad (28)$$

Channel is assume to slowly varying therefore we can assume that $h_{i-\tau_l} \approx h_i$. Under this assumption, since chaotic sequences are uncorrelated with it's shifted version therefore

$$\text{var}[Z_i^{bg}] = \{\alpha_i\hat{\alpha}_i \cos(\phi_i - \hat{\phi}_i)\}^2 N(J-1)P_c^2 \quad (29)$$

Real and Imaginary part of complex AWGN have power spectral density equal to $N_0/2$ and independent to chaotic sequences, therefore variance of Z_i^{cg} can be derived as

$$\text{var}[Z_i^{cg}] = \hat{\alpha}_i^2 N\frac{N_0}{2}P_c \quad (30)$$

Putting equations (26), (28), (29) and (30) in equation (11) and after rearranging the equation we have equation (12).

Appendix B
Derivation of Equation (17)

Rewriting equation (16) after solving derivative, we have

$$\mathbf{\hat{h}_i} = \left(\sigma_\mathbf{h}^{-\mathbf{2}} + \tfrac{1}{\sigma_w^2}\mathbf{C}_{j,i}^T\mathbf{s}_{j,i}\mathbf{s}_{j,i}^T\mathbf{C}_{j,i}\right)^{-1} \times \\ \left(\mathbf{m_h}\sigma_\mathbf{h}^{-\mathbf{2}} + \frac{\mathbf{C}_{j,i}^T\mathbf{s}_{j,i}y_i}{\sigma_w^2}\right) \quad (31)$$

Let

$$\sigma_\mathbf{h}^{-\mathbf{2}} + \frac{1}{\sigma_w^2}\mathbf{C}_{j,i}^T\mathbf{s}_{j,i}\mathbf{s}_{j,i}^T\mathbf{C}_{j,i} = \mathbf{T} \quad (32)$$

Hence equation (31) becomes

$$\mathbf{\hat{h}_i} = \mathbf{T}^{-1}\left(\mathbf{m_h}\sigma_\mathbf{h}^{-\mathbf{2}} + \frac{\mathbf{C}_{j,i}^T\mathbf{s}_{j,i}y_i}{\sigma_w^2}\right) \quad (33)$$

Put the value of $\sigma_\mathbf{h}^{-\mathbf{2}}$ from equation (32) to (33), we get equation (17)

Appendix C
Derivation of Equation (21) and (22)

Putting the value of y_i from equation (14) in equation (20), we have

$$z_i = \mathbf{s}_{j,i}^T\mathbf{h_i} + \left(\sum_{j=1,k\neq j}^{N}\mathbf{s}_{j,i}^T\mathbf{C}_{j,i}\right)\mathbf{h_i}\times \\ \frac{\mathbf{C}_{j,i}^H}{\mathbf{C}_{j,i}\mathbf{C}_{j,i}^H} + \frac{\xi_i\mathbf{C}_{j,i}^H}{\mathbf{C}_{j,i}\mathbf{C}_{j,i}^H} \quad (34)$$

Since the cross-correlation of two different chaotic signals is very small, hence we can neglect the second term i.e.

$$z_i \approx \mathbf{s}_{j,i}^T\mathbf{h_i} + \frac{\xi_i\mathbf{C}_{j,i}^H}{\mathbf{C}_{j,i}\mathbf{C}_{j,i}^H} \quad (35)$$

Now, following the same steps as in section IV-A, we get equation (21) and equation (22).

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