# Smooth Blind Extraction Method for Harmonic Signals Hidden in Chaos Energy Accumulation Zone

XIAOZHEN LIU, YUANSHUO ZHENG, XINWU CHEN, ERFU WANG*, QUN DING

***Abstract*—**Put forward a method for blind separation mixed signals hidden in chaotic energy accumulation zone. Take different chaotic systems' wavelet time-frequency analysis for reference to determine amplitude and frequency of harmonic signal. Then use JADE algorithm for adding noise mixed-signal blind separation realizing the hidden harmonic signal blind extraction. Finally the simulation realizes doing smooth processing to extracted harmonic signal which restores harmonic signals in frequency domain and time domain, verifying The validity and the applicability of the method.

***Keywords*—**Chaos, harmonic signal, time-frequency analysis, energy concentration, extract, smooth.

## I. Introduction

CHAOS is very active in nonlinear science and has quite broad application prospects. Since 1970s, good results have been obtained by many scholars. Chaotic signal processing has made great progress in theory and been applied in many disciplines and engineering .Chaotic signal used in secure transmission can achieve encryption and masking effects for the transmitted signal. In the case of source signal and channel conditions are unknown, receiving end analyze the observed signal and make blind extraction and separation for desired signal hidden in the chaos. It is of great theoretical significance and practical value. Interference signals can be monitored which can be used as a means of verifying secure transmission capabilities of chaotic signal. Chaos has a wide spectrum similar with noise, so if we want to separate the desired signal hidden in the chaos effectively, the key is to find an effective basis to distinguish between chaos and noise. For the actual transmission of multiple antenna systems and channels with additive noise, blind separation model for channel plus noise is presented in this paper.

Scholars has done many researches about chaos blind separation and noise added blind separation at home and abroad, such as extraction of fetal ECG from mother and extraction of small targets under the sea clutter signals. Empirical modal analysis and wavelet transform are used to implement blind separation of harmonic signals under chaotic background and so on [1-5]. Several good results have been obtained in this field internally. Cheng Xiefeng [6] proposed empirical mode analysis for chaotic signals (EMD) applying intrinsic mode functions to chaos signal as noise in order to achieve the extraction of chaotic signals.

Chaotic wavelet analysis, time-frequency domain characteristics analysis for interference signal and harmonic signal are mentioned. Put forward a method for harmonic signals extraction based on characteristic matrix joint approximate diagonalization (JADE). Compared with previous studies, it has a breakthrough that it can achieve harmonic signals extraction. which hidden in hidden in the chaotic energy in positive definite systems of noisy channel.

## II. Chaotic Signal

Set Liu and Lorenz chaotic system based on three-dimensional as examples to simulate and analyze. Lorenz chaotic system [7] is a classical chaotic system to implement easily. Its mathematical model is made up of three first order differential equations, expressed as follow:

$$\begin{cases} \dfrac{dx}{dt} = a(y - x) \\ \dfrac{dy}{dt} = cx - xz - y \\ \dfrac{dz}{dt} = xy - bz \end{cases} \tag{1}$$

Where $a, b, c$ are parameters, when they are selected as $a = 10, b = 8/3, c = 28$, Lorenz system is in a state of chaos, chaotic attractor of system is shown in Fig.1.

Liu chaotic system [8] is a chaotic system with quadratic nonlinearity. There is a square in its mathematical model, which

This work was supported by the National Natural Science Foundation of China under Grant No. 61571181(sponsor and financial support acknowledgment goes here).

Xiaozhen Liu. Author is with Electronic Engineering College, Heilongjiang University, Harbin, 150080 China (e-mail: 15046658153@163.com).

Yuanshuo Zheng. Author is with Electronic Engineering College, Heilongjiang University, Harbin, 150080 China (e-mail: 1248708457@qq.com).

Xinwu Chen. Author is with Electronic Engineering College, Heilongjiang University, Harbin, 150080 China (e-mail: 271819833@qq.com).

Erfu Wang*. Author is with Electronic Engineering College, Heilongjiang University, Harbin, 150080 China (corresponding author to provide phone: 86-137-6680-9595;e-mail: efwang_612@163.com).

