Locally anisotropic interpolation of wind fields

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***Abstract*—** This work is devoted to an approach to solving the task of restoring the wind field structure by the local measurement data in a certain set of points with a low density. The peculiarity of the proposed approach is to take into account the anisotropy of the wind field. In this case, the described algorithm allows us to take into account not only the scale of the anisotropy at different points, but the local variability of the anisotropy directions.

***Keywords*—** local anisotropic interpolation, low-density measuring network , wind field.

I. Introduction

WIND speed measurement date, like other data on atmospheric parameters, are usually available at a certain set of points. Therefore, in order to restore a reliable airflow structure, some form of spatial interpolation is required.

The wind speed, as a rule, is quite variable in space and strongly depends on local geographic features: the terrain, the presence of high vegetation, the proximity of the coast and many other factors.

In general, a qualitative reconstruction of the flow structure requires a high density of the observation network, which in many cases is impossible to provide.

On the other hand, in many practical applications, the information about the local structure of wind fields is critical. For example, these applications include the aviation.

The fact is that the local intensive variability of the wind field, called the wind shear, has a significant effect on the dynamics of the aircraft flight and can lead to adverse consequences [1].

In this connection, tools for monitoring wind fields in the vicinity of the aerodrome are developed, as well as algorithms for identifying the wind shear, in order to provide the meteorological support for flight safety. Such systems include low level wind shear alerting systems based on wind speed measurements at several points. However, these algorithms use the hypothesis of the linear variation of airflow parameters between measurement points, which, as already noted, cannot in general provide a reliable estimate of the wind speed gradients.

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At present time a wide range of deterministic interpolation algorithms is available for use to estimate values at points where there aren’t observational data [2-4]. The choice of one or another method essentially depends on the data spatial attributes, since different methods lead to different structures of the reconstructed field.

The peculiarity of the wind field structure is that the correlation of the wind field parameters in the wind direction and in the orthogonal directions is different even in a free atmosphere (far from the earth's surface). According to, for example, the generally accepted model of atmospheric turbulence, scales in longitudinal (wind speed) and orthogonal directions differ by a factor of two. Under the influence of the underlying surface, the scale ratio can also vary from point to point, depending on the local terrain features.

To take into account the anisotropy of the wind field while solving the problems of restoring its structure by the local measurements data, it is necessary to consider the direction from the point at which the measurement data are available to the point at which the wind speed parameters are estimated.

If the observation network has a sufficiently high density, the application of specialized algorithms that take into account anisotropy is not required, since at small distances between observation points the interpolation error caused by the use of the simplest linear algorithms turns out to be negligible.

However, if the observation network has a low density, taking anisotropy into account while restoring the wind field becomes critically important.

The application of specialized algorithms of anisotropic interpolation becomes especially actual with the use of expensive measuring instruments, for example, wind lidar profilers, the advantage of which is the ability to obtain data on wind speed at different altitudes, but on the other hand, the observation network always has an extremely low density.

However, the use of profilers in combination with specialized algorithms of the anisotropic interpolation potentially allows efficiently restoring the spatial structure of the air flow.

Therefore, there is an actual problem of restoring the structure of the wind field by the data of a low-density measuring network, taking into account the local anisotropy of the wind field. The algorithm for solving this problem is presented in this work.

II. Background

At present time there is a large number of methods for interpolating spatial data on an unstructured measurement network. These methods are most intensively used in solving tasks of a geophysical data interpretation. Herein we consider only a few basic methods, the modification of which is used by the developed algorithm.

In fact, the majority of interpolation methods are used to calculate the values of the target variable at arbitrary points

$$Z(\boldsymbol{r}) = \sum_{i=1}^{n} w_i Z(\boldsymbol{r}_i).$$

The principles used to specify the weighting coefficients w_i are differ and the expressions for their calculation are also different.

One of the popular interpolation methods is the method of Inverse Distance Weighting (IDW). In the method of inverse distances, the inverse proportion of the weighted coefficients on a certain degree of distance between the calculated point and the observation point d_i is used:

$$w_i = \frac{\dfrac{1}{d_i{}^p}}{\sum\limits_{i=1}^{n} \dfrac{1}{d_i{}^p}},$$

where the exponent p usually takes the value 1, 2 or 3.

