Legendre Wavelet and Particle Swarm Optimization for Power Amplifier Linearization

Xiaoyang Zheng, Yong Fu and Zhengyuan Wei

Abstract — This paper implements Legendre wavelet and particle swarm optimization (PSO) to linearize power amplifier (PA). The novel method proposed is very efficient and the pre-distorter (PD) shows stability and effectiveness. The reasons are mainly the Legendre wavelet base can offer piecewise lower order polynomial approximation at different level of resolution for the PA linearization, and the PSO is a powerful optimization tool based on stochastic searching technique. Furthermore, a quite significant improvement in linearity is achieved based on the measured data of the PA characteristics and out power spectrum has been compared.

Keywords—Power amplifier, Pre-distortion, Legendre wavelet, Particle swarm optimization.

I. Introduction

The power amplifier is a major source of nonlinearity in a communication system. Thus, the improvement of the linearity of the PA becomes an objective of first importance for mobile communication systems. Recently, a lot of efforts have been made to improve the linearity of the PA [1–5]. Among these methods, the polynomial model is widely used to predict and design the performance of the PA because of its simplicity and easiness of implementation. Especially, orthogonal polynomials have been proposed to model the PA and the PD [7-9]. It is very important that the coefficients of the orthogonal polynomial model can be extracted with much improved numerical stability than those of the conventional polynomials [5].

In this work, a novel technique of the PA and the PD models is presented by utilizing the Legendre wavelet and the PSO to discuss commonly employed models (the Wiener model and the Hammerstein model) consisting of a linear dynamic system and a static nonlinearity [10-19]. The linear dynamic system (LDS) is realized by an FIR filter, and the static nonlinearity (SNL) is characterized by AM/AM and AM/PM effects [6-10], which are approximated by the Legendre wavelet, respectively. In this paper, an LDMOS class-AB power amplifier with 50W output power at 2.14 GHz was used to assess the proposed technique for actual PA linearization. This PA is excited by a WCDMA signal with bandwidth of 20 MHz and chip rate of 3.84 Mcps. Applying the practical transmission signal, the Wiener model is used to model the PA with memory by using the Legendre wavelet, which can offer an efficient tool for the pre-distortion because of its rich properties. On the other hand, through computing the inversion of the Wiener model, the Hammerstein model is implemented to design the PD by using the PSO algorithm. The approach presented in this article has advantages: adaptive piecewise approximation at different decomposition level; the Legendre wavelet base functions are orthogonal and expressed in closed form and can be implemented with little demand on the computation resources; lower order approximation; an efficient pre-distortion linearization technique to overcome the stagnation in the nonlinear system identification.

The organization of this paper is as follows. In Section II, we relate the Legendre wavelet and illustrate its benefit in PA modeling and the inversion structure of the PA model and compare it with the conventional polynomial model. In Section III, we formulate a pre-distortion linearization algorithm with the Legendre wavelet and the PSO algorithm. Numerical examples are presented and a great improvement in linearity is achieved. Finally, the conclusion is obtained in Section IV.

II. PA Model And Its Inversion With Legendre Wavelet

In this section, we first relate the principle of the model known as PA model and inversion from a mathematical point of view[11]. In a second step, the Legendre wavelet is introduced. Finally, applying the practical transmission signal, the different polynomial bases including the Legendre wavelet are used to PA model and its inversion and compared with respect to the normalized mean squared error (NMSE) of least squares (LS) estimation.

A. Principle of PA model and its inversion

The PD, which has the inverse characteristic of the PA, is used to compensate for the nonlinearity in the PA. The most satisfied PA linearization is that the ultimate function of the system from the PD input to the PA output would ideally consist of a linear gain and 0° phase shift. Generally, let N_W , P_H denote the operators of the Wiener model for the PA and the Hammerstein model for the PD, respectively.