Qun Ding. Author is with Electronic Engineering College, Heilongjiang University, Harbin, 150080 China (e-mail: qunding@aliyun.com).

is different from other chaotic systems. Differential equations for its mathematical model is :

$$\begin{cases} \dfrac{dx}{dt} = a(y - x) \\ \dfrac{dy}{dt} = bx - kxz \\ \dfrac{dz}{dt} = -cz + hx^2 \end{cases} \tag{2}$$

Where $x, y, z$ are three variables, $a, b, c$ and $k$ are system parameters. When set $a = 10, b = 40, c = 2.5, k = 1, h = 4$ , Liu system is in chaos State. Chaotic attractors is shown in Fig.2.

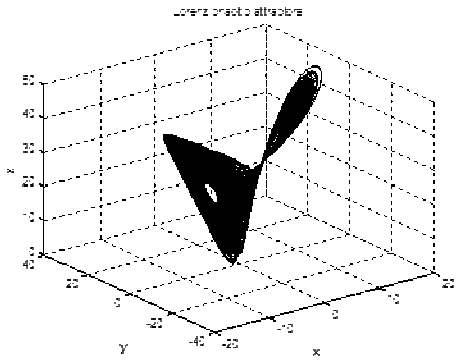


Fig.1 Chaotic attractors of Lorenz system

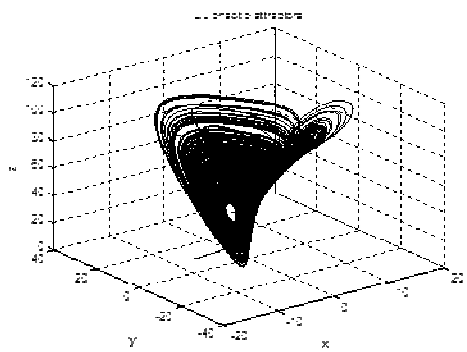


Fig.2 Chaotic attractors of Liu system

## III. HIDE HARMONIC SIGNALS

### A. Hide harmonic signals

In order to implement the extraction of harmonic signals hidden in chaos system, harmonic signals and interference signal are selected by the time-frequency analysis of chaotic characteristics. The waveforms of the two chaotic signals are shown in Fig.3 and Fig.4.

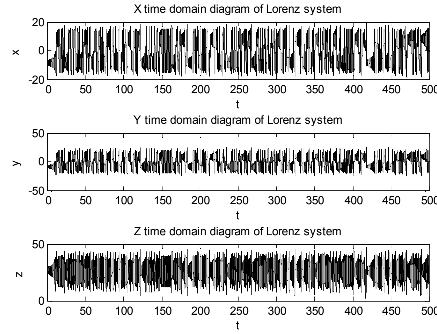


Fig.3 Three-way waveform diagrams of Lorenz chaotic system

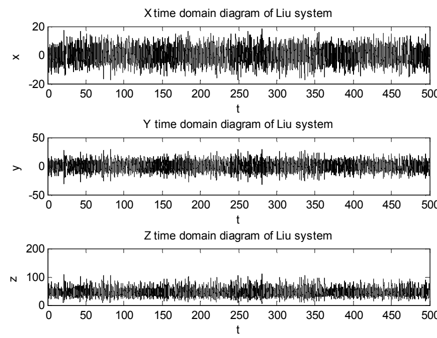


Fig.4 Three-way waveform diagrams of Liu chaotic system

When choose harmonic signal, we need to refer to wavelet transform of the chaotic signal. Wavelet transform distributes n sampling points on the x axis on both sides evenly, but just positive X axis used in it. The wavelet transform of the two chaotic signal is shown in Fig.5 and Fig.6. Experience has shown that if we want hide harmonic signal in the chaotic energy band, the frequencies and amplitudes of harmonic signals need some sort of limit.