It should be noted at once that the method does not allow us to reveal the structure of the variability of property, but it can serve for accurate interpolation. The basis is the assumption that the smaller the distance between the measurement points, the closer the values of the observed parameter at these points, and their connection weakens as the measurement points are removed from one another. The value of the physical quantity at the target point will be most similar to the values at nearby reference points, and less similar to the values at remote reference points.

The value of the exponent p is chosen in such a way as to minimize the root-mean-square interpolation error. The integral weights of the used reference points is, as can be readily appreciated equal to one.

The IDW function can be used in cases where the point set is sufficiently dense to capture the local variability of the analyzed data structure.

The values calculated by this method cannot be higher than the values at the reference points. This method does not allow us to estimate the interpolation error.

The Shepard method is analogous to the method of inverse distances. It also uses inverse distances while calculating weighted coefficients with the exponent p=2. The difference is that while constructing an interpolation function in local areas, the least squares method is used. This reduces the probability of appearance of false patterns around the observation points in the generated structure.

Anisotropy in the method of inverse weighted distances is considered by scaling the relative coordinates when calculating the distance between points:

$$d_i = \sqrt{(x - x_i)^2 + k_y{}^2 (y - y_i)^2 + k_z{}^2 (z - z_i)^2},$$

where (x, y, z), (x_i, y_i, z_i) are the respective coordinates of the interpolation and measurement points in a coordinate system whose axis of abscissas coincides with the main semiaxis of the anisotropy ellipsoid, k_y, k_z are scale factors that show how many times the length of the main semiaxis is greater than the semiaxis of the ellipsoid of anisotropy in other coordinates.

Another example of spatial data interpolation methods is the method of Radial basis functions (RBF). In this method, mathematical functions are used to estimate the values, which minimize the surface warp. The surfaces constructed with the use of these functions will pass through all the reference points. Therefore, this method of describing the surface does not reveal the structure of variation of the physical quantity, but it is an accurate interpolator. Each radial function has a different shape and results for different interpolation surfaces. The RBF methods are a form of artificial neural networks.

This method is most convenient for constructing slowly varying surfaces in the presence of a large number of reference points. A decrease in the number of reference points leads to a change in the shape of the isolines, but the general nature of the surface of the studied space (the position and intensity of the extrema, etc.) is preserved or changes to an insignificant degree.

In this method, the target function is found as a linear combination of a set of radial basis functions:

$$Z(\boldsymbol{r}) = a + \sum_{i=1}^{n} \mu_i B(\boldsymbol{r}_i),$$

where a is a constant, i is the index of the measurement point, μ_i is unknown coefficients, $B(\boldsymbol{r}_i)$ is basic functions that depend on the distance of the point $\boldsymbol{r}$ to the *i-th* observation point $\boldsymbol{r}_i$.

Several types of basic functions are used:

- Inverse Multiquadric

$$B(d_i) = \frac{1}{\sqrt{d_i{}^2 + \delta^2}},$$

- Multilog

$$B(d_i) = \log\left(d_i{}^2 + \delta^2\right)$$

- Multiquadric

$$B(d_i) = \sqrt{d_i{}^2 + \delta^2}.$$

Most often used;

- Natural Cubic Spline

$$B(d_i) = \left(d_i{}^2 + \delta^2\right)^{3/2},$$

- Thin Plate Spline

$$B(d_i) = \left(d_i^{\,2} + \delta^2\right)\log\left(d_i^{\,2} + \delta^2\right),$$

where δ^2 is the smoothing factor, the larger the parameter, the smoother the structure of the reconstructed field will be. Reasonable values of the indicator are in the range from the average point-to-point distance of the sample to half of this average mean.

III. Interpolation Algorithm

Suppose that the measurement data are available in a certain set of points arbitrarily located in space having known coordinates (fig. 1).

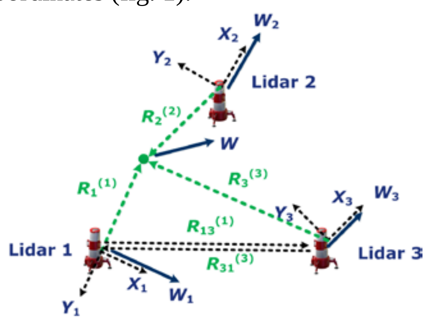

Fig. 1 local coordinate systems for considering the spatial anisotropy of the wind field

Without limitation of generality, it is assumed that the means for measuring the wind field parameters located in a horizontal plane with coordinates (x_i, z_i) allows measuring airflow parameters at several altitudes $\left\{y_1^{(i)},\ldots,y_{s_i}^{(i)}\right\}$. At the same time, the number of measurement heights and the actual height of the wind field measurements at points with different coordinates do not necessarily coincide.

