Cyclic Spectrum Reconstruction from sub-Nyquist Sampling with Dual Sparse Constraint

Xushan Chen, Jibin Yang, Xiongwei Zhang, and Jianfeng Li

***Abstract*—**For cyclostationary signal processing, cyclic spectrum reconstruction is an important and challenge task for efficient and robust spectrum sensing, since cyclic feature detection takes advantage of spectral correlation characteristics to identify signal parameters and make reliable spectrum access decisions under uncertain noisy environments, but requires high complexity sampling and computational cost especially for wideband spectrum sensing. This paper proposes a novel cyclic spectrum reconstruction approach by exploiting the intrinsic sparsity and the differential spectral domain sparsity of the two dimensional cyclic spectrum of communication signals. Based on the linear relationship between the time-varying data covariance of the compressive samples and the unknown cyclic spectrum, the proposed estimator utilizes the symmetry property of the cyclic spectrum to improve the smoothness of the recovered cyclic spectrum and then to detect the spectrum occupancy accurately and robustly. Simulation results demonstrate the effectiveness of the developed estimator under different compression ratio cases.

***Keywords*—**Cyclostationary Signal Processing, Cyclic Spectrum, Sub-Nyquist Sampling, Cyclic Feature Detection, Differential ℓ_1 minimization.

I. INTRODUCTION

FOR most manmade signals, such as communication, radar, remote sensing and sonar systems, some parameters vary periodically with time, even under certain circumstances contain multiple incommensurate periods. For example, sinusoidal carries in amplitude, phase and frequency modulation systems, or periodic keying of the amplitude, phase and frequency in digital modulation systems. The case that the statistical parameters vary in time with single or multiple periodicities can be interpreted as cyclostationary. The spectral lines exists when quadratically transforming the cyclostationary signals, which means that the autocorrelation function fluctuates with time periodically. This spectral correlation property, as well as spectral redundancy, reflects the fundamental characteristic of cyclostationary signals and describes the correlation exists between spectral components of the random signals in different spectral bands [1]. It is important to exploit the inherent spectral redundancy to perform various signal processing task including detecting the presence of signals corrupted by noise and interference, identifying the modulation and waveform patterns, estimating parameters such as carrier frequency, symbol period and occupied frequency bandwidth.

In this paper we aim to develop cyclic spectrum recovery techniques that are efficient in computation and feasible in practical implementation. The cyclic spectrum is able to separate multiple signals that have the same carrier frequency, keying frequency or symbol rate, which means they occupy identical band in time or frequency domain while there is no overlapping between the spectral correlation density functions (SCD) or, equivalently, cyclic spectrum of these signals. Compared to the conventional power spectrum estimation that ignore cyclostationary, the advantages of cyclic spectrum are in the following aspects:

- Reliable anti-interference ability. The power spectrum of signals and noise/interference may overlap each other in the frequency domain, while these spectral components can be successfully separated in the cyclic frequency domain. For example, Gaussian noise exhibits nonzero entries only at $\alpha = 0$, that is, the noise cannot affect the signal components at $\alpha \neq 0$.

- A good sense of identification. The distinct spectral correlation characteristics of different signals are easily distinguishable on two dimensional cyclic spectrum. For example, the magnitude of the SCD for a pulse-amplitude modulated (PAM) signal exhibits peaks at k/T_c, $k = 0, \pm1, \pm2, \cdots$, and the SCD for a binary phase-shift keyed (BPSK) signal contains periodic peaks at $\pm 2f_0 + (k/T_c)$, $k = 0, \pm1, \pm2, \cdots$.

- Abundant parameter information. Cyclic spectrum is obtained by measuring the temporal correlation of the filtered signals, which is identical to the spectral density of correlation at frequencies $f + \alpha/2$ and $f - \alpha/2$. Therefore cyclic spectrum contains more information including amplitudes and phases of the spectral components of the signals in order to perform multi-parameter estimation task.

Although cyclic spectrum is robust to uncertain noise and interference, and outperforms under low signal-to-noise ratio (SNR) conditions, and can identify various modulation types, it requires a sampling rate that is higher than Nyquist rate to

This work was supported in part by the National Natural Science Foundation (No. 61471394, 61402519) and the Natural Science Foundation of Jiangsu Province, China (No. BK20140071, BK20140074).