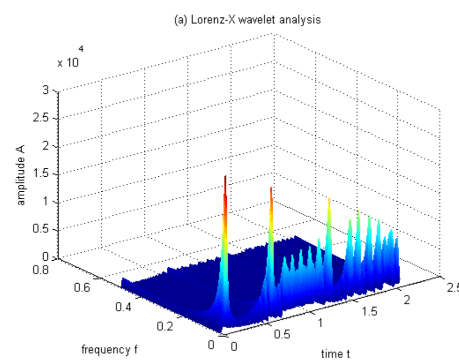


Fig.5 path wavelet transform of Lorenz chaotic system

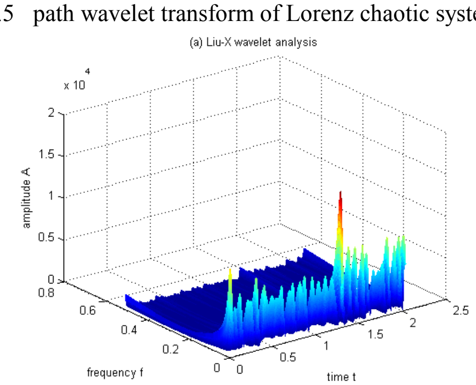

Fig.6 path wavelet transform of Liu chaotic system

Analysis wavelet transform of chaotic signal to select harmonic signal. Then it can be hidden in the chaotic energy accumulation zone. There are two aspects to consider. First is amplitude. Observe the height of chaos energy accumulation zone to determine the amplitude of the harmonic signal making sure it is below the height of chaotic energy. The next one is frequency range. Calculating the chaos energy accumulation zone determines the frequency of harmonic signals in the frequency range of chaos energy (energy accumulation zone of Lorenz chaotic is 0-60 Hz, while Liu chaotic is 0-50 Hz). Judging by these two aspects makes the harmonic signal successfully hidden. Meanwhile, add a BPSK signal as a interference signal to strengthen the algorithm performance. We can select BPSK the same way as harmonic signal.

In this paper, algorithm is proposed to find a way to extract harmonic signal hidden in the blindly. Three signals exist: harmonic signal, chaotic signal, interference signal(BPSK). The mixed signals transferred through unknown channel and at the receiving end data obtained which can be separated to complement algorithm. The main information of harmonic signal information is mainly reflected in the frequency. So as long as extract the proper frequency of harmonic signal which is consistent with the source signal, which can prove algorithm is feasible. Effect of chaotic signal in the algorithm must be considered according to its spectrum is completely submerged in the spectrum of the noise signal. So it is hard to solve this problem through spectral domain. In the practical transmission process, the noise in the channel is not negligible. The impact of noise on the signal extraction is given in the following article.

### *B. Noise plus interference proof*

Some literature[9,10] has shown that additive white Gaussian noise signal has a additive effect on extraction. There is a certain amount of noise attached to the extracted signals. Concrete evidence is as follows：

Assume variance of the stationary additive white Gaussian noise is $\sigma^2$ , blind source separation models are :

$$X(t) = AS(t) \tag{3}$$

$$X(t) = AS(t) + N(t) \tag{4}$$

The separation matrix of no noise model is $W$ , the equation (6) is multiplied by the matrix on both sides

$$WX(t) = WAS(t) + WN(t) \tag{5}$$

The first item on the right side is the same as separation signal noise-free, then take the second into consideration, $W$ is also equal

$$W = UQ = U\Lambda_s^{-\frac{1}{2}}V_s^T \tag{6}$$

Where $U$ is orthogonal separation matrix, $\Lambda_s^{-\frac{1}{2}}V_s^T$ is prewhitening matrix's orthogonal solution, $\Lambda_s$ , $V_s$ are eigenvalue diagonal matrix and the corresponding eigenvector matrix of the observed signal covariance matrix $R_{xx}$ . $Wn(t)$ is

$$WN(t) = U\Lambda_s^{-\frac{1}{2}}VN(t) \tag{7}$$

Its covariance matrix is

$$E\left\{WN(t)\left[WN(t)\right]^T = U\Lambda_s^{-\frac{1}{2}}V_s^T E\left\{N(t)N(t)^T\right\}V_s\Lambda_s^{-\frac{1}{2}}U^T\right\} \tag{8}$$

Due to $E\left\{N(t)N(t)^T\right\} = \sigma^2 I$ , $V_s$ is column orthogonal, $U$ is orthogonal matrix. Let

$$\Lambda_s^{-\frac{1}{2}}\Lambda_s^{-\frac{1}{2}} = \sigma_s^2 I \tag{9}$$

Then

$$E\left\{WN(t)\,[WN(t)\,]^T\right\} = \left(\frac{\sigma^2}{\sigma_s^2}\right) I \tag{10}$$

Equation (10) shows $WN(\mathrm{t})$ , whose respective components is not relevant, is linear combination of component of $N(t)$ . By the Central limit theorem, we know that $WN(\mathrm{t})$ tends to be the Gaussian distribution. Therefore each component is independent.

