Subspace based Model Predictive Control for Linear Parameter Varying Systems

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Abstract**—**A novel model predictive control method based on subspace identification for linear parameter varying (LPV) systems is presented. The LPV systems in innovations are introduced to derive the subspace prediction output with the subspace identification algorithm. The subspace prediction output is composed of the time-varying system matrices and the transition matrices. Next, the subspace prediction output is transformed to the appropriate one for the design of the controller. Through RQ factorization, the R matrix can be obtained from the above subspace prediction output. The subspace predictors are derived from R matrix using the orthogonal projection. Then, the subspace predictors are used to design the model predictive controller. The controller is to get the control sequence which can be obtained by minimizing the cost function and the control input is calculated from the control sequence. It's shown that the integrated action is incorporated in the control effect to eliminate the steady error. The simulation example is a model of the out-of-plane dynamics of a flexible rotor blade of a fixed speed wind turbine and it can be represented as the LPV system state-space model. The simulation results are provided to illustrate the performance of this method.

Keywords**—**Linear parameter varying systems, Subspace identification, Model predictive control, Flapping dynamics of a wind turbine

I. Introduction

THE model predictive control has been attractive for decades in control theory field and has become one of the main methods of modern control and achieved wide applications in industry processes [1], [2], [3], [4]. The traditional industrial predictive control is based on input-output model, including parametric and nonparametric ones. In order to improve the control performance, a state space model should be adopted, so the modern filter theory and the design method of controller developed in recent years can play a role [5]. However, it was unable to obtain the accurate state-space model among the complex industrial targets due to the limitation of identified method. Subspace identification method has changed this situation perfectly, the control workers may relieve completely from the tedious modeling by mechanism. The accurate state-space model can be obtained when there are enough process input and output data [6], [7], [8]. By combining the merits of subspace identification and MPC, subspace based model predictive control (SPC) was formed [9], [10], [11].

Linear parameter varying (LPV) systems are a particular class of nonlinear systems which can be thought of as time-varying systems, for which the variation depends explicitly on a time-varying parameter referred to as the scheduling or weight sequence [12]. LPV systems are widely used in control field, especially in gain-scheduling, and robust control techniques [13], [14]. The applicability of subspace techniques to periodically time-varying systems has already been shown in some researchers. Vincent and Verhaegen [15] presented kernel methods for subspace identification which have much smaller dimensions in LPV systems. Felici et al. [16] presented the periodic scheduling sequence to determine the column space of the time-varying observability matrices. Wingerden and Verhaegen [17] proposed subspace identification method with affine parameter dependence of LPV systems for the open- and closed-loop data. But these papers only solve the identification problem. It's a powerful technique using subspace identification to design model predictive controller in LPV systems and the SPC for LPV systems has never been seen in any other publications.

In this paper, the main contribution is that we present a novel model predictive control method based on subspace identification for LPV systems. The subspace prediction output of the LPV system is first derived by recursive substitution. Through RQ factorization, the subspace predictors can be obtained from R matrix. Next, construct the incremental form of cost function in MPC and the subspace predictors are incorporated to get the control input.

The outline of the paper is arranged as follows. We start in Section 2 with the prediction output for LPV systems using subspace identification. In Section 3, we give the subspace predictive control method for LPV systems. In Section 4 the simulation example is presented that show the potential of the proposed method. Section 5 ends with the conclusions.

II. Subspace prediction output for LPV systems

Consider the LPV system described by innovations form

This work was supported by the Major State Basic Research Development Program 973 (No. 2012CB215202), the National Natural Science Foundation of China (No.61134001), Key Laboratory of Dependable Service Computing in Cyber Physical Society (Chongqing University), Ministry of Education and Scientific and Technological Research Program of Chongqing Municipal Education Commission (No.KJ1503008).

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$$x_{k+1} = A_k x_k + B_k u_k + K_k e_k \tag{1}$$
$$y_k = C_k x_k + D_k u_k + e_k \tag{2}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^r$, $y_k \in \mathbb{R}^l$, are state, input and output vectors respectively. $e_k \in \mathbb{R}^l$ is zero-mean white Gaussian sequence which is independent of the input u_k and of the initial state x_0.