This work is funded by Fundamental and Advanced Research Project of Chongqing CSTC of China, the project No. are CSTC2013JCYJA00022 and cstc2012jjA00018 and cstc2014jcyjA0080.
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Then in the discrete time case, the PA linearization is implicitly given

$$y[q] := N_W\{P_H\{x[q]\}\} = g \cdot x[q-q_0] \quad (1)$$

with some real valued constant gain $g > 0$ and some constant delay $q_0 \in N_0$. Here $x[q]$ and $y[q]$ are the discrete-time input and output signals, respectively, and $q = 1,2,\cdots,Q$. Or if the inverse of the behaviour of the target system exists, the explicit definition of the PD reads

$$y[q] := P_H\{\{x[q]\} = N_W{}^{-1}\{g \cdot x[q-q_0]\}. \quad (2)$$

Fig. 1 demonstrates the structures of the Wiener model and the Hammerstein model, respectively [11].

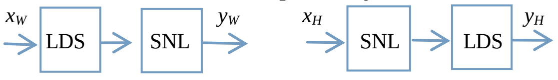

Fig.1 (a) Wiener model (PA) (b) Hammerstein model (PD)

In this article, we utilize the Legendre wavelet base to approximate the two operators N_W and P_H by applying a set of 73920 samples of the practical transmission signal.

B. Legendre wavelet

In this context, we briefly review the Legendre wavelet base constructed by Alpert [12]. It is pointed out that the Legendre wavelet can effectively approximate the above two operators with respect to the pre-distortion due to its nice properties.

Let $L_k(x)$ denote the Legendre polynomial of degree k, which is defined as follows

$$L_0(x) = 1, \qquad L_1(x) = x,$$
$$L_{k+2}(x) = \frac{2k+3}{k+2} x L_{k+1}(x) - \frac{k+1}{k+2} L_k(x). \quad (3)$$

Let $\phi_k(x)$ denote the Legendre scale function defined as

$$\phi_k(x) = \begin{cases} \sqrt{2k+1} L_k(2x-1), & x \in [0,\ 1], \\ 0, & x \notin [0,\ 1]. \end{cases} \quad (4)$$

For $n = 0,1,2,\cdots$ and $l = 0,1,2,\cdots,2^n - 1$, define interval $I_{nl} = [2^{-n} l,\ \ 2^{-n}(l+1)]$. For $p = 1,2,\ldots$, define $V_{p,n}$ as a space of piecewise polynomial functions,

$V_{p,n} = \{f : f|_{I_{nl}}$ is a polynomial of degree strictly less than p; and f vanishes elsewhere$\}$,

which constitutes a linear space. The whole set $\{\phi_k\}_{k=0}^{p-1}$ forms an orthonormal basis for $V_{p,0}$.

Generally, the subspace $V_{p,n}$ is spanned by $2^n p$ functions which are obtained from $\phi_0,\cdots,\phi_{k-1}$ by dilations and translations,

$$V_{p,n} = \text{span}\,\{\phi_{k,nl}(x) = 2^{n/2}\phi_{k,n}(2^n x - l),\ \ 0 \le k \le p-1,\ \ 0 \le l \le 2^n - 1\}$$

,which forms an orthonormal basis. In order to intuitively understand this base, we let $p = 5$ and $n = 1$, respectively. Fig. 2 plots the functions $\phi_{k,nl}(x)$ for $V_{5,1}$.