Xushan Chen and Jianfeng Li are with People's Armed Police Institute of Politics, Shanghai, 201703 China (corresponding author to provide phone: +86-15618573363; e-mail: chenxushan87@ 163.com).

Jibin Yang and Xiongwei Zhang are with PLA University of Science and Technology, Nanjing, Jiangsu 210007 China. (e-mail: yjbice@ sina.com).

induce cyclostationarity and also needs a long observation time to acquire reliable cyclic statistics [1][2]. Especially when the bandwidth of the signals becomes much wider from 300 MHz to several GHz, the wideband receivers require ultra-high analog-to-digital (A/D) sampling rate, in combination with large computation and storage cost, these drawbacks limit the dynamic range of the receivers in practical applications [3]. However, due to low simultaneous occupancy utilization in contrast to the whole spectrum width [4], it is able to exploit the sparsity property of the cyclic spectrum of communication signals in order to not only significantly reduce the sampling rate but also robustness against uncertain noise and low SNR conditions.

A series of research of wideband spectrum sensing were presented in [5][6][7][8], where the authors focused on determining spectrum occupancy by spectrum estimation first. These methods are still be affected by the spectrum estimation accuracy that is easily influenced by noise uncertainty. Accordingly, compressive sampling based cyclic spectrum reconstruction techniques have been developed, which directly and effectively recovery the sparse cyclic spectrum from sub-Nyquist samples. The work of [9] expressed the linear relationship between the correlation matrix of non-uniform-sampled samples and the signal correlation matrix. The authors exploited the block Toeplitz structure of the wide sense cyclostationary signal correlation matrix so that the linear relationship can be presented as an overdetermined system. The innovation of [10] was to derive a method for cyclic spectrum reconstruction for two different sparse multiband signals sampling schemes: multicoset sampling [11] and modulated wideband converter [12], and provided the minimal sampling rate allowing for perfect reconstruction for both sparse and non-sparse signals. A transformed linear system has been established in [13] to connect the time-varying cross-correlation of compressive measurements with the cyclic spectrum. Furthermore in some special cases this method still maintained valid without taking any sparsity constraint into account. Another similar study was presented in [14]. The authors also formulated the time-varying cross-correlation of compressive samples as a function of the cyclic spectrum, and took advantage of the folded replicable property of the cyclic spectrum of the digital samples to acquire the original cyclic spectrum.

Compared to prior work, our contribution is essentially to propose a cyclic spectrum estimator by providing an additional differential sparsity constraint as well as the unique sparsity property of the cyclic spectrum. Based on the linear relationship between the compressive measurements and the cyclic spectrum in [14], the cyclic spectrum can be feasibly solved via convex ℓ_1 norm minimization. The two terms of sparsity constraints represent the intrinsic sparsity and smoothness of the optimizer, respectively. Afterwards, the spectrum occupancy can be detected from the recovered cyclic spectrum by using a multi-cycle generalized likelihood ratio test (GLRT) [15].

II. Preliminaries and Problem Formulation

A. Review Stage

Consider k active primary users signals over a wide band in the frequency range $[-f_{\max}, f_{\max}]$, we have

$$x(t) = \sum_{i=1}^{k} x_i(t) + n(t) \tag{1}$$

where $x_i(t)$ denotes the i th signal, $i = 1, \cdots, k$, $n(t)$ is independently and identically distributed zero-mean Gaussian noise with noise variance σ^2.

According to the definition of the cyclic autocorrelation function $R_x(\alpha,\tau) = \int x(t+\tau/2)x^*(t-\tau/2)e^{-j2\pi\alpha t}dt$, the cyclic spectrum is the Fourier transform of $R_x(\alpha,\tau)$ with respect to the time-delay τ

$$S_x(\alpha, f) = \int R_x(\alpha,\tau)e^{-j2\pi f\tau}d\tau \tag{2}$$

where α denotes the cyclic frequency and f represents the frequency. The sequence of digital samples of $x(t)$ is defined as $\{x(nT_s): n = 0, \pm1, \pm2, \cdots\}$, where T_s is the periodic time sampling interval, $T_s = 1/f_s$, f_s denotes the sampling rate. Therefore the power spectral density $\tilde{S}_x(f)$ of $x(nT_s)$ is related to the power spectral density $S_x(f)$ of the waveform by the aliasing formula [1]

$$\tilde{S}_x(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} S_x\left(f - \frac{n}{T_s}\right) \tag{3}$$