When speaking about the task of restoring the wind field, the solution of the problem will be considered with respect to a single component of the wind speed, since the problem is solved similarly and independently for other components.

At each measurement point (x_i, y_i, z_i), the velocity vector of the air flow is known $\boldsymbol{W}_i = (u_i, v_i, w_i)$.

Let us introduce into consideration at each measurement point a local coordinate system whose direction of the abscissa axis $\boldsymbol{\xi}_i$ coincides with the local wind speed $\boldsymbol{W}_i$ at this point, and the direction of the applicator axis is calculated as a vector product

$$\boldsymbol{\zeta}_i = \boldsymbol{\xi}_i \times \boldsymbol{e}_y,$$

while the ordinate axis is calculated as a vector product in the form of

$$\boldsymbol{\eta}_i = \boldsymbol{\zeta}_i \times \boldsymbol{\xi}_i,$$

where $\boldsymbol{e}_y$ is a single unit vector of the vertical axis in the Cartesian terrestrial coordinate system.

Let us assume that the scale coefficients of the anisotropy $(k_{\eta i}, k_{\zeta i})$ at each measurement point are known. As a first approximation, the turbulence scales of the wind field can be used to calculate these coefficients, for example, according to the model MIL-HDBK-1797B or MIL-F-8785C [5, 6].

Let us assume that the radius vector from the measurement point (x_i, y_i, z_i) to the point (x, y, z) at which the interpolated value of the wind speed parameters is calculated is:

$$\boldsymbol{r}_i = (x - x_i, y - y_i, z - z_i).$$

The coordinates of the vector $\boldsymbol{r}_i$ in the local coordinate system $(\boldsymbol{\xi}_i, \boldsymbol{\eta}_i, \boldsymbol{\zeta}_i)$ associated with the measurement point (x_i, y_i, z_i) are calculated as:

$$s_{\xi i} = (\boldsymbol{r}_i, \boldsymbol{\xi}_i),\ s_{\eta i} = (\boldsymbol{r}_i, \boldsymbol{\eta}_i),\ s_{\zeta i} = (\boldsymbol{r}_i, \boldsymbol{\zeta}_i).$$

Herein the right-hand side of the relations presented is the scalar product of the corresponding vectors.

Then the value of the wind velocity components at the point (x, y, z) is calculated by the formula:

$$\widetilde{W}(x, y, z) = \frac{\sum\limits_i \dfrac{\mu_i}{d_i^{\,p}}}{\sum\limits_i \dfrac{1}{d_i^{\,p}}}, \tag{1}$$

where

$$d_i = \sqrt{\delta^2 + s_{\xi i}^{\,2} + k_{\eta i}^{\,2} s_{\eta i}^{\,2} + k_{\zeta i}^{\,2} s_{\zeta i}^{\,2}}.$$

Recommended value of the exponent p=2.

The formula (1) features coefficients μ_i that do not usually coincide with the measured values of the wind field parameters at the measurement point. These coefficients are calculated from a system of equations of the following form:

$$\frac{\sum\limits_j \dfrac{\mu_j}{d_{ji}^{\,p}}}{\sum\limits_j \dfrac{1}{d_{ji}^{\,p}}} = W_i, \tag{2}$$

where

$$d_{ji} = \sqrt{\delta^2 + s_{\xi ji}^{\,2} + k_{\eta j}^{\,2} s_{\eta ji}^{\,2} + k_{\zeta j}^{\,2} s_{\zeta ji}^{\,2}}, \tag{3}$$

$$s_{\xi ji} = (\boldsymbol{r}_{ij}, \boldsymbol{\xi}_j),\ s_{\eta ji} = (\boldsymbol{r}_{ij}, \boldsymbol{\eta}_j),\ s_{\zeta ji} = (\boldsymbol{r}_{ij}, \boldsymbol{\zeta}_j), \tag{4}$$

$$\boldsymbol{r}_{ij} = (x_i - x_j, y_i - y_j, z_i - z_j).$$

It is clear that, in accordance with formulas (3) and (4) the inequality $d_{ji} \neq d_{ij}$ present in the general case:

- in the formula (3) various scale factors are present depending on the measurement point,
- coordinates $(s_{\xi ji}, s_{\eta ji}, s_{\zeta ji})$, $(s_{\xi ij}, s_{\eta ij}, s_{\zeta ij})$ of the

same radius vector $\boldsymbol{r}_{ij}$ are calculated in different local coordinate systems (ξ_j, η_j, ζ_j) and (ξ_i, η_i, ζ_i) correspondingly.

