Effect of Multirate and Suppression Filter in the Performance of Wavelet Packet Multicarrier Multicode CDMA System

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Abstract**—**Many ways can be considered to improve the performance of Multicarrier Multicode Code-Division Multiple Access (MC\MCD-CDMA) systems. Some of them are to use: Multirates services, suppression filters, wavelet packets, diversity and others. Multirates (MR) services can be provided in MC\MCD-CDMA system by varying the number of multicodes for each user according to his data rate but the processing gain of all users must be fixed. Suppression filter (SF) can improve the performance for the system by rejecting the narrow-band jammer interference. Since wavelet packets (WPs) have lower sidelobes compared with sinusoidal carriers, then systems which use WPs as subcarriers are very effective in reducing the problem of intercarrier interference Also, to reduce multiple access interference diversity techniques can be used. In this paper, the effects of MR , WPs and SF on the performance of MC\MCD-CDMA system that uses WPs as subcarrier were investigated. The system is denoted as WP-MC\MCD-CDMA system, and the study includes the effects of number of the rates, SF and its number of taps, WP family type and filter length.

Keywords**—**Diversity, Multirate, Suppression filter and Wavelet.

I. Introduction

Multirate (MR) services can be provided in Coded Division Multiple Accesses (CDMA) systems by two schemes. One is the multiprocessing gain (MPG) scheme, which is also called variable spreading length (VSL) and the other one is multicode multirate (MCR) scheme. In MPG scheme, the processing gain varies from one user to another, but the number of multicodes and chip rate are the same. In MCR scheme, the processing gain of all users is fixed but the number of multicodes for each user varies according to data rate [1], [2]. In [3], the MCR scheme is used to allow each user to transmit multiple numbers of orthogonal codes.

The narrowband interference signal (jamming signal) can degrade the performance of communication systems [4]. In CDMA system, the inherent processing gain in many cases provides the system with su¢cient degree of narrow-band interference rejection capability. But, if the interference signal is powerful enough, the conventional receiver is ine¤ective in mitigating this problem. In this case, an interference suppression filter (SF) can be employed to reject the jamming signal. In [5], a wiener-type filter, described in [6] is used to reject the jamming signal and thus improve the system performance.

The Wavelet Packets Multicarrier Multicode Coded-Division Multiple Access (WP-MCnMCDCDMA) system [3]-[5] and [7] uses WPs as subcarrier instead of sinusoidal one. Since the WPs have lower sidelobes and negligible sidelobe energy leakage compared with that for sinusoid carrier, the system which employs WPs as subcarrier is very effective in suppressing inter-carrier interference and multiple access interference**.**

The performance of the above system is affected by the many factors, such as WP family and their filter length, diversity techniques, variance of path gain, number of users, number of multicodes and others. The aim of this paper is to explore the impact of WPs families and their filter length, MR services, SF and variance of path gain on the performance of WP-MC\MCD-CDMA system. The paper is organized as follows. In the next section, an explanation for the WPs which are used as a subcarrier in this paper is given. In section III, we propose the transceiver system of WP-MC\MCD-CDMA with MCR and SP. In section IV, signal to interference plus jamming and noise ratio is presented. Section V presents the bit error rate (BER) and outage probability (P_{out}) based on the optimum diversity combining techniques which is the maximum ratio combining diversity. Determination for SF coefficients is given in section VI, while in section VII the determination of the variances of the path gain of uniform, exponential and Gaussian multipath intensity profile are presented. Some numerical results of the system BER and P_{out} performances are presented in section VIII and finally the conclusions are given in section IX.

II. Wavelet Packets

The h^{th} wavelet packet, $wp_h(t)$, which used as a subcarrier in this paper is given by:

$$wp_h(t) = \sum_i w_h(t - iT_n)\,,$$

where

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$$w_h(t) = \sqrt{N/T_n}\, p_h(Nt/T_n),$$

and $p_h(\bullet)$ is the wavelet function which is defined recursively by a pair of quadrature mirror filters (QMF) $h_0(k)$ and $h_1(k)$ as follows:

$$p_{2h}(x) = \sqrt{2}\sum_{k=0}^{2N-1} h_0(k)\, p_h(2x-k),$$

$$p_{2h+1}(x) = \sqrt{2}\sum_{k=0}^{2N-1} h_1(k)\, p_h(2x-k).$$

The support length, time and energy of $wp_h(t)$ are $2N-1$, T_n and 1, respectively. Furthermore

$$\left\langle w_h(t-(kT_n/N)), w_h(t-(nT_n/N)) \right\rangle = \delta_{kn},$$

and

$$\left\langle w_{2h}(t-(kT_n/N)), w_{2h+1}(t-(nT_n/N)) \right\rangle = 0,$$

for any integers k and n. δ is the Dirac delta function.

III. System Model and Description

The system for WP-MC\MCD-CDMA with MCR that employs a SF is shown in Fig. 1.