This aliasing formula can be generalized to cyclic spectrum [1] as follows

$$\tilde{S}_x^{\alpha}(f) = \frac{1}{T_s}\sum_{m,n=-\infty}^{\infty} S_x^{\alpha+m/T_s}\left(f - \frac{m}{2T_s} - \frac{n}{T_s}\right) \tag{4}$$

It is known that the support regions in the bifrequency for the cyclic spectrum that is aliased by periodic time sampling consist of multiple diamond-shaped regions. As is shown in Fig. 1, the cyclic spectrum exhibits nonzero entries only for $|f| + |\alpha/2| \le f_{\max}$, while the cyclic spectrum of digital samples contains multiple folded replicas of the original cyclic spectrum [14], Fig. 1 shows the folded cyclic spectrum within $0 \le \alpha, f \le f_s$. Therefore we can unfold the original cyclic spectrum from the folded cyclic spectrum of digital samples by mapping the subregions I, II and III into corresponding positions.

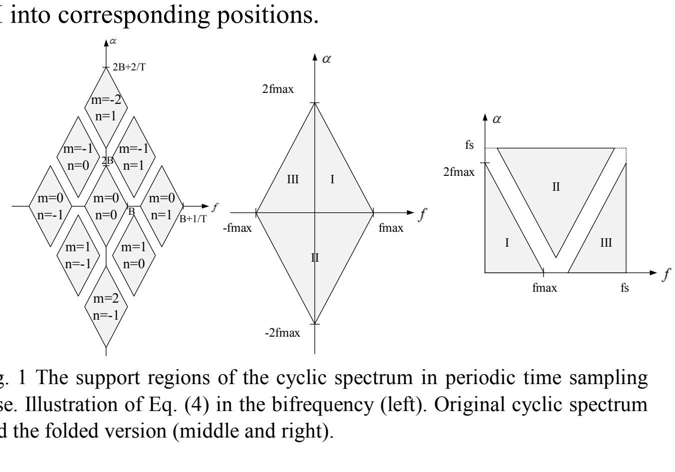

Fig. 1 The support regions of the cyclic spectrum in periodic time sampling case. Illustration of Eq. (4) in the bifrequency (left). Original cyclic spectrum and the folded version (middle and right).

III. Cyclic Spectrum Reconstruction

A. Obtain Compressive Measurements from CS Sampling Systems

In many practical compressed sensing (CS) sampling systems that target continuous-time spectrally-sparse signals, such as Random Demodulator (RD) [16][17] and the Modulated Wideband Converter (MWC) [12]. To acquire the samples, the continuous-time signal is taken to be passed through a set of filters, where each filter $a_m(t)$ constructs a row of the sampling matrix $\mathbf{A} \in \mathbb{R}^{M\times N}$. The matrix $\mathbf{A}$ maps the analog signal to the discrete set of measurements. In compressed sensing framework the matrix $\mathbf{A}$ satisfies restricted isometry property (RIP) [18] whose entries are chosen independent and identically distributed according to a Gaussian, Bernoulli, or more generally any sub-gaussian distribution. Therefore the compressive measurements can be obtained from the CS sampling system

$$\mathbf{z} = \mathbf{A}\mathbf{x} \tag{5}$$

The digital representation of the continuous-time signal is given by $\mathbf{x} = \left[x[0],\cdots,x[N-1]\right]^{\mathrm{T}}$, and the compressive samples is $\mathbf{z} = \left[z[0],\cdots,z[M-1]\right]^{\mathrm{T}}$. That is, the Nyquist sampling rate is $T_s = 1/f_s$, where T_s is the uniform sampling interval, and the compressive sampling rate is $f_{sc} = (M/N)f_s$, where $M \ll N$.