The time-varying system matrix is now given by

$$A_k = \sum_{i=1}^{m} A^{(i)} \mu_k^{(i)} \tag{3}$$

and B_k, C_k, D_k and K_k are similar to A_k. The matrices $A^{(i)} \in \mathbb{R}^{n\times n}$, $B^{(i)} \in \mathbb{R}^{n\times r}$, $C^{(i)} \in \mathbb{R}^{l\times n}$, $D^{(i)} \in \mathbb{R}^{l\times r}$, $K^{(i)} \in \mathbb{R}^{n\times l}$. The model weights $\mu_k^{(i)} \in \mathbb{R}$.

Define the output vector y_k^d as

$$y_k^d = \begin{bmatrix} y_k^{\mathrm{T}} & y_{k+1}^{\mathrm{T}} & \cdots & y_{k+d-1}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{4}$$

and the input vector u_k^d, the noise vector e_k^d are similar to y_k^d where d is defined as the window size. Define the transition matrix $\Phi_A(k,j) \in \mathbb{R}^{n\times n}$ for $k > j$:

$$\Phi_A(k,j) = A_{k-1} A_{k-2} \cdots A_j \tag{5}$$

where $\Phi_A(k,k) = I_n$, I is the identity matrix.

The subspace prediction output of the LPV system can be derived by recursive substitution of Eqs. (1)-(2):

$$y_k^d = \Gamma_k^d x_k + H_k^d u_k^d + F_k^d e_k^d \tag{6}$$

with

$$\Gamma_k^d = \begin{bmatrix} C_k \\ C_{k+1}\Phi_A(k+1,k) \\ \vdots \\ C_{k+d-1}\Phi_A(k+d-1,k) \end{bmatrix},$$

$$H_k^d = \begin{bmatrix} D_k & 0 & \cdots & 0 \\ h_{k,2,1}^d & D_{k+1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ h_{k,d,1}^d & \cdots & h_{k,d,d-1}^d & D_{k+d-1} \end{bmatrix},$$

$$F_k^d = \begin{bmatrix} I_k & 0 & \cdots & 0 \\ f_{k,2,1}^d & I_k & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ f_{k,d,1}^d & \cdots & f_{k,d,d-1}^d & I_k \end{bmatrix}.$$

where $h_{k,i,j}^d = C_{k+i-1}\Phi_A(k+i-1,k+j)B_{k+j-1}$, $f_{k,i,j}^d = C_{k+i-1}\Phi_A(k+i-1,k+j)K_{k+j-1}$ with $i = 2,\cdots,d$, $j = 1,\cdots,d-1$, and $i > j$.

To use the subspace prediction output to design model predictive controller, define the period is p and N samples in system. The equation (6) can be transformed as

$$Y_{i,d,N}^k = \Gamma_k^d X_{i,N}^k + H_k^d U_{i,d,N}^k + F_k^d E_{i,d,N}^k \tag{7}$$

where

$$Y_{i,d,N}^k = \begin{bmatrix} y_{k+ip}^d & y_{k+(i+1)p}^d & \cdots & y_{k+(i+N-1)p}^d \end{bmatrix},$$
$$U_{i,d,N}^k = \begin{bmatrix} u_{k+ip}^d & u_{k+(i+1)p}^d & \cdots & u_{k+(i+N-1)p}^d \end{bmatrix},$$
$$E_{i,d,N}^k = \begin{bmatrix} e_{k+ip}^d & e_{k+(i+1)p}^d & \cdots & e_{k+(i+N-1)p}^d \end{bmatrix},$$
$$X_{i,N}^k = \begin{bmatrix} x_{k+ip} & x_{k+(i+1)p} & \cdots & x_{k+(i+N-1)p} \end{bmatrix}.$$

III. Subspace predictive control for LPV systems

Construct the following instrumental variable matrix W_N:

$$W_N^{k+p-d} = \begin{bmatrix} U_{0,d,N}^{k+p-d} \\ Y_{0,d,N}^{k+p-d} \end{bmatrix} \tag{8}$$

where

$$U_{0,d,N}^{k+p-d} = \begin{bmatrix} u_{k+p-d} & u_{k+2p-d} & \cdots & u_{k+N-d} \\ u_{k+p-d+1} & u_{k+2p-d+1} & \cdots & u_{k+N-d+1} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k+p-1} & u_{k+2p-1} & \cdots & u_{k+N-1} \end{bmatrix},$$

$$Y_{0,d,N}^{k+p-d} = \begin{bmatrix} y_{k+p-d} & y_{k+2p-d} & \cdots & y_{k+N-d} \\ y_{k+p-d+1} & y_{k+2p-d+1} & \cdots & y_{k+N-d+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k+p-1} & y_{k+2p-1} & \cdots & y_{k+N-1} \end{bmatrix}.$$