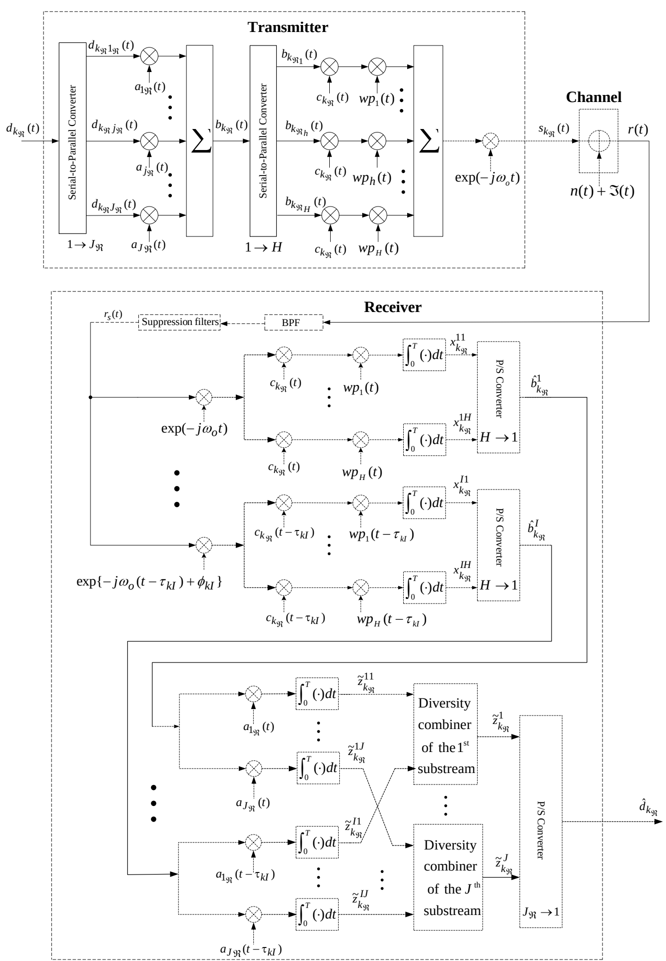

Fig. 1 WP-MC\MCD-CDMA system with multirate and SF

The transmitter consists of two parts which are multirate\multicode part and spreading\wavelet packet modulation part. The receiver consists of four parts which are, filtering part, despreading\wavelet packet correlator part, coding correlator part and the diversity combiner part. The total number of MR service users in this system is.

$$K = \sum_{\Re=1}^{\aleph} \sum_{k_\Re=1}^{K_\Re} k_\Re ,$$

where $\aleph$ represents the number of multicode sets, $K_\Re$ is the number of MR users in the service group $\Re$. Each user in this system is indexed by two variables: k and $\Re$, which indicates respectively, user number and his service group. Each MR user can transmit at a time only one kind of the $\aleph$ codes he has. The k^{th} user of the service group $\Re$ has a bit stream

$$d_{k_\Re}(t) = \sum_{i=-\infty}^{\infty} d_{k_\Re}^{i} \prod(t-(iT/J_\Re H))$$

where $d_{k_\Re}^{i}$ is the i^{th} value of the bit stream, $J_\Re$ is the number of substreams for the service group $\Re$, H is the number of superstream in WP part. The symbol duration of each data stream = T while the bit duration $T/(J_\Re H)$. The bit duration decreases as the number of substreams increases. The number of substreams is assumed to be multiple of certain number, J_1, that is $J_\Re = J_1 L_\Re$ where $L_\Re$ is an integer number and $1 = L_1 < L_2 \cdots < L_\aleph$. The users of service group $\Re$ provides $L_\Re$ bits with bit duration $T_\Re = T_1/L_\Re$ during the bit duration T_1 for the service group 1. At the transmitting side, the bit stream $d_{k_\Re}(t)$ is serial-to-parallel (S/P) converted into $J_\Re$ parallel substreams and then coded by an orthogonal code signal $a_{j\Re}(t) = \sum_{i=0}^{N_c-1} a_{k_\Re}^{i} \Pi(t-iT_c)$. The coded signal has a bit duration $T_c = T/(HN_c)$ and code length N_c. The coding process reduces the inter-substream interference resulting from the interference between the $J_\Re$ substreams. The coded substreams are then added and the resulting signal $b_{k_\Re}(t) = \sum_{j_\Re=1}^{J_\Re} d_{k_\Re j_\Re}(t) a_{j_\Re}(t)$ is again S/P converted into H superstreams. This means that, the whole signal spectrum is divided into H disjoint superstream subbands, with the h^{th} superstream being given by

$$b_{k_\Re h}(t) = \sum_{j_\Re=1}^{J_\Re} d_{k_\Re j_\Re h}(t) a_{j_\Re}(t), \qquad h = 1, \cdots, H,$$

where $d_{k_\Re j_\Re h}$ which has a rate = $1/T$ is the data symbol of the k^{th} user, $j_\Re$ substream of the h^{th} superstream. To maintain orthogonality between the coding signals, the maximum number of $J_\Re$ is limited to N_c.

The next step is the spreading of the coded superstreams by the processing gain sequence, $c_{k_{\Re}}(t)=\sum_{i=0}^{N_n-1} c_{k_{\Re}}^{i}\Pi(t-iT_n)$. The length of the code and the chip duration are N_n and $T_n=T/N_n$, respectively. Thus, the spread-spectrum system has a bandwidth $B_s=2/T_n$. The spreading superstreams then will modulate a wavelet packets, $wp_h(t)$. Note that, the WPs with different h indices represent different subbands. Due to flexibility of wavelet packets the bandwidth of each subband can be arbitrarily chosen and partition of subbands is not limited by a minimum frequency distance but it is determined by the channel characteristic. The modulated superstreams are added, modulated by a sinusoidal carrier, $\exp(j\omega_o t)$ and then transmitted. Assuming perfect power control such that the power for all users is the same and equal to P, then the transmitted signal for the k^{th} user from the $\Re^{th}$ service group is given by

$$s_{k_{\Re}}(t)=\sqrt{2P}\sum_{h=1}^{H}\sum_{j_{\Re}=1}^{J_{\Re}}\mathrm{Re}[d_{k_{\Re}j_{\Re}h}(t)a_{j_{\Re}}(t)\times c_{k_{\Re}}(t)wp_h(t)\exp(j\omega_o t)], \tag{1}$$

The above signal, $s_{k_{\Re}}$, passes through a noisy channel before being detected by the receiver. In this paper, the channel path is assumed to have Nakagami distribution [8] with impulse response given by:

$$h_c(t)=\sum_{l=1}^{L}\beta_{k_{\Re}l}\exp(j\phi_{k_{\Re}l})\delta(t-\tau_{k_{\Re}l}), \tag{2}$$

where L is the number of propagation paths and $\beta_{k_{\Re}l}$, $\phi_{k_{\Re}l}$, and $\tau_{k_{\Re}l}$ are respectively, gain, phase and time delay for the l^{th} path. The phase delays is assumed to be uniformly distributed over $[0,2\pi]$. The output of the channel for the $k_{\Re}$ user is given by

$$y_{k_{\Re}}(t)=h_c(t)*s_{k_{\Re}}(t)=\sum_{l=1}^{L}\beta_{k_{\Re}l}\exp(j\phi_{k_{\Re}l})s_{k_{\Re}}(t-\tau_{k_{\Re}l}).$$