B. Relationship between Compressive Samples with Cyclic Spectrum

Consider the covariance matrix of compressive samples $\mathbf{z}$, we have $\mathbf{R}_z = E\left\{\mathbf{z}_t\mathbf{z}_t^{\mathrm{T}}\right\}$. Since the elements of $\mathbf{R}_z$ are symmetric, the upper triangular matrix of $\mathbf{R}_z$ can be stacked into a vector $\mathbf{r}_z$, which is given by

$$\begin{aligned}\mathbf{r}_x = \big[&r_x(0,0), r_x(1,0),\cdots,r_x(N-1,0),\\ &r_x(0,1), r_x(1,1),\cdots,r_x(N-2,1),\cdots,r_x(0,N-1)\big]^{\mathrm{T}}\end{aligned} \tag{6}$$

According to Eq. (5) it can be obtained that $\mathbf{R}_z = \mathbf{A}\mathbf{R}_x\mathbf{A}^{\mathrm{H}}$. It is shown in [14] that the vectorized version of $\mathbf{R}_x$ is in relation to $\mathbf{r}_z$ in the following way

$$\mathbf{r}_z = \mathbf{Q}_M vec\left\{\mathbf{R}_z\right\} \tag{7}$$

where $\mathbf{Q}_M \in \{0,1/2,1\}^{(M(M+1)/2)\times M^2}$ [14]. Therefore Eq. (7) can be represented as

$$\mathbf{r}_z = \mathbf{Q}_M vec\left\{\mathbf{A}\mathbf{R}_x\mathbf{A}^{\mathrm{H}}\right\} = \mathbf{Q}_M\left(\mathbf{A}\otimes\mathbf{A}\right)vec\left\{\mathbf{R}_x\right\} \tag{8}$$

Now we want to relate $\mathbf{r}_x$ to the cyclic spectrum so that the linear relationship can be formulated between each other. As is shown in Eq. (2), denote (a,b) be the digital representation of the bifrequency (α, f) , we have

$$\alpha = (a/N)f_s,\ f = (b/N)f_s \tag{9}$$

where a is the digital cyclic frequency and b is the digital frequency. Therefore the discrete cyclic spectrum with respect to time-lag v is given by

$$s_x^{(c)}(a,b) = \sum_{v=0}^{N-1}\tilde{r}_x^{(c)}(a,v)e^{-j\frac{2\pi}{N}bv} \tag{10}$$

where $\tilde{r}_x^{(c)}(a,v)$ is the cyclic autocorrelation

$$\tilde{r}_x^{(c)}(a,v) = \left\{\frac{1}{N}\sum_{n=0}^{N-1-v} r_x(n,v)e^{-j\frac{2\pi}{N}an}\right\}e^{-j\frac{\pi}{N}av} \tag{11}$$

Eq. (9) and (10) can be written in the following matrix form [14]

$$\tilde{\mathbf{R}}_x^{(c)} = \sum_{v=0}^{N-1}\mathbf{G}_v\mathbf{R}\mathbf{D}_v \tag{12}$$

$$\mathbf{S}_x^{(c)} = \tilde{\mathbf{R}}_x^{(c)}\mathbf{F} \tag{13}$$

where $\mathbf{F}$ is the discrete fourier transform matrix, the (v,v) th diagonal elements of $\mathbf{D}_v$ are 1, and $\mathbf{G}_v$ satisfies

$$\left[\mathbf{G}_v\right]_{(a,n)} = \left[\frac{1}{N}e^{-j2\pi a\left(n+\frac{v}{2}\right)/N}\right]_{(a,n)}$$

Similarly we can obtain

$$vec\left\{\tilde{\mathbf{R}}_x^{(c)}\right\} = \sum_{v=0}^{N-1}\left(\mathbf{D}_v^{\mathrm{T}}\otimes\mathbf{G}_v\right)vec\left\{\mathbf{R}\right\} \tag{14}$$

Using Eq. (12) it holds that

$$vec\left\{\tilde{\mathbf{R}}_x^{(c)}\right\} = vec\left\{\mathbf{S}_x^{(c)}\mathbf{F}^{-1}\right\} = \left(\mathbf{F}^{-\mathrm{T}}\otimes\mathbf{I}_N\right)vec\left\{\mathbf{S}_x^{(c)}\right\} \tag{15}$$

From Eq. (13) and Eq. (14) we have

$$\mathbf{r}_x = \mathbf{H}^{\dagger}\mathbf{W}\mathbf{s}_x^{(c)} \tag{16}$$

where $\mathbf{H} = \sum_{v=0}^{N-1}\left(\mathbf{D}_v^{\mathrm{T}}\otimes\mathbf{G}_v\right)\mathbf{B}$ and $\mathbf{W} = \mathbf{F}^{-\mathrm{T}}\otimes\mathbf{I}_N$, $\mathbf{I}_N$ is $N\times N$ identity matrix.