According to the above analysis conclusion, when source signal is independent, the noise can not be ignored, separate source signals may be obtained using separation matrix produced by no-noise source separation algorithm. Because of the separate signals is correct separation signal superimposed by a certain variance of Gaussian white noise, the performance is not very good. This paper also gives a way to solve this problem.

### *C. Smooth signal*

The initialization source signal will be data packaged and transfer in the unknown channel and contaminated by additive white Gaussian noise. Receiver will observe mixed signal which will be extracted blindly to obtain harmonic signal. The algorithm used in this paper is called JADE. Do Fast Fourier transform (FFT) on extracted signal vector, judgethe size of the correlation coefficient between transformed signal and source signal. When the correlation coefficient is less than 0.9, we can make the extracted signal into smoothing processing which can remove the burrs caused by additive noise interference on the signal, reduce the effect of noise and extract frequency information accurately. Then restore original signal in both frequency domain and time domain.

After the analysis, we get the mathematical model for the algorithm:

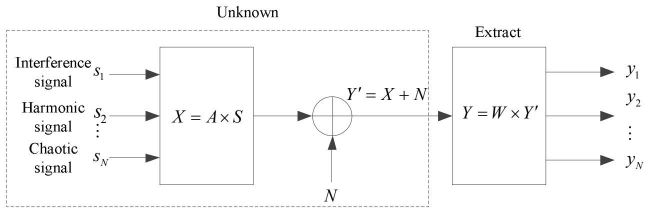


Fig.7 Mathematical model for the system

Harmonic signals' (the desired signal) extraction belongs to the blind extraction. This channel plus noise model of subdefinite model approximates to the actual situation. Instantaneous mixture model for the blind source extraction is:

$$Y(\mathrm{t}) = A \times S(t) + N(t) \quad (11)$$

Where $A_{n\times n}$ is unknown channel mixing matrix, which is randomly generated. $S(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T$ represents unknown source signals consisting of vectors（in this paper $N = 3$ ). $N(t) = [n_1(t), n_2(t), \cdots, n_N(t)]^T$ represents the additive white Gaussian noise channel vector. Separation model can be expressed as

$$Y = W \times Y' = W \times A \times S \quad (12)$$

$W_{n\times n}$ is the receiver separation matrix, $Y(t) = [y_1(t), y_2(t), \cdots, y_N(t)]^T$ represents the vector of observed signals at the receiver. Blind extraction aims at looking for estimation for harmonic signals from the observed signal.

Therefore, the flow chart of this algorithm is shown in Fig.8.