The received signal is given by

$$r(t)=\sum_{\Re=1}^{\aleph}\sum_{k_{\Re}=1}^{K_{\Re}}y_{k_{\Re}}(t)+n(t)+\Im(t), \tag{3}$$

where $n(t)$ is the Additive White Gaussian Noise (AWGN) and $\Im(t)$ is the Binary Phase Shift Keying (BPSK) narrow-band interference jammer (NBIJ) given by [4]

$$\Im(t)=\sqrt{2\Im}\zeta(t)\cos[2\pi(f_o+\Delta)+\psi]. \tag{4}$$

The NBIJ has $\Im$ power, $\zeta(t)$ information sequences, Δ offset for the interference carrier frequency with respect to signal carrier frequency and ψ phase. Also,the NBIJ has a bandwidth $B_{\Im}=2/T_{\Im}$, we assume that $B_{\Im}<B_s$. The information sequence $\zeta(t)$ has bit width $T_{\Im}$ and a bit rate $1/T_{\Im}$. The ratio of the NBIJ bandwidth to the system bandwidth is given by $p=B_{\Im}/B_s=T_n/T_{\Im}$.

To remove the out-of band noise, the $r(t)$ signal first passed through a Band-Pass Filter (BPF) having bandwidth B_s. The desired signal and inferences will pass without any distortion. The filtered signal is then passed through the suppression filter (SF), whose impulse response is given by [6]

$$h_s(t)=\sum_{m=-M_1}^{M_2}\alpha_m\delta(t-mT_n), \tag{5}$$

- M_1 and M_2 represents the number of filter taps on the left hand side and right hand side of the center tap. Also, $M_1\geq 0$ and $M_2\geq 0$. If M_1 or $M_2=0$, the filter is said to be Single Sided (SS) suppression filter, otherwise the filter is Double Sided (DS) suppression filter. In this paper DS suppression filter is used.
- $\alpha_m's$ are the filter coefficients with $\alpha_0=1$.

The output of the filter for each tap is given by

$$r_s(t)=\left(\sum_{\Re=1}^{\aleph}\sum_{k_{\Re}=1}^{K_{\Re}}y_{k_{\Re}}(t)+\hat{n}(t)+\Im(t)\right)*h_s(t) \tag{6}$$

where $\hat{n}(t)$ is the filtered AWGN. At the despreading\wavelet packet correlator part of the receiver, each $k_{\Re}^{th}$ user received signal from the filter output at the I diversity branches is demodulated by a locally generated carrier, depressed by $c_{k_{\Re}}(t)$, demodulated by $wp_h(t)$ and correlated over a period T to recover the superstream which is then parallel-to-serial (P/S) converted. Consider the first user of the $\Re^{th}$ service rate is our reference user also the first WP is our reference WP, then the output of the first correlator, first P/S converter in this part of the receiver, $x_{1\Re}^{11}$, is given by

$$\begin{aligned}x_{1\Re}^{11}&=\int_0^T r_s(t)c_{1_{\Re}}(t)wp_1(t)[\cos(\omega_o t)-j\sin(\omega_o t)]dt\\&=x_{DS}^{11}+x_{DSI}^{11}+x_{MPI}^{11}+x_{CDI}^{11}+x_{WPI}^{11}\\&\quad+x_{MUI}^{11}+x_{MSI}^{11}+n^{*1}+\Im^{*1},\end{aligned} \tag{7}$$

- x_{DS}^{11} is the desired user signal at the zeroth tap of the suppression filter,
- x_{DSI}^{11} is the self-interference due to the reference user and caused by the taps of suppression filter excluding the zeroth tap,
- x_{MPI}^{11}, x_{CDI}^{11}, x_{WPI}^{11} and x_{MUI}^{11} are respectively, the multipath interference, the multicode interference, the wavelet packets interference and the multiuser interference due to users other than the desired user,
- x_{MSI}^{11} is the multiservice interference,
- n^{*1} is suppressed correlated AWGN,
- $\Im^{*1}$ is the suppressed narrowband interference.

The output signal of the first P/S converter in WPs part for reference user of the $\Re^{th}$ service group is given by:

$$\hat{b}^1_{k_{\Re}} = x^{11}_{DS} + x^{11}_{DSI} + \sum_{h'=1}^{H}[x^{1h'}_{MPI} + x^{1h'}_{CDI} + x^{1h'}_{WPI} + x^{1h'}_{MUI} + x^{1h'}_{MSI} + n^{*h'} + \Im^{*h'}] \quad (8)$$

At the third part of the receiver which is, the multicode correlator part, the output signal from each P/S converter is despread by the MR user code $a_{j_{\Re}}$, and then correlated over a period *T*. The output signal for the reference user of the $\Re^{th}$ service group of the first correlator, $\tilde{z}^{11}_{1\Re}$, is given by:

$$\tilde{z}^{11}_{1\Re} = \int_0^T \hat{b}^1_{k_{\Re}} \times a_{1_{\Re}}(t)dt = z^{11}_{DS} + z^{11}_{DSI} + z^{11}_{MPI} + z^{11}_{CDI} + z^{11}_{WPI} + z^{11}_{MUI} + z^{11}_{MSI} + \tilde{n}^{11} + \tilde{\Im}^{11}. \quad (9)$$

The outputs of the correlators are combined by diversity combiner and at last, the output of each combiner for the $J_{\Re}$ substreans is P/S converted to recover the data signal, $\hat{d}_{k_{\Re}}(t)$.

IV. Signal-to-Interference plus Jamming and Noise Ratio

The performance of the system is tested by two methods which are, the Bit Error Rate (BER) and the outage probability, P_{out}. To find them we need to find the output signal to Interference plus jamming and Noise Ratio (SIJNR). The SIJNR depends on the power for the desired signal, the variances for the interferences, jamming and noise. The desired signal, the interferences, the jamming and the noise terms consists of two parts, the Inphase part and Quadrature part [7]. Without loss of generality, in this paper the Inphase part is considered.