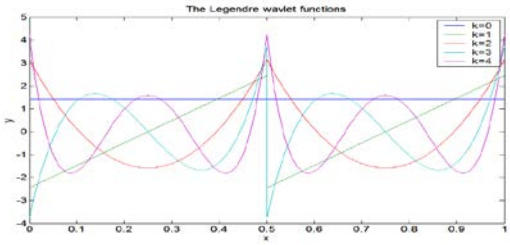

Fig.2. Legendre wavelet bases with p=5 and n=1

According to the above properties, the approximation of a function $f \in L_2[0,1]$ in $V_{p,n}$ is represented as follows

$$f(x) = \sum_{l=0}^{2^n-1} \sum_{k=0}^{p-1} s_{k,nl}\phi_{k,nl}(x).$$

In this paper, we use the discrete-time signals $x[q]$ and $y[q]$ as training data to identify the coefficients $s_{k,nl}$. The approximate estimation satisfies [12]

$$\left\| f - \sum_{l=0}^{2^n-1}\sum_{k=0}^{p-1} s_{k,nl}\phi_{k,nl}(x) \right\|_2 \sim O(2^{-np}), \quad (5)$$

which demonstrates that the approximation error exponentially converges with the level n of the resolution and the order p of the Legendre wavelet.

C. PA model and its inversion

The models of the PA linearization can be significantly simplified by splitting up the elaborate dynamic nonlinearities into compositions of the LDS and SNL [11], which are described by operators L and N respectively. Then the continuous- time description of the Wiener model, i.e., PA model reads

$$|y[q]| = N_{A,BB}\{|(h_L * x[q])|\}, \quad (6)$$
$$\arg\{y[q]\} = N_{\Phi,BB}\{|(h_L * x[q])|\} + \arg\{x[q]\}, \quad (7)$$

where h_L is a finite impulse response of the LDS with length L_0, and

$$x[q] = [x[q], x[q-1], \cdots, x[q-L_0+1]]^T, \quad (8)$$
$$h_L = [h_L[0], h_L[1], \cdots, h_L[L_0-1]]^T. \quad (9)$$

Furthermore, the SNL of PA is characterized by the combination of the AM/AM and the AM/PM conversions. The operator $N_{A,BB}$ in (6) is termed the AM/AM conversion and describes the (nonlinear) relation between the amplitude of the input baseband signal $x[q]$ and the amplitude of the output baseband signal $y[q]$, and the operator $N_{\Phi,BB}$ in (7) denotes the AM/PM conversion, which characterizes the phase offset of the output baseband signal depending on the input amplitude.

According to Fig.1 and by computing the inversions of three operators H_L, $N_{A,BB}$ and $N_{\Phi,BB}$, we can obtain the I/O relation of the Hammerstein model, which is given by the similar combination of the AM/AM and the AM/PM conversions. Fig.3 illustrates the SNL model by using the AM/AM and AM/PM conversions.

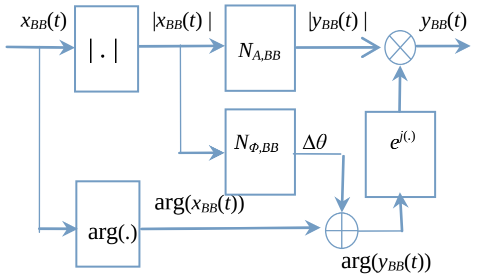

Fig.3. A static baseband structure

In Fig.3, the $\Delta\theta$ represents the time-dependent phase offset. Consequently, the response $y[q]$ of the SNL to the input signal $x[q]$ has the form

$$y[q] = |y[q]| e^{j(\arg(x[q]) + N_{\Phi,BB}(|h_L * x[q]|))}. \tag{10}$$

With (6), (7) and (10), we can elaborate methods to find three parameter sets for the PA model.

In this subsection, we implement a common approach in the context of PA identification to estimate the parameters of the LDS and the SNL part separately based on the measured data [9]. Firstly, the filter coefficients are randomly generated uniformly distributed on the complex square [-0.5, 0.5) + j[-0.5, 0.5) and normalized such that the magnitude of the largest weight equals one. Then we estimate the weights of the FIR filter with an additional rough compensation of the nonlinearity. For the Hammerstein model in Section III, the linear filter is again inverted by the root search method. In the second step, the SNL is identified more accurately based on the results of the first phase and by using the Legendre wavelet base, which is defined on the subinterval I_{nl}, i.e., piecewise polynomials which are represented by a linear combination of basis functions and can be approximated be different level n of resolution with required accuracy.