Therefore we can obtain the linear relationship between the vectorized cyclic spectrum and the compressive samples by combining Eq. (8) with Eq. (16)

$$\mathbf{r}_z = \mathbf{\Phi}\mathbf{H}^{\dagger}\mathbf{W}\mathbf{s}_x^{(c)} \tag{17}$$

where $\mathbf{\Phi} = \mathbf{Q}_M\left(\mathbf{A}\otimes\mathbf{A}\right)$.

C. Estimating the Cyclic Spectrum via Sparse Constraints

Consider the signal $x(t)$ contains multiple BPSK signals, the original cyclic spectrum and the folded replicas are shown in Fig. 2. It is shown from the support regions labeled by solid line that the cyclic spectrum of BPSK signals in periodic time sampling case exhibits two sparsity properties.

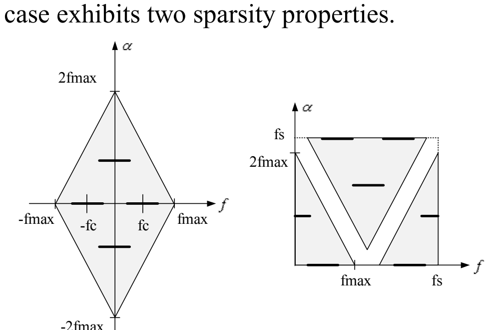

Fig. 2 Original cyclic spectrum of two BPSK signals (left). The folded version of original cyclic spectrum (right).

- *Intrinsic bifrequency domain sparsity*. Only the entries at the support regions are nonzero and all the other entries are zero, which implies that $\mathbf{s}_x^{(c)}$ in Eq. (17) is sparse.

- *Differential frequency domain sparsity*. Since the cyclic spectrum of the digital samples consists of folded replicas of the original cyclic spectrum, the folded cyclic spectrum remains symmetric with respect to $f_s/2$ on the frequency direction. Therefore the symmetrical column-wise differences are sparse, this differential sparsity can be taken into account to improve the smoothness of the estimated cyclic spectrum.

Hence we consider a $\left(N^2/2\right)\times N^2$ matrix $\mathbf{D}$, $\mathbf{D}(i,i)=-1$ for any row index $i\in\left[1,N^2/2\right]$. When $i\in\left[kN,(k+1)N\right]$, $k\in[0,N-1]$, only the column index j satisfies $j=i+(N-2k-1)N$ elements being 1 and all other elements being 0. For example, denote $N=4$, the digital cyclic spectrum is a 4×4 matrix, the differential matrix $\mathbf{D}\in\mathbb{R}^{8\times 16}$ is defined as follows

$$\mathbf{D}=\begin{bmatrix} -1 & 0 & 0 & \cdots & & & & & \cdots & 0 & 1 & & \cdots & 0 \\ 0 & -1 & & & & & & & & & & 1 & & \vdots \\ 0 & \ddots & -1 & & & \ddots & & & & & & & 1 & \\ \vdots & & & -1 & & & 0 & & & & & & & 1 \\ & & & & -1 & & & 1 & & & & & & \\ & & & & & -1 & & & 1 & & & & & \vdots \\ \vdots & & & & & \ddots & -1 & & & 1 & & & & 0 \\ 0 & & & & & & 0 & -1 & & & 1 & 0 & \cdots & 0 \end{bmatrix}$$

The entries of $\mathbf{D}\mathbf{s}_x^{(c)}$ reflects the difference of the corresponding two columns as mentioned earlier, therefore $\mathbf{D}\mathbf{s}_x^{(c)}$ is a sparse vector. According to Eq. (17) and the two sparsity property, the cyclic spectrum reconstruction can be formulated by the following optimization problem

$$\hat{\mathbf{s}}_x^{(c)}=\arg\min_{\mathbf{s}_x^{(c)}}\left\|\mathbf{r}_z-\mathbf{\Psi}\mathbf{s}_x^{(c)}\right\|_2^2+\mu_1\left\|\mathbf{s}_x^{(c)}\right\|_1+\mu_2\left\|\mathbf{D}\mathbf{s}_x^{(c)}\right\|_1 \tag{18}$$

where $\mathbf{\Psi}=\mathbf{\Phi}\mathbf{H}^{\dagger}\mathbf{W}$. The second and the third ℓ_1 norm penalty term in Eq. (18) represent the intrinsic sparsity and the smoothness of the optimizer respectively. The choices of the regularization parameters μ_1 and μ_2 play a key role in affecting the performance of the cyclic spectrum reconstruction, so in order to find reasonable guidelines for choosing the parameters, we rely on the bound results of these parameters which was derived in [19]. Assume the maximum thresholds of μ_1 and μ_2 are given by μ_1^* and μ_2^*, the actual values of μ_1 and μ_2 are generally set to 5%-10% of μ_1^* and μ_2^* in the following simulations, which will lead to robust and satisfied performance.