The desired Inphase signal power (S), is the power of the signal for the first user of the first rate, of the first wavelet packet which propagate via first path at the zeroth filter tap, it is given by [7]

$$S = [z^{11}_{DS}]^2 = \frac{P(N_nT)^2}{2}\beta^2_{1_11} \quad (10)$$

The interference variance consists of six variances which are:

- $\sigma^2_{DSI} = \text{var}[z^{11}_{DSI}]$: self-interference variance due to reference user at taps of SF other than the zeroth tap,
- $\sigma^2_{MPI} = \text{var}[z^{11}_{MPI}]$: multipath interference variance due to paths other than the desired path,
- $\sigma^2_{MCDI} = \text{var}[z^{11}_{MCDI}]$: multicode interference variance,
- $\sigma^2_{WPI} = \text{var}[z^{11}_{WpI}]$: wavelet packets interference variance,
- $\sigma^2_{MUI} = \text{var}[z^{11}_{MUI}]$: multiuser interference variance,
- $\sigma^2_{MSI} = \text{var}[z^{11}_{MSI}]$: multiservice interference variance.

To calculate the above variances for BPSK modulation, it is assumed that all of them are zero mean independent random variables. Also, note that $d^i_{k_{\Re}}$, $a^i_{k_{\Re}}$, $c^i_{k_{\Re}}$ which are the i^{th} bit of user data bit stream, the i^{th} bit of orthogonal code and the i^{th} bit of the processing gain, respectively, $\in\{\pm 1\}$ with probabilities $P(1) = P(0) = 0.5$. Invoking the results in [3]-[5] and [7], the total interference variance, σ^2_{TI}, can be shown to be equal to:

$$\sigma^2_{TI} = \sigma^2_{DSI} + \sigma^2_{MPI} + \sigma^2_{MCDI} + \sigma^2_{WPI} + \sigma^2_{MUI} + \sigma^2_{MSI} = \frac{P(N_nT)^2}{2}MI \quad (11)$$

Where *MI* is given by

$$MI = \frac{\beta^2_{1_11}}{N_n}\sum_{\substack{m=-M_1\\ m\neq 0}}^{M_2}\alpha_m + \frac{\Omega}{12T_n(N_nH)^2} \times\left[2\sum_{m=-M_1}^{M_2}\alpha^2_m + \sum_{m=-M_1}^{M_2}\alpha_m\alpha_{m+1}\right]\left[\frac{Q_xK_1}{J_1}\sum_{h=1}^{H}\sum_{h'=1}^{H}\xi_{hh'} - \frac{1}{J_1^2}\sum_{h'=1}^{H}\xi_{h1} + Q_x\left(\sum_{h=1}^{H}\sum_{h'=1}^{H}\xi_{hh'}\right)\sum_{\Re=2}^{\aleph}\frac{K_{\Re}}{J_{\Re}}\right], \quad (12)$$

where

- $\xi_{hh'} = \int_0^{T_n}\left\{(r_{h'h}(\rho))^2 + (\hat{r}_{h'h}(\rho))^2\right\}d\rho$, $0 < \rho < T_n$, with $r_{h'h}(\rho) = \int_0^{\rho}w_h(t)w_{h'}(t)dt$ and $r_{h'h}(\rho) = \int_0^{\rho}w_h(t)w_{h'}(t)dt$ being the partial cross-correlation functions between WPs [7],
- $\text{var}\left[\sum_{l=1}^{L}\beta_{k_{\Re}l}\right] = \sum_{l=1}^{L}\text{var}\left[\beta_{k_{\Re}l}\right] = \Omega\sum_{l=0}^{L-1}\text{var}\left[\beta_{k_{\Re}l}\right] = \Omega\, Q_x$ with $\Omega = \text{var}[\beta_{k_{\Re}1}]$ and Q_x represents the sum of amplitude levels of all multipath components for *x* intensity profile. It depends on the nature of the Multipath Intensity Profile (MIP).

The noise variance, $\sigma^2_n = \text{var}[\tilde{n}^{11}]$, and the jamming variance, $\sigma^2_{\Im} = \text{var}[\tilde{\Im}^{11}]$, are respectively, given by [4]-[5] by

$$\sigma^2_n = \frac{P(N_nT)^2}{2}\left[\frac{H\Omega}{N_n(E_s/N_o)}\sum_{m=-M_1}^{M_2}\alpha^2_m\right] = \frac{P(N_nT)^2}{2}NI, \quad (13)$$

$$\sigma^2_{\Im} = \frac{P(N_nT)^2}{2}\times\left[\frac{H\Omega(\Im/S)}{N_n^2}\sum_{m=-M_1}^{M_2}\sum_{m=-M_1}^{M_2}\alpha_{m_1}\alpha_{m_2}\sigma^2_J(m_1,m_2)\right] = \frac{P(N_nT)^2}{2}\Im I, \quad (14)$$

where $N_o/2$ is the double sided power spectral density for the AWGN, $E_s = 2P\Omega T$ is the mean received symbol energy, $S' = 2P\Omega$ and $\sigma^2_{\Im}(m_1,m_2)$ is given by [6, eq. (19)] as

$$\sigma_J^2(m_1, m_2) \approx \int_{-1}^{1} \left[sign\left[1 - p\left|xN_n - m_1 + m_2\right|\right] \times \cos\left[2\pi q\left(xN_n - m_1 + m_2\right)\right]\left(1 - |x|\right)\right] dx,$$

given that $N_n >> 1$, $q = \Delta T_n = 2\Delta/B_s$ (ratio of the offset of interference carrier frequency to half-spread spectrum bandwidth) and $sign[x] = x$ or zero for $x \geq 0$ or $x < 0$, respectively. Using (10), (11), (13) and (14), the SIJNR, γ, can be written as

$$\gamma = \frac{S}{\sigma_{TI}^2 + \sigma_n^2 + \sigma_{\Im}^2} = \beta_{1_1 1}^2 Y, \tag{14}$$

$$Y = \left[MI + NI + \Im I\right]^{-1}. \tag{15}$$

V. Bit Error Rate and Outage Probability Performances

The average bit error rate, $\overline{P}_e$ is obtained by averaging the instantaneous BER of SIJNR, γ, over the channel fading functions. The outage probability represents the probability of unsatisfactory reception of the signal over the intended coverage area. In this paper, we define the P_{out} as the probability that γ falls below γ_{th} [9, page 5], i.e. $P_{out} = \Pr(\gamma < \gamma_{th}) = \int_0^{\gamma_{th}} f_\gamma(\gamma) d\gamma$. $\overline{P}_e$ and P_{out} depend on the method of diversity combining employed. Since Maximal Ratio Combining is the optimum diversity combining technique [10], this diversity technique is used in this paper.