Now, let $x_{BB}[q] = |h_L * x[q]|$ and $y_{BB}[q] = |y[q]|$, respectively. With (6) and (7), we obtain the approximations of the AM/AM and AM/PM conversions satisfying

$$y_{BB}[q] = \sum_{l=0}^{2^n-1} \sum_{k=0}^{p-1} s_{k,nl} \phi_{k,nl}(x_{BB}[q]), \tag{11}$$

$$\arg(y[q]) = \sum_{l'=0}^{2^{n'}-1} \sum_{k'=0}^{p'-1} d_{k',n'l'} \phi_{k',n'l'}(x_{BB}[q]) + \arg(x[q]), \tag{12}$$

where $s_{k,nl}$ and $d_{k',n'l'}$ are the Legendre wavelet approximation coefficients. Due to the similar means to approximate the AM/PM characteristic, here we explicitly discuss the approximation of the AM/AM conversion. Let us define the $Q \times 1$ input data vector $\mathbf{x} = [x[1], \cdots, x[Q]]^T$, the $Q \times 1$ output data vector $\mathbf{y} = [y[1], \cdots, y[Q]]^T$, and the $2^n p \times 1$ parameter vector $\mathbf{s} = [s_1, \cdots, s_{2^n p}]^T$. For $u = 1, 2, \cdots, 2^n p$, define $\phi_u(x) = [\phi_u(x[1]), \cdots, \phi_u(x[Q])]^T$, and the $Q \times 2^n p$ matrix $\mathbf{\Phi} = [\phi_1(x), \cdots, \phi_{2^n p}(x)]^T$. Samples $\{x[q], y[q]\}_{q=1}^{Q}$ correspond to points on the PA transfer characteristics, which may or may not be ordered or equally spaced. Then we can now represent (11) as

$$\mathbf{y} = \mathbf{\Phi s}. \tag{13}$$

In this subsection, we utilize the common LS method to optimize the coefficients the vectors in (13) such that

$$\mathbf{s} = (\mathbf{\Phi}^T \mathbf{\Phi})^{-1} \mathbf{\Phi}^T \mathbf{y}. \tag{14}$$

For conventional polynomials, the inversion of the $2^n p \times 2^n p$ matrix $\Phi^T \times \Phi$ in formula (14) can experience a numerical instability problem, whereas for the Legendre wavelet method proposed in this paper, the parameter estimation in (11) can be improved substantially and the numerical instability problem can be alleviated because of the orthogonality and adaptive piecewise approximation on the subinterval I_{nl} of the Legendre wavelet base.

In this work, to give a quantitative measure of the approximation accuracy, we use the NMSE of the form

$$NMSE(dB) = 10\log_{10}\left(\frac{\sum_{q=1}^{Q} |z[q] - y[q]|^2}{\sum_{q=1}^{Q} |z[q]|^2}\right) \tag{15}$$

as metric to choose the optimum dimensions of the PA model with different polynomials. Here $z[q]$ is the measured PA response, $y[q]$ is the PA model response. Let the decomposition level $n = 0$, $n = 1$ respectively, and compared with different polynomials such as conventional polynomials and orthogonal polynomials, the nonlinearity order of the PA model is obtained as shown in Fig. 4.

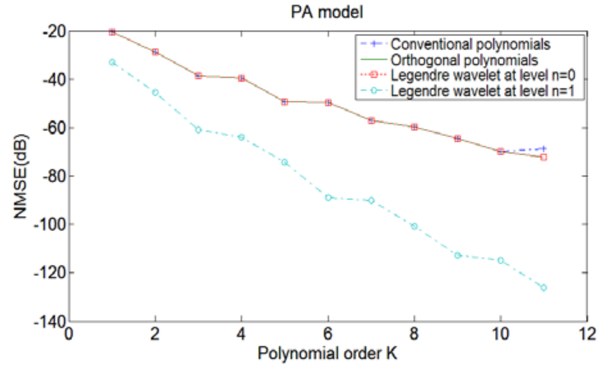

Fig.4. Estimate NMSE of the AM/AM conversion of the PA model

Fig.5 shows the AM/PM conversion of the PA with different polynomial order.