The necessity to calculate the coefficients μ_i in the formula (1) from the system of equations (2) instead of directly using the measured values is due to the fact that the direct use of $\boldsymbol{W}_i$ can lead to the fact that the values of the interpolated wind field at the measurement points will differ from the measured values.

IV. Discussion

To estimate the effect of taking the anisotropy into account, calculations were made on the wind field interpolation from two measurement points lying in one plane. An examination of such a simple case allows estimating the influence of accounting the anisotropy on the structure of the interpolated field. In the considered model problem the measurement points are located at a distance of 2. The distance is taken in dimensionless units. The scale factor is assumed equal to $k_{\zeta i} = 2$.

In the first example considered, it was assumed that the exponent is p = 2. The measured wind speed values at both points were assumed to be the same, and only the flow direction differs. The difference between the flow directions at two measurement points was assumed equal to 45° and the flow direction at the first measurement point varied in the range 0°…75°.

Fig. 2 shows the spatial distribution of the interpolated flow direction as an orientation increment of the velocity vector depending on the distance from the first measurement point:

$$\Delta\varphi(d_i) = \varphi(d_i) - \varphi(d = 0).$$

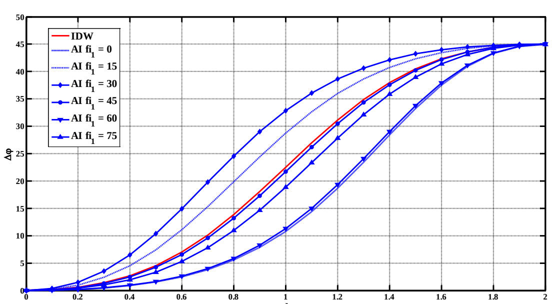

Fig.2 change in the anisotropic flow direction at a distance from the first measurement point, depending on its direction at this point.

The orientation angle of the velocity vector 0° corresponds to the wind speed direction along the radius vector between two measurement points. As a comparison, the solid line shows the calculation performed by the method of inverse distances.It is seen that the method of inverse distances gives the same structure of the change in the flow direction without distinction of its initial orientation (direction at the first measurement point). While taking the anisotropy into account, the dynamics of the change essentially depends on the initial direction of the wind speed vector. It should be noted that the change in the structure turns out to be essentially nonlinear when the initial orientation of the wind speed vector changes. If the corresponding curve of the direction increment shown by the dashed line lies below the curve calculated by the method of inverse distances taking into account the initial orientation of the air flow in the direction from the first measurement point to the second one, it is located higher even at angles of the flow 15°…30°. When the flow is oriented at an angle of 45°, the direction change curve roughly coincides with the curve calculated by the method of inverse distances. At angles of 60°…75° the corresponding dependencies are relocated below the IDW curve, but the graph corresponding to the direction of 75° passes higher than the graph corresponding to the direction of 60°.

Such a discontinuous variation of the structure is resulting from the different orientation of the anisotropy ellipsoids, which determine the degree of influence of each measurement point in different directions relative to each other.

It should also be noted that if the change in the flow direction at the first and second halves of the distance ([0,1] and [1,2], respectively) is approximately the same while calculating with the use of the method of inverse distances, the changes are more non-linear while taking the anisotropy into account: a change in direction may have a sharper character either at the first or second distance depending on the initial orientation of the flow. This may lead to the appearance of the wind shear phenomenon mentioned at the beginning of the article.

V. Conclusion

Therefore, even the simplest example shows that the variability of the wind speed direction at the measurement points can lead to a different structure of the restored wind field by the measurement data taking its anisotropy into account. In the general case, this picture turns out to be more nonlinear both from the viewpoint of the dynamic pattern of the wind field parameters in space and the variability of the flow structure as a whole, depending on the measurement data.

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