IV. Spectrum Occupancy Detection based on Recovered Cyclic Spectrum

After we obtain the recovered cyclic spectrum by solving the optimization problem Eq. (18), the spectrum occupancy can be detected from the vectorized cyclic spectrum $\hat{\mathbf{s}}_x^{(c)}$. The multi-cycle generalized likelihood ratio test (GLRT) framework, which is feasible for all types of modulation and waveform types, is used to detect the presence of primary signals on the whole wideband simultaneously [14]. The amounts of the sub-bands that need identify whether exists primary signals or not depend on the frequency resolution of the digital cyclic spectrum. Denote the $f^{(n)}=(n/N)f_s$, $n\in[0,N/2]$, the detection task is to acquire the carrier frequency and bandwidth of each signals after making a decision for all $f^{(n)}$.

V. Experiments and Analysis

The goal of this section is to evaluate the reconstruction property of the optimization problem (18) obtained by taking and performing recovery in noiseless and different compression ratios cases. In all experiments below, we randomly generate the sampling matrices $\mathbf{A}$ by drawing from i.i.d Gaussian matrices and normalizing the columns to have norm 1. The Normalized Mean Square Error (NMSE) is used as a performance measure of cyclic spectrum reconstruction, which is defined by $\left\|\hat{\mathbf{s}}_x^{(c)}-\mathbf{s}_x^{(c)}\right\|_2^2\Big/\left\|\mathbf{s}_x^{(c)}\right\|_2^2$.

Intuitively the two-dimensional cyclic spectrum is sparse, so that ℓ_1 norm penalty function can be exploited to describe the sparsity of $\mathbf{s}_x^{(c)}$ and furthermore induce a sparse solution from the following optimization problem, which is given by [14].

$$\min_{\mathbf{s}_x^{(c)}}\left\|\mathbf{r}_z-\mathbf{\Psi}\mathbf{s}_x^{(c)}\right\|_2^2+\lambda\left\|\mathbf{s}_x^{(c)}\right\|_1 \tag{19}$$

In order to show the superiority compared to the reconstruction performance of the above ℓ_1 norm regularized least squares problems, we generate two BPSK signals as the primary signals, whose center frequencies also locate at 187.5MHz and 375MHz respectively. The bandwidth of the frequency spectrum is $f_{\max}=500\text{Hz}$, the Nyquist rate is $f_s=1\text{GHz}$, the symbol period of each signal is $0.1\mu s$.

Note that in the practical implementation, such as MWC [12], the signal $x(t)$ enters M channels simultaneously. In the i th channel, $x(t)$ is multiplied by a T_p-periodic mixing function $p_i(t)$. Therefore the total observation time T can be divided into $L=T/T_B$ intervals, where T_B is the length of each interval. A compressive sample vector $\mathbf{z}_t(\ell)$ is obtained in each interval, which provide a useful way to estimate the covariance matrix $\hat{\mathbf{R}}_z$ of the compressive samples by $\hat{\mathbf{R}}_z=(1/L)\sum_{\ell=0}^{L-1}\mathbf{z}_t(\ell)\mathbf{z}_t^{\mathrm{H}}(\ell)$, and then obtain $\hat{\mathbf{r}}_z$ derived by Eq. (18). The estimation error reduces when the segmentation numbers L of the total observation time increases, but deteriorate the frequency resolution of $\hat{\mathbf{s}}_x^{(c)}$.

In Fig. 3 we plot the cyclic spectrum recovery performance as a function of the compression ratio based on 500 Monte-Carlo simulations. The compression ratio is from 0.1 to 1, and the segmentation numbers of the total observation time are $L = 40,200$. It can be seen that the NMSEs of our estimator outperform the estimator Eq. (19) owing to the additional differential frequency domain sparsity. The NMSEs of the cyclic spectrum recovery become stable when the compression ratio is larger than 0.5, that is, the recovery performance of the estimator tends to robust. Furthermore we find that the cyclic spectrum recovery in the $L=200$ case has slightly better performance than it in the $L=40$ case, because the estimation error of $\hat{\mathbf{r}}_z$ reduces when L increase.