The Maximal Ratio Combining (MRC) depends on the idea that: the components of the received signal with high amplitudes contain relatively low noise power level. Thus, their effect on the decision process can be increased by squaring their amplitude [11]. In MRC technique, the diversity branches are weighted according to their amplitude and then combined. In [7] it was found that the average BER $\overline{P}_e^{MRC}$ and P_{out}^{MRC} are given respectively as.

$$\overline{P}_e^{MRC} = \frac{1}{\Gamma(Im)} \int_0^\infty \left[\left(\frac{m}{\Omega Y}\right)^{Im} \gamma^{Im} \exp\left(-\frac{m\gamma}{\Omega Y}\right) \times \left(\frac{1}{2} erfc\left(\sqrt{\frac{Y}{2}\sum_{i=1}^{I} \beta_{1_1 1_i}^2}\right)\right)\right] d\gamma, \tag{17}$$

$$P_{out}^{MRC} = \tilde{G}\left(\frac{m\gamma_{th}}{\Omega Y}, Im\right) \tag{18}$$

where $\Gamma(m)$ is the gamma function, $\tilde{G}(x, m)$ is the incomplete gamma function and Y is given by (16).

VI. Determination of Suppression Filter Coefficients

It is shown in [6] that the coefficients of the suppression filter can be determined using

$$\sum_{m=-M_1, m\neq 0}^{M_2} \alpha_m \rho(n - mT_n) + \rho(nT_n) = 0, \quad n = -M_1, \cdots, -1, 1, \cdots M_2 \tag{19}$$

where $\rho(\upsilon T_n)$ is a lowpass autocorrelation function consists of three components

$$\rho(\upsilon T_n) = \rho_s(\upsilon T_n) + \rho_n(\upsilon T_n) + \rho_{\Im}(\upsilon T_n) \tag{20}$$

where $\rho_s(\upsilon T_n)$ is the lowpass version of the desired signal, $\rho_n(\upsilon T_n)$ is due to noise and $\rho_{\Im}(\upsilon T_n)$ is due to narrowband interference. Using the same method as [5] we can show that $\rho_s(\upsilon T_n)$ for certain service rate is given by

$$\rho_s(\upsilon T_n) = 2P\Omega \aleph K_{\Re} LHJ \tag{21}$$

The $\rho_n(\upsilon T_n)$ and $\rho_{\Im}(\upsilon T_n)$ are given in [6] by

$$\rho_n(\upsilon T_n) = \begin{cases} 2N_o/T_n, & \upsilon = 0 \\ 0, & \upsilon \neq 0 \end{cases} \tag{22}$$

$$\rho_{\Im}(\upsilon T_n) = \begin{cases} \Im(1 - |\upsilon|\, p)\cos(2\pi\upsilon q), & |\upsilon| \leq \mathrm{int}[1/p] \\ 0, & |\upsilon| > \mathrm{int}[1/p] \end{cases} \tag{23}$$

where $\mathrm{int}[x]$ is defined as the integer part of x. Substitute (21), (22) and (23) in (20) we obtains

$$\Psi = \begin{cases} \aleph K_{\Re} LHJ + 2N_n(E_s/N_o)^{-1} + \Im/S', & \upsilon = 0 \\ (\Im/S')(1 - |\upsilon|\, p)\cos(2\pi\upsilon q), & |\upsilon| \leq \mathrm{int}[1/p] \\ 0, & |\upsilon| > \mathrm{int}[1/p] \end{cases}$$

$$\rho(\upsilon T_n) = 2P\Omega\Psi \tag{24}$$

From (19) and (24) we can find the coefficients α_m.

VII. Determination of the Variance of the Path Gain

The variance of the path gain $\left(\mathrm{var}\left\lfloor \sum_{l=1}^{L} \beta_{k_{\Re} l} \right\rfloor\right)$, depends on the nature of the Multipath Intensity Profile (MIP). Three kinds of MIP are used in this paper, namely exponential, uniform and Gaussian MIPs. The values of them are shown below:

A. Uniform MIP

For the uniform MIP, all multipath components amplitude levels are the same. Therefore,

$$\mathrm{var}\left\lfloor \sum_{l=1}^{L} \beta_{k_{\Re} l} \right\rfloor = \sum_{l=1}^{L} \mathrm{var}[\beta_{k_{\Re} l}] = \mathrm{var}[\beta_{k_{\Re} l}] \sum_{l=1}^{L} 1 = \Omega Q_u$$

where $Q_u(\bullet)$ is the sum of amplitude levels of all multipath components for the uniform MIP and is given by

$$Q_u = \sum_{l=0}^{L-1} 1 = L \tag{25}$$

B. Exponential MIP

In this case, the different amplitude levels of different paths have exponential relation with the first arrived signal component [12]. Assume the amplitude decaying factor of is δ and the path gains are independent identically distributed random variable, then

$$\mathrm{var}\left\lfloor \sum_{l=1}^{L} \beta_{k_{\Re}l} \right\rfloor = \Omega\, Q_e(L,\delta),$$

where $Q_e(\bullet)$ denotes the sum of amplitude levels of all multipath components for the exponential MIP and is given by [13, page 87]

$$Q_e(L,\delta) = \sum_{l=0}^{L-1} e^{-\delta l} = \frac{1-e^{-\delta l}}{1-e^{-\delta}} \tag{26}$$

C. Gaussian MIP

For the Gaussian MIP profile, the successive amplitude levels will follow a Gaussian distribution. Let the mean and variance of the distribution be, respectively μ and σ^2, then

$$\mathrm{var}\left\lfloor \sum_{l=1}^{L} \beta_{k_{\Re}l} \right\rfloor = \Omega Q_G(L,\mu,\sigma^2)$$

where $Q_G(\bullet)$ denotes the sum of amplitude levels of all multipath components for the Gaussian MIP and it is given by

$$Q_G(L,\mu,\sigma^2) = \sum_{l=0}^{L-1} \frac{1}{\sqrt{2\pi}\sigma} e^{-(1-\mu)^2/2\sigma^2} \tag{27}$$

The above expression does not have a closed form solution. But, it can be noted that if $\mu = 0$ and $\sigma = 1/\sqrt{2\pi}$ then the value of the first term of $Q_G = 1$. This matches exactly the cases of Q_u and Q_e. Because of that, Gaussian distribution of $(0, 1/\sqrt{2\pi})$ is assumed for the Gaussian profile in all comparisons.