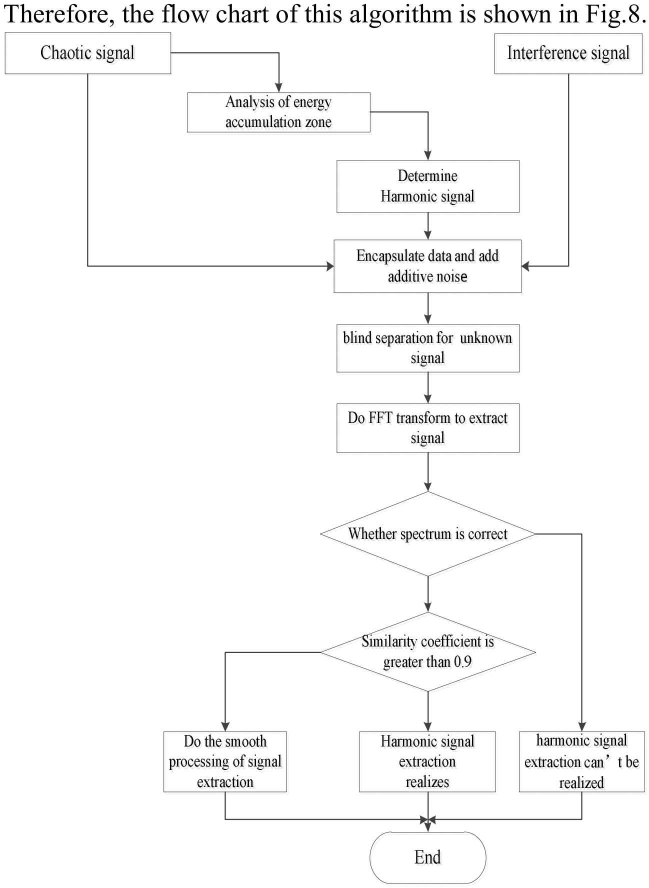


Fig.8 Algorithm flow chart

## IV. Simulation and Performance Analysis

The simulation of the algorithm is mainly in two aspects. The first is when the three signals on the sending end are harmonic (sinusoidal) signal, interference (BPSK) signal and chaotic signal. The main purpose is to determine whether the algorithm can extract harmonic signal hidden within energy accumulation band effectively. The second is when there exists two harmonic signals without interference signal to determine the minimum frequency space that can realize extraction effectively. In general, the mathematical expression for harmonic signal is as follows:

$$S_{\sin}(t) = A_0 \sin(2\pi f_0 t + \varphi) \quad (13)$$

Where $A_0$ is the amplitude of harmonic signal, $f_0$ is the frequency of harmonic signal, $\varphi$ is the initial phase of the harmonic signal (the simulation is set to be zero), mathematical expression of interference signal (BPSK) is:

$$S_{BPSK}(t) = A[\sum_n a_n g(t - nT_s)] \cos \omega_c t \quad (14)$$

Where $A$ is amplitude of the carrier signal, $a_n = \begin{cases} 1 \\ -1 \end{cases}$ is pseudo random sequence, $g(t)$ is unit mode function when door width is $T_s$ . $\omega_c$ is carrier frequency. Setting parameters of one or two signal should base on chaotic signal and specific parameters will be given in the following. The third is three-dimensional chaotic signal's $X$ path component and mathematical expressions and parameter have been given, no further explanation.

The other is the need to identify the channel mixing matrix. Because the performance of blind separation algorithm is not under the influence of channel mixing matrix, thus randomly generate a full rank matrix as the channel matrix. Full rank matrix is $rank(A) = n$ . $n$ is determined by the number of source signals. If there are three input source signals, then $n = 3$ . Channel mixing matrix is

$$A = \begin{bmatrix} 0.0326 & 0.5442 & 0.2423 \\ 0.5525 & 0.0886 & 0.0616 \\ 0.1006 & 0.2916 & 0.5505 \end{bmatrix}$$

Following the case of Lorenz chaos as detailed analysis the algorithm of extracting results.

### *A. Lorenz chaotic system*

Simulation first: through the wavelet transform analysis of Lorenz chaotic system's $X$ component, set the frequency of harmonic signal to be $f_0 = 10$ Hz, $A = 20$ . The carrier frequency of the interference signal BPSK is $\omega_c = 10$ Hz. The sampling frequency is 65 times the carrier frequency, the carrier amplitude is $A = 25$ . The waveforms and spectrums of interference signal (BPSK) and harmonics signal are shown in Fig.9 and Fig.10. Set channel noise variance $sgma = 2$ as example (range of SNR in voice communication is 10-20dB , when $sgma = 2, SNR = 17dB$ , ) , there are three source signals, channel matrix is $n = 3$ .Waveform and spectrum of extracted and smoothed harmonic signal in JADE algorithm are shown in Fig.11 and Fig.12.