We also evaluate the detection probability versus different compression ratio in order to demonstrate the effectiveness that the recovered cyclic spectrum can improve the detection accuracy for determining the spectrum occupancy. Whether the frequency $f^{(n)}$ contains primary signals or not depends on the GLRT statistic, and the false alarm rate is fixed at $P_{fa} = 0.1$. As is shown in Fig. 4 that based on 500 Monte-Carlo simulations when the compression ratio is larger than 0.25 the detection probability can achieve no less than 95%

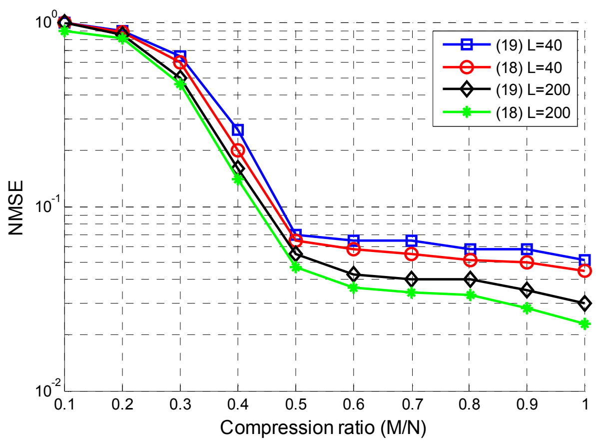

Fig. 3 Comparisons of NMSE in different compression ratio cases

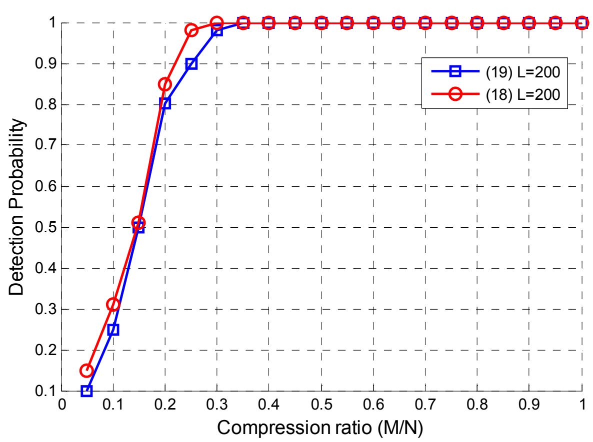

Fig. 4 The detection probability in different compression ratio cases

VI. Conclusion and Future Work

A novel cyclic spectrum reconstruction method is introduced based on the intrinsic bifrequency domain sparsity and the differential frequency domain sparsity. The main contribution of this work is to formulate an underdetermined minimization problem with a dual sparsity penalty. Simulations results demonstrate the effectiveness of the proposed estimator that further improve the detection probability of spectrum occupancy. Interesting aspects of future research include investigating the d ($d \geq 2$)-dimensional off-grid signal recovery and parameter estimation [21].

References

[1] Gardner W A. Exploitation of spectral redundancy in cyclostationary signals [J]. IEEE Signal Processing Magazine, 1991, 8(2):14-36.

[2] Dandawate A V, Giannakis G B. Statistical tests for presence of cyclostationarity [J]. IEEE Transactions on Signal Processing, 1994, 42(9):2355-2369.

[3] Davenport M A, Laska J N, Treichler J, et al. The pros and cons of compressive sensing for wideband signal acquisition: Noise folding vs. Dynamic range [J]. IEEE Transactions on Signal Processing, 2011, 60(9):4628 - 4642.

[4] Chen X, Yu Z, Hoyos S, et al. A sub-Nyquist rate sampling receiver xxploiting compressive sensing [J]. IEEE Transactions on Circuits & Systems I Regular Papers, 2011, 58(3):507-520.

[5] D. Romero, R. López-Valcarce, G. Leus. Compressive wideband spectrum sensing with spectral prior information [C]. 2013 IEEE International Conference on Acoustics, Speech and Signal Processing: 4469-4473.

[6] Zhi Tian. Compressed wideband sensing in cooperative cognitive radio networks [C]. 2008 IEEE Global Telecommunications Conference: 1-5.