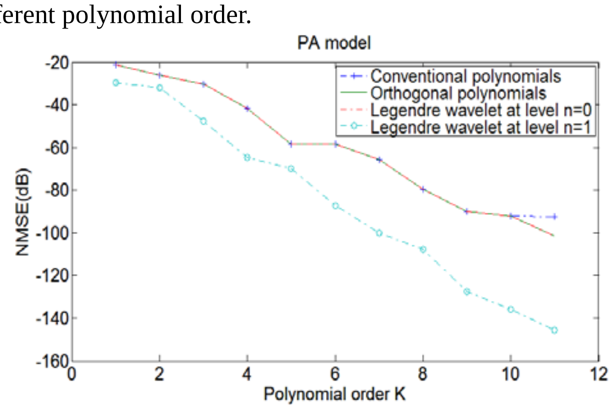

Fig.5. Estimate NMSE of the AM/PM of the PA model

Fig.4 and Fig.5 illustrate that the Legendre wavelet method for the pre-distortion is very efficient because of its lower order approximation than that of other polynomials. Furthermore, this technique offers approximations of the PA and its inversion at different level n of resolution. Now, we discuss the description of the inverse conversion of the SNL. In order to reduce computation load, an efficient method for the inversion model is obtained, when the input $x[q]$ and the output $y[q]$ are interchanged. Hence, the direction of the signal flow in the branch is reversed. The reader is referred to the literature for further details [9].

Fig.6 depicts the inversion of the PA model by using the Legendre wavelet and LS algorithm.

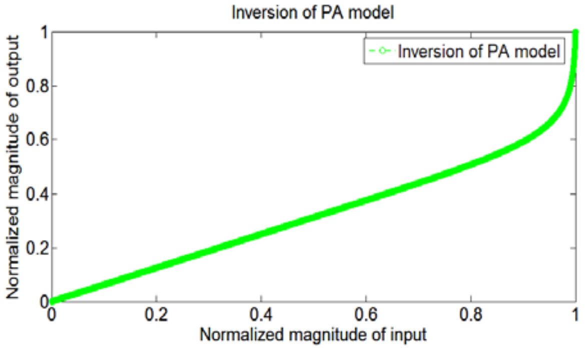

Fig.6. The inversion of the PA model

It is known that the pre-distortion technique involves the creation of an inverse characteristic complementary to the PA nonlinearity. Nevertheless, the behavior of the PA changes with time in practice. Therefore, the PD usually needs to be adjusted adaptively. Now, we turn to discuss the estimation algorithm of the inverse structure of the PA model, i.e., the Hammerstein model.

III. Digital Pr-distortion Using Legendre Wavelet And Pso

In Section II, we mainly focus on the representation and estimation of the SNL of the PA model and its inversion based on the Legendre wavelet base. The remaining task necessary to linearize the out of the PA is to find an adequate system, i.e., the PD which pre-processes the transmit signal in such a way that the overall response of the cascade PD-PA is linear. In this section, a complete digital pre-distortion linearization technique is developed based on the Legendre wavelet and the PSO algorithm.

A. Pre-distortion via indirect learning architecture

Most of the pre-distortion for PA linearization are based on indirect learning architecture, which is depicted in Fig.7 and more flexible and robust than direct learning architecture [11]. In addition, performances of the PA linearization can be improved by adding a feedback control to cope with external perturbations, parameter variations or operating frequency modifications.