VIII. Results and Discussions

Using the above analytical results and by means of the MATLAB program, we evaluate the BER performance and P_{out} performance for the system. The performance of the system is tested with and without suppression filter in presence of narrow-band interference. The effect of service rates, wavelet packet families and the length of their filter, $\Im/S'$ and the multiple number of service rates are illustrated. Unless otherwise mentioned, the numerical results were generated using:

- *Daubechies* wavelet packets with order 6 (*db*6),
- MRC diversity with mean path gain, $\Omega = 10$, diversity order $(D) = 3$ and Nakagami parameter $(m) = 1$,
- Exponential MIP with $\delta = 0.5$ and $L = 3$,
- Processing gain length $(N_n) = 512$ with chip duration $(T_n) = 10^{-8}\,s$,
- Ratio of the interference bandwidth to the system bandwidth $(p) = 0.1$,
- Offset for the interference carrier frequency with respect to signal carrier frequency $(\Delta) = 0$. This means that the narrowband interference exists at the middle of the CDMA spectrum,
- Interference jamming power to signal power $(\Im/S') = 40$ dB,
- Threshold of SINJR $(\gamma) = 35$ dB,
- Number of superstreams $(H) = 3$,
- Double Sided Suppression Filter: DS3 has $M_1 = 1$ and $M_2 = 1$, thus it has three taps and DS5 has $M_1 = 2$ and $M_2 = 2$, thus it has five taps,
- Three service rates:
  - Low Rate with number of substreams $(J_{LR}) = 2$ and number of users $(K_{LR}) = 20$,
  - Medium Rate with number of substreams $(J_{MR}) = 4$ and number of users $(K_{MR}) = 10$,
  - High Rate with number of substreams $(J_{HR}) = 8$ and number of users $(K_{HR}) = 5$.

A. Effect of Suppression Filter and Its number of Tap

The effect of using DS3 SF and DS5 SF on HR service group is illustrated in Fig. 2 using BER versus E_s/N_o and in Fig. 3 using P_{out} versus threshold value of E_s/N_o (γ_{th}). As expected, using SF and increasing the number of taps improve the performance of the system.

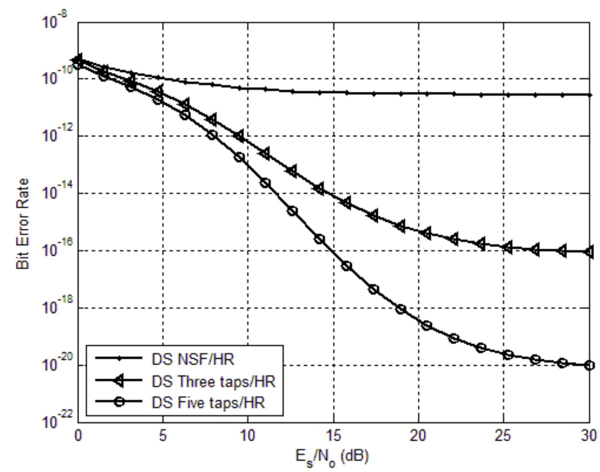

Fig. 2 BER performance versus E_s/N_o for HR service with and without SF

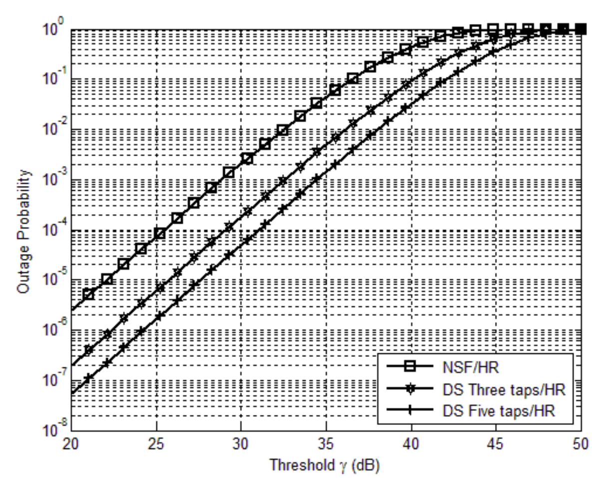

Fig. 3 P_{out} performance versus γ_{th} for HR service with and without SF

B. Effect of Service Rate and Number of Filter Taps

Fig. 4 and Fig. 5 shows respectively, the BER versus E_s/N_o and P_{out} versus γ_{th} for the three service groups using DS3 and DS5. As it can be seen from the two figures, increasing the service rate improve the BER and P_{out}. This is because as the number of service rate increases the number of substreams increases, thus the symbol duration increases. Accordingly, the percentage of distorted information is getting smaller resulting in better system performance. Also, as the number of taps increases the two performances are improved.