[7] Zhi Tian. Cyclic Feature Based Wideband Spectrum Sensing Using Compressive Sampling [C]. 2011 IEEE International Conference on Communications (ICC): 1-5.

[8] Sabahi M F, Masoumzadeh M, Forouzan A R. A Novel Algorithm for Frequency Domain Wideband Compressive Spectrum Sensing [J]. IET Communications, 2016.

[9] Dyonisius Dony Ariananda, Geert Leus. Non-uniform sampling for compressive cyclic spectrum reconstruction [C]. 2014 IEEE International Conference on Acoustic, Speech and Signal Processing: 41-45.

[10] Cohen, D, Rebeiz, E, Eldar, Y.C, et al. Cyclic spectrum reconstruction and cyclostationary detection from sub-Nyquist samples [C]. 2013 IEEE 14th Workshop on Signal Processing Advances in Wireless Communications: 425-429.

[11] Mishali M, Eldar Y C. Blind Multiband signal reconstruction: compressed sensing for analog signals [J]. IEEE Transactions on Signal Processing, 2007, 57(3): 993-1009.

[12] Mishali M, Eldar Y C. From theory to practice: sub-Nyquist sampling of sparse wideband analog signals [J]. IEEE Journal of Selected Topics in Signal Processing, 2009, 4(2): 375 - 391.

[13] Leus G, Tian Z. Recovering second-order statistics from compressive measurements[C] IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing: 337 - 340.

[14] Tian Z, Tafesse Y, Sadler B M. Cyclic feature detection with sub-Nyquist sampling for wideband spectrum sensing [J]. IEEE Journal of Selected Topics in Signal Processing, 2012, 6(1): 58 - 69.

[15] Lundén J, Koivunen V, Huttunen A, et al. Collaborative cyclostationary spectrum sensing for cognitive radio systems [J]. IEEE Transactions on Signal Processing, 2009, 57(11): 4182-4195.

[16] Kirolos S, Laska J, Wakin M, et al. Analog-to-Information Conversion via Random Demodulation[C] 2006 IEEE Dallas/CAS Workshop on Design, Applications, Integration and Software: 71-74.

[17] Laska J N, Kirolos S, Duarte M F, et al. Theory and Implementation of an Analog-to-Information Converter using Random Demodulation[C] 2007 IEEE International Symposium on Circuits and Systems: 1959-1962.

[18] Baraniuk R, Davenport M, Devore R, et al. A Simple Proof of the Restricted Isometry Property for Random Matrices [J]. Constructive Approximation, 2015, 28(28): 253-263.

[19] Angelosante D, Giannakis G B, Sidiropoulos N D. Estimating multiple frequency-hopping signal parameters via sparse linear regression [J]. IEEE Transactions on Signal Processing, 2010, 58(10): 5044-5056.

[20] Chi Y, Chen Y. Compressive recovery of 2-D off-grid frequencies[C] 2013 Asilomar Conference on Signals, Systems and Computers: 687-691.

[21] Shiau L C, Chang J T. Precise Semidefinite Programming Formulation of Atomic Norm Minimization for Recovering d-Dimensional ($d \geq 2$) Off-the-Grid Frequencies[C] 2014 Information Theory and Applications Workshop (ITA): 1-4.

Xushan Chen received his Bachelor, Master and Ph.D. degree in network engineering, system engineering and information and communication engineering from PLA University of Science and Technology, China, in 2009, 2012 and 2016. He is currently a Lecturer at the Lab of Politics Informationization, People's Armed Police Institute of Politics. His major research interests include compressive sampling, analog-to-information converter and sparse signal processing.

Jibin Yang is currently an associate professor at PLA University of Science and Technology. He received his Ph.D. degree from PLA University of Science

and Technology in 2006. His current research interests include DSP algorithms and multimedia signal processing.

Xiongwei Zhang is currently a professor at PLA University of Science and Technology. He received his Ph.D. degree from Nanjing Institute of Communication Engineering in 1992. His current research interests cover intelligence information processing, speech and image signal processing and telecommunication systems.

Jianfeng Li is currently a Lecturer at the Lab of Politics Informationization, People's Armed Police Institute of Politics. His research interests are related to library information and documentation. He has published research papers at national and international journals, conference proceedings as well as chapters of books.