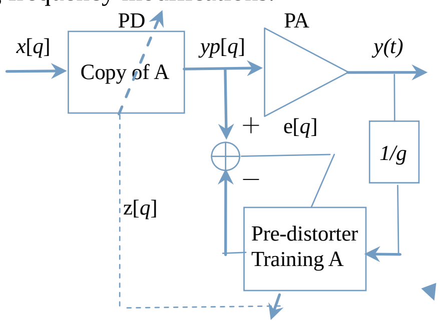

Fig.7. Pre-distortion architecture

In this section, we use this learning structure to adjust the parameters of the PD based on the inversion model proposed in Section II and PSO algorithm.

B. Learning behaviour of PSO algorithm

To estimate the parameters of the Hammerstein model in an optimal way, it is required to jointly optimize the coefficients of the inversion of the Wiener model. However, this minimum issue is not linear-in-parameters and the objective function most probably has several local minima. It is well-known that the PSO is a population-based stochastic searching technique developed by Kennedy and Eberhart [20]. Owing to its implementation simplicity and fewer adjustable parameters than the other global optimization algorithms, the PSO is an efficient approach to solving complex and large-scale problems. Especially, by using the PSO, the convergence can be accelerated and the risk of stalling at a local minimum can be reduced.

Thus, in this subsection, the PSO algorithm is adopted to find the optimal coefficients of the PD for system identifications. In the pre-distortion linearization, the goal is to minimize the error between desired outputs and trained outputs of the PA, and then the NMSE in (15) will be defined as the fitness function. With the $2^n p$ number of decision parameters of this optimal problem, the position and velocity of each particle i in the swarm are defined as $S_{i,knl} = (s_{i,1}, s_{i,2}, \cdots, s_{i,2^n p})$ and $V_i = (v_{i,1}, v_{i,2}, \cdots, v_{i,2^n p})$, respectively. The best previous position of each particle is defined as $P_i = (p_{i,1}, p_{i,2}, \cdots, p_{i,2^n p})$, and the global best position of all particles is represented as $P_g = (p_{g,1}, p_{g,2}, \cdots, p_{g,2^n p})$. Therefore, the velocity and position of each particle are updated as follows

$$V_i(j+1) = \omega(j)V_i(j) + c_1(j)r_1(P_i(j) - S_{i,knl}(j)) + c_2(j)r_2(P_g - S_{i,knl}(j)), \quad (16)$$

$$S_{i,knl}(j+1) = S_{i,knl}(j) + V_i(j+1), \quad (17)$$

where j is iteration step and $\omega(j)$ denotes the inertia weight at iteration j. In the above two equations $c_1(j)$ and $c_2(j)$ are cognitive parameter and social parameter at iteration j, respectively. Also r_1 and r_2 are two random numbers that are uniformly generated between 0 and 1.

In this context, the initial positions are based on the coefficients of the inversion of the PA model proposed in Section II. That is to say, the expansion coefficients of the inversion model are used as reference system for the adaptive system identification, i.e., the Hammerstein model. In addition, the velocities of all particles in the swarm randomly generate, respectively. Furthermore, in order to improve computational efficiency, a linearly adaptable inertia weight and linearly time-varying acceleration coefficients [20-22] over the evolutionary procedure of PSO algorithm are adopted. Firstly, the inertia weight ω starts with a high value $\omega_{\max}$ and linearly decreases to $\omega_{\min}$ at the maximal number of iterations and has the form

$$\omega(j+1) = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \cdot iter(j), \quad (18)$$

where $iter_{\max}$ is the maximal number of iterations (generations) and *iter* is the current number of iterations. Secondly, the cognitive parameter c_1 starts with a high value $c_{1\max}$ and linearly decreases to $c_{1\min}$, whereas the social parameter c_2 starts with a low value $c_{2\min}$ and linearly increases to $c_{2\max}$. Therefore, the acceleration coefficients $c_1(j+1)$ and $c_2(j+1)$ satisfy

$$c_1(j+1) = c_{1\max} - \frac{c_{1\max} - c_{1\min}}{iter_{\max}} \cdot iter(j), \tag{19}$$

$$c_2(j+1) = c_{2\min} - \frac{c_{2\min} - c_{2\max}}{iter_{\max}} \cdot iter(j). \tag{20}$$