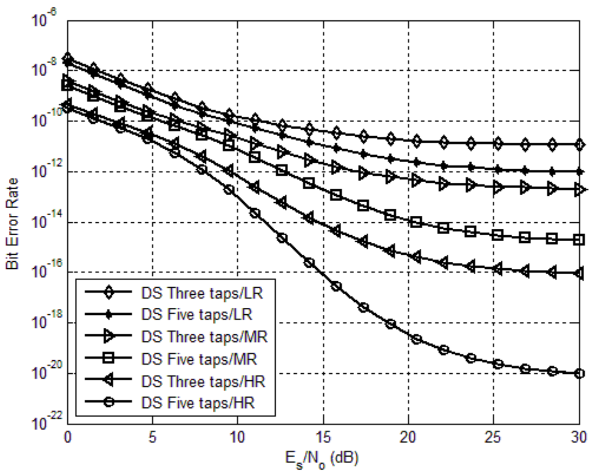

Fig. 4 BER performance versus E_s/N_o for the three service rates using the DS3 and DS5

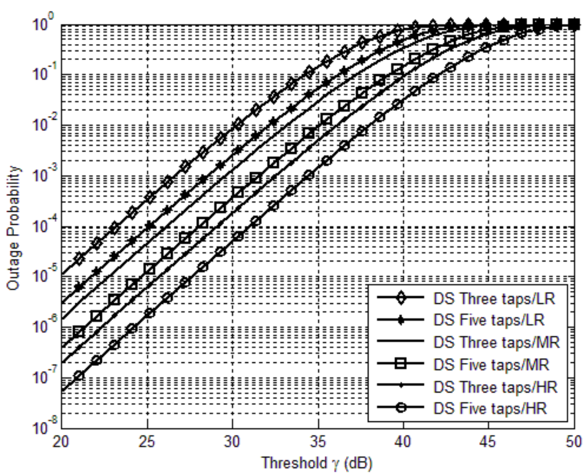

Fig. 5 P_{out} versus γ_{th} for the three service rates by means of DS3 and DS5

C. Effect of Wavelet Family and its Filter Length

The variance *MI* as given in (12) depends on the cross-correlation of the wavelet packets. The properly choosing of wavelet packets can decrease the cross-correlation of the wavelet packets, therefore reducing the *MI* variance and improving the system performance. Several factors affect the cross-correlation properties of the wavelet packets, such as the family of the wavelet packets and the filter length. The filter length, for any wavelet packets depends on number of scaling numbers that are used to present the scaling signals of the wavelet packets family and the order of the wavelet packets. For *Daubechies*, *Symmlets* and *Coiflets* if the order of the wavelet packets is N, then the filter length is $2N$, $2N$ and $6N$, respectively. Two wavelet packets from three different families are used for this simulation, namely

- *Daubechies* wavelet packets with order 6 (*db*6) and 9 (*db*9)[14],
- *Symmlets* wavelet packets with order 6 (*Sym*6) and 9 (*sym*9) [15],
- *Coiflets* wavelet packets with order 2 (*coif*2) and order 3 (*coif*3) [15].

These families are chosen because they are orthogonal and have compact support, that is, fast decaying time which results in good localization in both frequency and time.

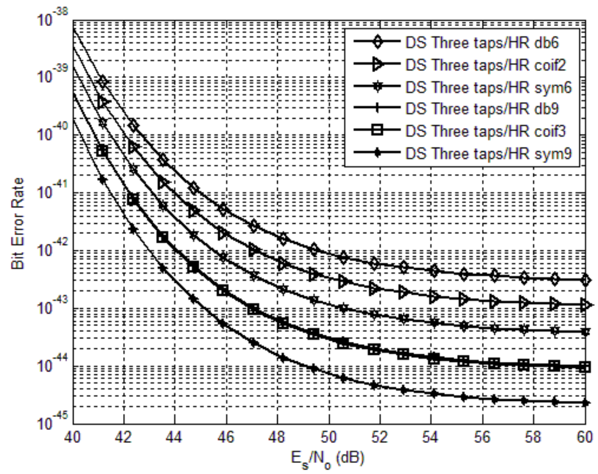

Fig. 6 BER performance versus E_s/N_o for three WPs family and two filter lengths for each one of them

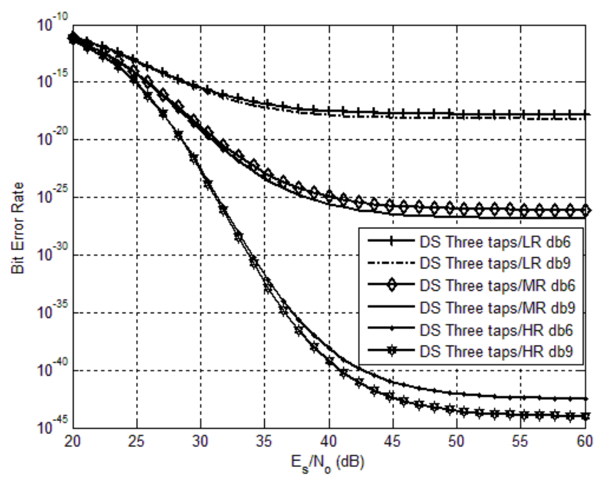

Fig. 7 BER performance versus E_s/N_o for the three service rates using *db6* and *db9* wavelet packets.

Fig. 6 illustrates the BER performance as a function of E_s/N_o for the three wavelet families. The filter lengths are taken to be 12 and 18. It is clear from the figure that as the filter length increases the BER performance improved. This is because, as filter length increases the filter characteristics approach the ideal filter characteristics, so the wavelet packets cross-correlation decreases and BER improved. Also, notice that *sym*6 has better performance than *db*6 and *coif*2, and *sym*9 has better performance than *db*9 and *coif*3 although the differences are not significant. The reason is that for certain filter length, these wavelet packets have approximately equal cross-correlation properties.

Fig.7 shows the BER performance as a function of E_s/N_o for the three service rates by means of two *Daubechies* wavelet packets, namely *db6* and *db9*. It is clear from the figure that as the filter length and the rate increases the performance level increases.

D. Effect of Multipath Intensity Profile

Fig. 8 shows the effects of uniform MIP, exponential MIP and Gaussian MIP on BER performance for the three service rates. For fair comparison, the first path is assumed to arrive at the receiver with unity gain. Thus, $\mu = 0$ and $\sigma = 1/\sqrt{2\pi}$ is taken for Gaussian profile. The decay factor for exponential MIP is taken equal to $\delta = 0.5$. The MIP performance is affected by the Q_x parameter, the values of this parameter are 3 for uniform MIP, 1.9744 for exponential MIP and 1.0432 for Gaussian MIP which are calculated using (25), (26) and (27) for uniform MIP, exponential MIP and Gaussian MIP, respectively. The less the value of Q_x, the less the variance for noise plus interferences, the better is the BER. Since Q_x for Gaussian profile is less than Q_x for exponential and uniform profiles, then Gaussian MIP possesses faster decrement of successive amplitude levels of multipath than exponential MIP and uniform MIP. Because of that Gaussian MIP has better performance as it can be noted from Fig. 8.