The $U_{1,d,N}^k$ and $Y_{1,d,N}^k$ are represented as

$$U_{1,d,N}^k = \begin{bmatrix} u_{k+p} & u_{k+2p} & \cdots & u_{k+N} \\ u_{k+p+1} & u_{k+2p+1} & \cdots & u_{k+N+1} \\ \vdots & \vdots & \ddots & \vdots \\ u_{k+p+d-1} & u_{k+2p+d-1} & \cdots & u_{k+N+d-1} \end{bmatrix},$$

$$Y_{1,d,N}^{k}=\begin{bmatrix} y_{k+p} & y_{k+2p} & \cdots & y_{k+N} \\ y_{k+p+1} & y_{k+2p+1} & \cdots & y_{k+N+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k+p+d-1} & y_{k+2p+d-1} & \cdots & y_{k+N+d-1} \end{bmatrix}.$$

A series of algebraic calculations are carried out on (7) and take the following RQ factorization:

$$\begin{bmatrix} W_N^{k+p-d} \\ U_{1,d,N}^{k} \\ Y_{1,d,N}^{k} \end{bmatrix}=\begin{bmatrix} R_{11}^{k} & 0 & 0 \\ R_{21}^{k} & R_{22}^{k} & 0 \\ R_{31}^{k} & R_{32}^{k} & R_{33}^{k} \end{bmatrix}\begin{bmatrix} Q_1^{\mathrm{T}} \\ Q_2^{\mathrm{T}} \\ Q_3^{\mathrm{T}} \end{bmatrix} \tag{9}$$

The optimal prediction $\hat{Y}_{1,d,N}^{k}$ can be found from the orthogonal projection of the row space of $Y_{1,d,N}^{k}$ onto the row space of the matrix $\begin{bmatrix} W_N^{k+p-d} \\ U_{1,d,N}^{k} \end{bmatrix}$:

$$\hat{Y}_{1,d,N}^{k}=Y_{1,d,N}^{k}\,/\begin{bmatrix} W_N^{k+p-d} \\ U_{1,d,N}^{k} \end{bmatrix} \tag{10}$$

The optimal prediction $\hat{Y}_{1,d,N}^{k}$ also can be written as

$$\hat{Y}_{1,d,N}^{k}=L_w^k W_N^{k+p-d}+L_u^k U_{1,d,N}^{k} \tag{11}$$

where L_w^k is the subspace predictor that corresponds to the past input-output data and L_u^k is the subspace predictor that corresponds to the future input data.

Through the implementation of the orthogonal projection, by letting

$$\begin{bmatrix} L_w^k & L_u^k \end{bmatrix}=\begin{bmatrix} R_{31}^k & R_{32}^k \end{bmatrix}\begin{bmatrix} R_{11}^k & 0 \\ R_{21}^k & R_{22}^k \end{bmatrix}^{\dagger} \tag{12}$$

where superscript † represents the Moore-Penrose pseudo-inverse. We can get the L_w^k and L_u^k.

Define an incremental form of cost function which has an integrated action to eliminate the steady error:

$$\begin{aligned} J &= \sum_{i=1}^{N_2}(r_{k+i}-\hat{y}_{k+i|k})^{\mathrm{T}}Q(r_{k+i}-y_{k+i|k}) \\ &\quad+\sum_{j=1}^{N_u}\Delta u_{k+j-1}^{\mathrm{T}}R\Delta u_{k+j-1} \\ &= (r_f-\hat{y}_f)^{\mathrm{T}}Q(r_f-\hat{y}_f)+\Delta u_f^{\mathrm{T}}R\Delta u_f \end{aligned} \tag{13}$$

where N_2 and N_u are the prediction and control horizon respectively, r_{k+i} is the reference setpoint signal at the current time $k+i$, Q and R are the weighting matrices, the subscript f represents "future". The prediction output of MPC can be expressed as