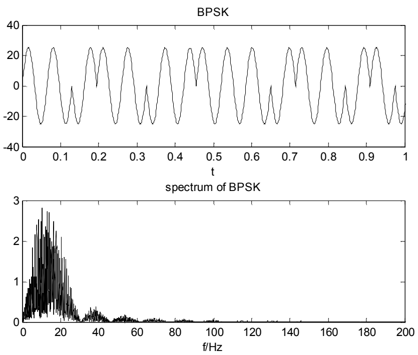


Fig.9 Spectrum of interference signal

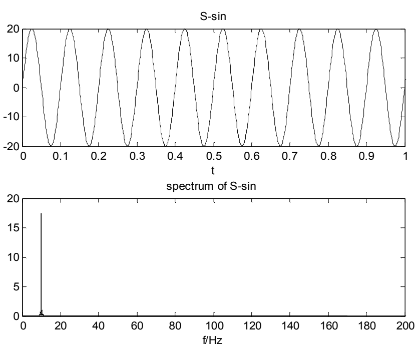


Fig.10 Spectrum of harmonic signal

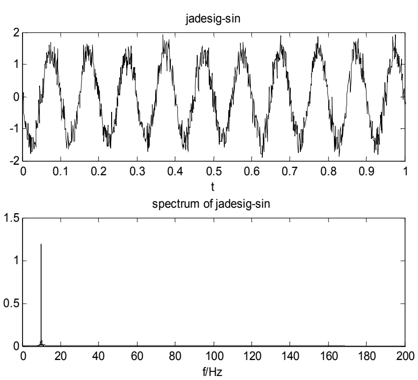


Fig.11 Spectrum of extracted harmonic signal

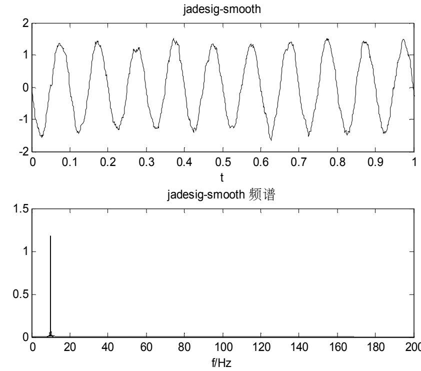


Fig.12 Spectrum of the smoothed harmonic signal

In simulation, harmonic signal extracted by JADE algorithm, there are serious glitches in extracted signal before smoothing, which verifies the additive white Gauss noise influences harmonic signal and the effect is additive. But the spectrum analysis of extracted signal can determine the frequency of the signal is consistent with harmonic signal. Smoothed harmonic signal can reflect the original harmonic signal which proves the algorithm can effectively implement blind extraction harmonic signal hidden in the chaotic energy band.

Simulation second: source signal consists of two harmonic signals and chaotic signal. Two harmonic signals are $s_1(t) = A\sin(2\pi f_1)$ , $s_2(t) = A\sin(2\pi f_2)$ , and $f_2 = f_1 + \Delta f$ .According to Lorenz chaotic spectrum analysis determines frequency of harmonic signal is within the range of 0-60Hz. $A = 20$ and $f_1 = 10$ Hz, which is at the peak position of the chaotic signal spectrum, and take the arbitrary channel noise variance to be $sgma = 2$ . Three signals' mixed order are $s_1(t), s_2(t)$ , chaos and channel mixing matrix $n = 3$ . Simulate under different $\Delta f$ , we get the following data table:

Table 1 data before smoothing

Table 1 data before smoothing

| Frequency interval $\Delta f$ /Hz | Before smoothing | After smoothing |
|---|---|---|
| 5 | $\begin{bmatrix} 0.9676 & 0.0049 & 0.0146 \\ 0.0031 & 0.9489 & 0.1140 \\ 0.0049 & 0.0086 & 0.8313 \end{bmatrix}$ | $\begin{bmatrix} 0.9959 & 0.0050 & 0.0152 \\ 0.0033 & 0.9870 & 0.1190 \\ 0.0058 & 0.0101 & 0.9730 \end{bmatrix}$ |
| 2 | $\begin{bmatrix} 0.9668 & 0.0059 & 0.0170 \\ 0.0005 & 0.9491 & 0.1258 \\ 0.0001 & 0.0078 & 0.8264 \end{bmatrix}$ | $\begin{bmatrix} 0.9959 & 0.0061 & 0.0178 \\ 0.0005 & 0.9869 & 0.1301 \\ 0.0001 & 0.0093 & 0.9727 \end{bmatrix}$ |
| 1 | $\begin{bmatrix} 0.9668 & 0.0081 & 0.0131 \\ 0.0062 & 0.9489 & 0.1497 \\ 0.0062 & 0.0001 & 0.8247 \end{bmatrix}$ | $\begin{bmatrix} 0.9960 & 0.0111 & 0.0205 \\ 0.0034 & 0.9869 & 0.1468 \\ 0.0003 & 0.0102 & 0.9719 \end{bmatrix}$ |

We can see from the table, when Smoothing factor is consistent, the effect of extraction doesn't have a big difference between $\Delta f = 1{,}2$ Hz and 5Hz. And if we can regulate the smoothing factor, this difference can be smaller. When $\Delta f \geq 5$ Hz, smoothing the two extracted signals can effectively reflect characteristic of harmonic signal.