Now we utilize the above PSO algorithm to identify the parameters of the PD. In each iterative step, the particles exchange information to update their movements toward the global minimum. After adaptation, the estimated parameters approximate the point-inverse of the PD are obtained. Compared with the conventional orthogonal polynomial method, Fig.8 shows the better approximation of NMSE by using the Legendre wavelet approach and the PSO algorithm at $iter_{\max} = 50$.

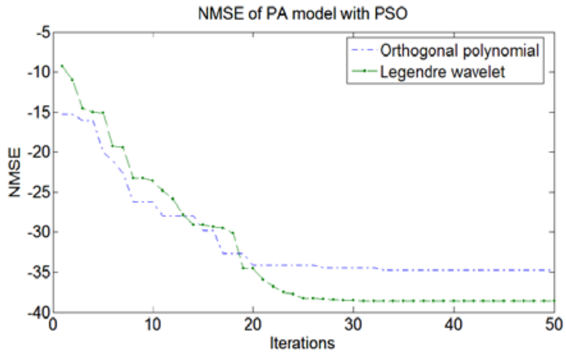

Fig.8. PD model with Legendre wavelet and PSO

It is pointed that the new technique proposed in this article enjoys inherent implementation/cost advantages over other PA linearization techniques discussed previously. The reasons are mostly in the rich properties of Legendre wavelet base and the strong optimal ability of the PSO algorithm.

C. Linearization performances

In this subsection, in order to evaluate the quality and the effectiveness of the proposed PA linearization technique in suppressing spectral regrowth, we compare the power spectral density of the PA output without linearization and with memory pre-distortion. The identification performance of the digital pre-distortion scheme based on the Legendre wavelet and the PSO methods is evaluated by measured I/O data signals. Fortunately, an efficient improvement of the PA linearization is obtained and the PSD both without and with the PD are illustrated in Fig.9, respectively.

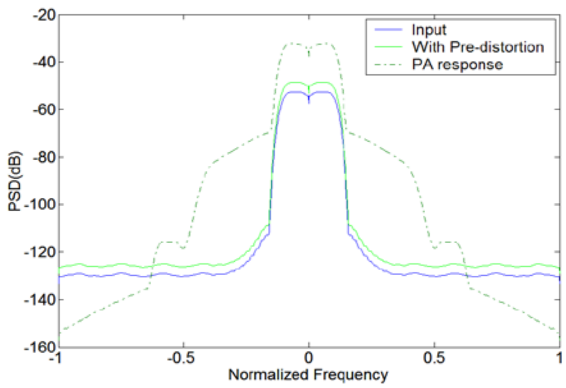

Fig.9. Comparison of the PSD with the Legendre wavelet method

In Fig. 9, the green line shows the linearized spectrum at the PA output. For comparison the PA response without pre-distortion is represented by the dashed line. It is remarkably suppressed with compensation by the proposed method to the level of -125 dB or less and reduced about 30dB, which shows that the Legendre wavelet and the PSO approaches have successfully compensated the nonlinearity of the PA system with high accuracy.

IV. CONCLUSION

In this paper, the PA model, i.e., the Wiener model with the LDS and the SNL and its inversion are firstly established to describe the behavior of the PA. Secondly, the Legendre wavelet and the PSO algorithm are applied to identify the PD and compared to other standard pre-distortion methods. Especially, it can be concluded from the above obtained results that the Legendre wavelet and the PSO methods require lower order adaptive polynomial approximation at different level of resolution and less computational complexity compared with other polynomial approaches. In another work, we shall describe the memory Legendre wavelet structures and realize the full potential of the Legendre wavelet and the PSO approaches.

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