$$\hat{y}_f=F_l y_k+\Gamma_l L_w^k\Delta w_p+\Gamma_l L_u^k\Delta u_f \tag{14}$$

where

$$F_l=\begin{bmatrix} I_l \\ I_l \\ \vdots \\ I_l \end{bmatrix},\ \Gamma_l=\begin{bmatrix} I_l & 0 & \cdots & 0 \\ I_l & I_l & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_l & I_l & \cdots & I_l \end{bmatrix},\ \hat{y}_f=\begin{bmatrix} y_{k+p+1} \\ y_{k+p+2} \\ \vdots \\ y_{k+p+d} \end{bmatrix},$$

$$\Delta u_f=\begin{bmatrix} \Delta u_{k+p} \\ \Delta u_{k+p+1} \\ \vdots \\ \Delta u_{k+p+d-1} \end{bmatrix},\ \Delta w_p=\begin{bmatrix} \Delta y_{k+p-d+1} \\ \Delta y_{k+p-d+2} \\ \vdots \\ \Delta y_{k+p} \\ \Delta u_{k+p-d} \\ \Delta u_{k+p-d+1} \\ \vdots \\ \Delta u_{k+p-1} \end{bmatrix}.$$

Substituting the $\hat{y}_f$ into the cost function J and minimize the J. The control sequence Δu_f can be obtained:

$$\begin{aligned} \Delta u_f &= -\left((\Gamma_l L_u^k)^{\mathrm{T}}Q(\Gamma_l L_u^k)+R\right)^{-1} \\ &\quad\times\left(\Gamma_l L_u^k\right)^{\mathrm{T}}Q\left(\Gamma_l L_w^k\Delta w_p+F_l(y_k-r_k)\right) \end{aligned} \tag{15}$$

At each time sample, only the first element of Δu_f is used for calculating the control input. So the control input u_k is

$$u_k=u_{k-1}+\Delta u_k \tag{16}$$

At the next time sample, we measure the new input-output data and the new control input will be calculated using above procedure.

IV. Simulation Example

The simulation example is a model of the out-of-plane dynamics of a flexible rotor blade of a fixed speed wind turbine [18]. Among other phenomena of the wind, gravity will lead to a nonlinear description of the flapping dynamics. The produced nonlinearity can be described by an LPV model using the scheduling sequence as the cosine of the blade rotation angle. After choosing some wind turbine parameters, the LPV system state-space model can be obtained from the following differential equations governing the flapping dynamics:

$$
\begin{aligned}
&\left[A^{(1)}\middle|A^{(2)}\right]=\left[\begin{array}{cc|cc}0 & 0.0734 & -0.0021 & 0\\ -6.5229 & -0.4997 & -0.0138 & 0.5196\end{array}\right],\\
&\left[B^{(1)}\middle|B^{(2)}\right]=\left[\begin{array}{c|c}-0.07221 & 0\\ -9.6277 & 0\end{array}\right],\\
&\left[C^{(1)}\middle|C^{(2)}\right]=\left[\begin{array}{cc|cc}1 & 0 & 0 & 0\end{array}\right],\\
&\left[D^{(1)}\middle|D^{(2)}\right]=\left[\begin{array}{c|c}0 & 0\end{array}\right],\\
&\left[K^{(1)}\middle|K^{(2)}\right]=\left[\begin{array}{c|c}0.1 & 0\\ 0.02 & 0\end{array}\right].
\end{aligned} \tag{17}
$$

First, we introduce the identification step to verify the model accuracy. The flapping dynamics is excited using a constant wind speed with added turbulence modeled by a white noise input signal. The parameters are given as follows, the samples $N = 5000$, the period $p = 10$, the window size $d = 6$. The eigenvalues of $A^{(i)}$ matrices are showed in Fig. 1. It can be seen as a satisfactory model.

To test the cross validation, a form of prediction error is defined as:

$$\text{VAF}=\max\{1-\frac{\operatorname{var}(y_k-\hat{y}_k)}{\operatorname{var}(y_k)},0\}\times 100 \tag{18}$$

where y_k and $\hat{y}_k$ are the values at instant k of process and model output respectively. The LTI (Linear Time Invariant) method of subspace identification is introduced as a comparison. The VAF on the validation data set can be seen in Table 1. We can get from (18) that with the increase