### *B. Liu chaotic system*

Based on the wavelet transform analysis of Liu chaotic system，the frequency of harmonic signal is set $f_0 = 12$ Hz, amplitude $A_0 = 20$ , carrier frequency of interference signal(BPSK) is $\omega_c = 12$ Hz and the sampling frequency is 60 times the carrier frequency and carrier amplitude is .Simulate after determining the parameter settings, we can get simulation map as below. Waveform and spectrum analysis of the interference signal and harmonic signal are shown in Fig.13 and Fig.14. Fig.15 is waveform and spectrum analysis diagram for separation of JADE. Fig.16 is separated signal's (after smoothing) waveform and spectrum analysis diagram which in JADE.

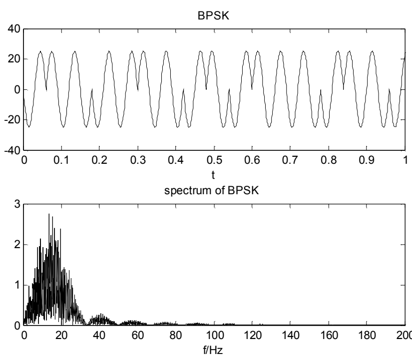


Fig.13 Waveform and spectrum of the interference signal

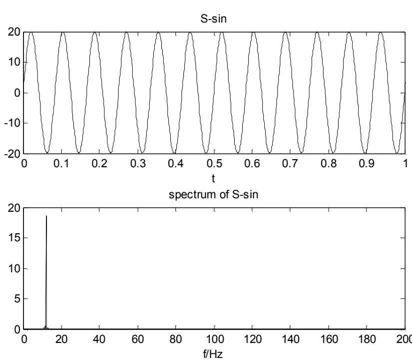


Fig.14 Waveform and spectrum of the harmonic signal

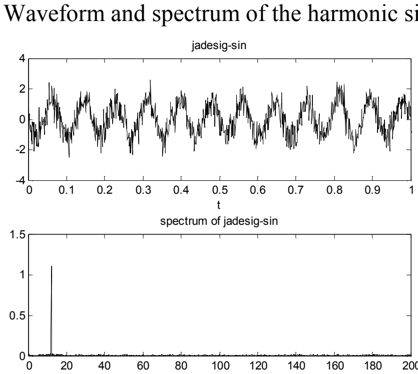


Fig.15 waveform and spectrum of extracted harmonic signal

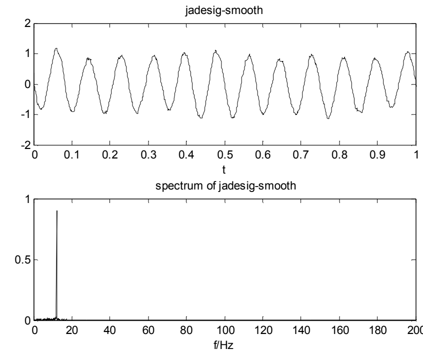


Fig.16 Waveform and spectrum of the smoothed harmonic signal

It can be seen from the simulation diagram, additive white Gauss noise has a additive influence on extracted harmonic signal, which is consistent with Lorenz chaotic system. It can be seen from the smooth waveform that it is necessary to smooth the extracted waveform.

Set channel's SNR $\mathrm{sgma} = 2$ as an example, $f_1 = 36$ Hz, $f_2 = f_1 + \Delta f$ (mixed in the same order as Lorenz simulation II), analyze the minimum frequency spacing of harmonic signal and a table obtained:

The similarity coefficient in Liu chaotic system proves the conclusion obtained in Lorenz chaotic system.