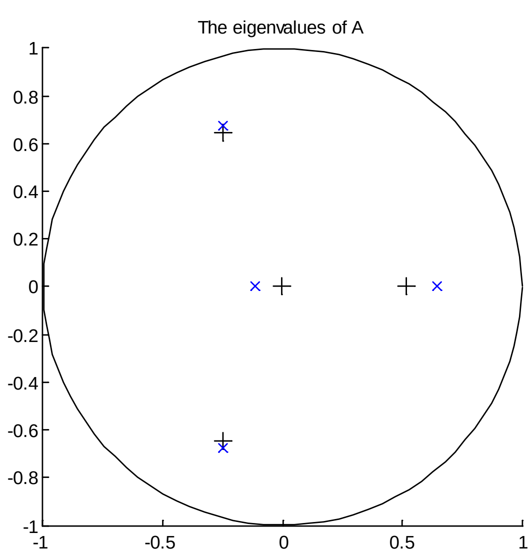

Fig. 1 The eigenvalues of $A^{(i)}$ matrices

Table 1. The VAF of LPV and LTI identified model

Identified method	LPV	LTI
VAF	98.0371	86.5233

of VAF, the model is more satisfactory. The cross validation results indicate that the LPV identified model is more accurate than LTI identified model.

Then, the identified LPV model is used to design the subspace predictive controller. The parameters of LPV subspace predictive controller (LPV-SPC) are tuned as follows. The prediction horizon $N_2 = 20$ and the control horizon $N_u = 10$. The weighting matrices are respectively represented as $Q = 0.5 * I_{20}$, $R = 0.1 * I_{10}$. Totally 900s are conducted in simulation. For comparison, the LTI subspace predictive controller (LTI-SPC) is used to control this

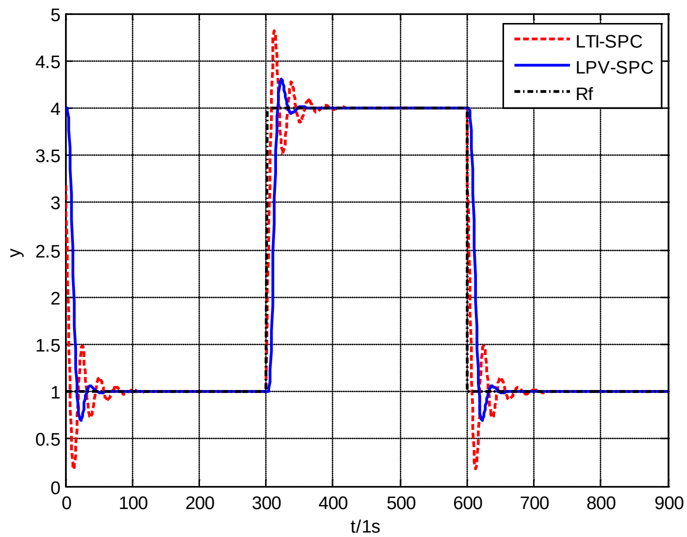

Fig. 2 The output tracking performance

system [19]. In the set-point test, Rf is defined as the reference output. The performance of output using the proposed LPV-SPC and the LTI-SPC is shown in Fig. 2.

For the analysis of Fig. 2, a form of prediction error ξ is conducted to verify the performance of output:

$$\xi=\sqrt{\frac{\sum_{i=1}^{N}\left(y_i-y_i^p\right)^2}{\sum_{i=1}^{N}\left(y_i\right)^2}} \tag{19}$$

where y_i and y_i^p are the values at instant i of reference and process output respectively. The smaller the value of ξ, the better the performance of the controller. The compared ξ is presented in Table 2 and we could conclude that LPV-SPC expresses better comparing with

Table 2. The ξ of LPV-SPC and LTI-SPC methods

Control method	LPV-SPC	LTI-SPC
ξ	0.03672	0.05731

the LTI-SPC. This is attributed to the system nonlinear characteristic which can be represented by the LPV model.

V. Conclusion

In the article, we presented a subspace based model predictive control method for linear parameter varying (LPV) systems. The time-varying matrices are used to get the subspace prediction output through recursive substitution. Then, the subspace predictive controller applied to LPV systems is designed using the subspace predictors. The method was implemented on flapping dynamics of a wind turbine. The primary contribution of this article is that we are able to obtain the subspace predictors to design the model predictive controller in a LPV system and we do not need to obtain the state space model of the system.

In the future, we will focus on using some effective calculation tools to reduce the computation of the method. Further, we will extend the application object to a more complex wind turbine system.

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