Table 2 data before smoothing

Table 2 data before smoothing

| Frequency interval $\Delta f$ /Hz | Before smoothing | After smoothing |
|---|---|---|
| 5 | $\begin{bmatrix} 0.9677 & 0.0058 & 0.0581 \\ 0.0049 & 0.9552 & 0.1795 \\ 0.0004 & 0.0007 & 0.6968 \end{bmatrix}$ | $\begin{bmatrix} 0.9959 & 0.0059 & 0.0591 \\ 0.0051 & 0.9856 & 0.1828 \\ 0.0005 & 0.0010 & 0.9368 \end{bmatrix}$ |
| 2 | $\begin{bmatrix} 0.9671 & 0.0093 & 0.0594 \\ 0.0021 & 0.9552 & 0.1746 \\ 0.0031 & 0.0038 & 0.6955 \end{bmatrix}$ | $\begin{bmatrix} 0.9958 & 0.0096 & 0.0605 \\ 0.0022 & 0.9863 & 0.1762 \\ 0.0043 & 0.0053 & 0.9356 \end{bmatrix}$ |
| 1 | $\begin{bmatrix} 0.9675 & 0.0084 & 0.0625 \\ 0.0027 & 0.9551 & 0.1595 \\ 0.0060 & 0.0007 & 0.6984 \end{bmatrix}$ | $\begin{bmatrix} 0.9959 & 0.0086 & 0.0635 \\ 0.0028 & 0.9860 & 0.1599 \\ 0.0084 & 0.0010 & 0.9380 \end{bmatrix}$ |

In theory, additive white Gauss noise has a additive influence on extracted signal. We will quantitatively describe the noise variance change effects on similar coefficient. Specific data are shown as below:

Table 3: similarity coefficient of extracted signals of two chaotic systems with different $sgma$

| | Before smoothing | | After smoothing | |
|---|---|---|---|---|
| *sgma* | Lorenz | Liu | Lorenz | Liu |
| 2 | 0.9670 | 0.9672 | 0.9958 | 0.9958 |
| 3 | 0.9310 | 0.9309 | 0.9912 | 0.9902 |
| 4 | 0.8852 | 0.8869 | 0.9844 | 0.9832 |
| 5 | 0.8361 | 0.8381 | 0.9739 | 0.97613 |

From the table we can see, the similarity coefficient is greatly improved after smoothing than before, which proves that after smoothing the waveform has a great improvement and is more similarer to harmonic signal. As channel noise variance increases, similar coefficient of extracted harmonic signals becomes smaller, which means the validity of the extracted signal is affected by the impact of channel conditions. In addition, the difference of similarity coefficient between Lorenz and Liu is quite small, which implies different chaotic systems have little impact on harmonic signal extraction.

## V. Conclusion

In this paper, we have realized extracted harmonic signal hidden in chaotic energy accumulation zone in noisy channel and positive definite system. The next step is to realize blind extraction harmonic signal and related signal in underdetermined system and realize blind extraction for multiple harmonic signals in the short term to provide effective separation method for speech secure communication.

## References

[1] Schreiber T and Kaplan D T, Signal separation by nonlinear projections: The fetal electro- cardiogram [J]. Phys, Rev. E. 53 R4326, 1996.
[2] M Richter, T Schreiber and DT Kaplan, Fetal ECG Extraction with Nonlinear State-Space Projections. IEEE Transactions on Biomedical Engineering, Vol.45, No.1, pp. 133-137.
[3] Haykin S, Li X B, Detection of signals in chaos [J]. Proceedings of IEEE, Vol.83, No.1, 1995, pp. 94-122.
[4] E F Wang, D Q Wang, Q Ding, Harmonic signal extraction from chaotic secure communication system of joint time-frequency domain method [J]. Journal of Communication, Vol.32, No.12, 2011, pp. 60-64.
[5] Araki S, Sawada H, Mukai R, Underdetermined blind sparse source separation for arbitrarily arranged multiple sensors. Signal Process, Vol.87, 2007, pp. 1833-1847.
[6] X F Cheng, L Xu, R Y Yan, Research of chaotic mixed-signal underdetermined blind source separation method. Journal of Nanjing University of Posts and Telecommunications, Vol.30, No.1, 2011, pp. 29-34.
[7] Lorenz, Edward Norton, Deterministic non-periodic flow [J]. Journal of the Atmospheric Sciences, Vol.20, No.2, 1963, pp. 130-141.
[8] C X Liu, T Liu, L Liu, A new chaotic attractor [J]. Chaos Solit Fract, Vol.22, No.1, 2004, pp. 31-103.
[9] W D Jiao, S X Yang, S T Wu, Study of noise removal technique based on independent component analysis [J]. Journal of Zhejiang University, Vol.38, No.7, 2004, pp. 872-876.
[10] Q Xiang, C S Lin, J F Cheng, Algorithm of blind source separation in noisy backgrounds [J]. Data acquisition and processing, Vol.21, No.1, 2006, pp. 42-45.