Note that the following parameters are used to generate Fig. 6, Fig. 7 and Fig. 8:

- $\Im/S' = 15$ dB,
- $H = 7$,
- $J_{LR} = 1$, $K_{LR} = 100$, $J_{MR} = 2$, $K_{MR} = 50$K, $J_{HR} = 4$ and $K_{HR} = 25$.

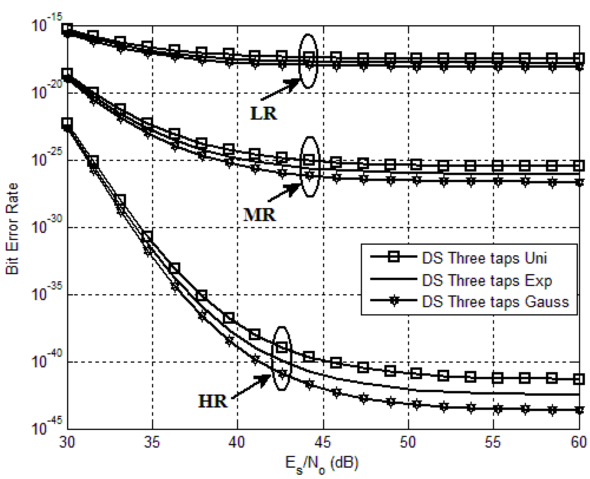

Fig. 8 Effect of MIP on BER performance for the three service rates

E. Effect of Jamming

With number of superstream $H = 5$, $J_{LR} = 1$, $K_{LR} = 100$, $J_{MR} = 2$, $K_{MR} = 50$K, $J_{HR} = 4$ and $K_{HR} = 25$ the effect of $\Im/S'$ and number of filter taps on the three service rates is demonstrated on Fig. 9. The following can be noted from the figure:

- As $\Im/S'$ increases the performance of the system degraded.
- The HR service outperforms the other two services and MR service outperform the LR service.
- For $\Im/S' < 30$ dB and for all rates the DS3 SF outperform the DS5 SF but for $\Im/S' > 30$ the performance for DS5 SF is much better than that for DS3 SF.

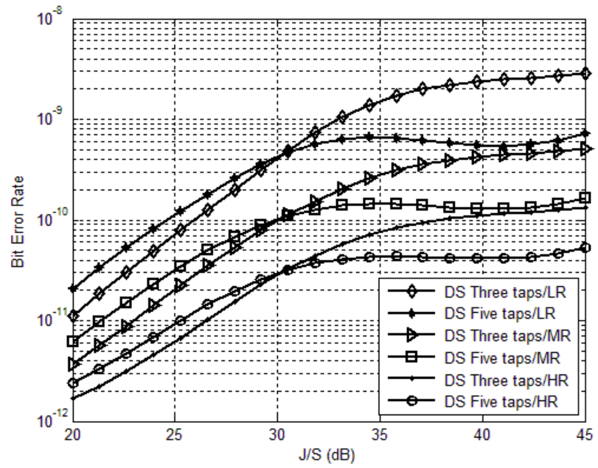

Fig. 9 Effect of $\Im/S'$ and filter taps on BER for the three service rates

F. Effect of number of Code

Fig. 10 shows the effect of increasing the number of codes (substreams) on BER while Fig. 11 shows the same effect on P_{out} using $\gamma_{th} = 30$ dB. The two figures demonstrate the effects at the three service groups using DS3 and DS5 SFs. The number of codes for all service groups are multiplied by $(2, 3, \cdots, 10)$. For example, when the number of codes is multiplied by 2 means that number of codes in LR, MR and HR are 40, 20 and 10, respectively. We can notice that the performance is improved as the number of codes increased. This is expected since as the number of substreams increases, the symbol duration increases. Accordingly, the percentage of distorted information is getting smaller resulting in better performance. Also, from (12) we can note that as *J* increases MI variance decreases, thus the performance of the system improved. This is a trade-off between cost and quality, since increasing the number of substreams will leads to more complex and expensive system but with better performance.

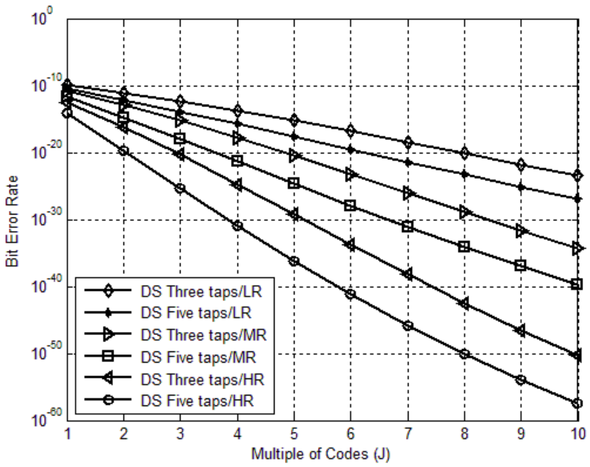

Fig. 10 Effect of multiple number of codes on BER

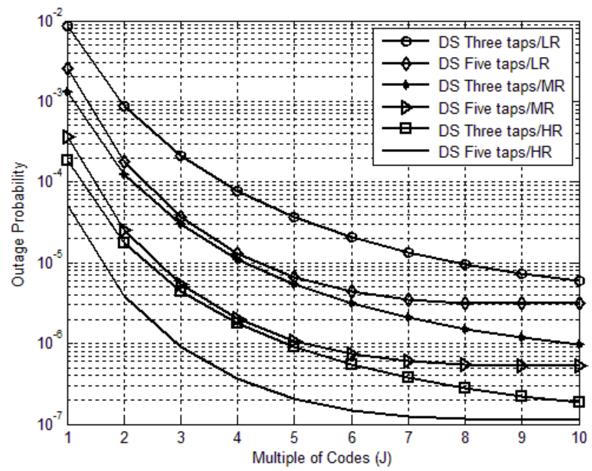

Fig. 11 Effect of multiple number of codes on P_{out}

IX. Conclusion

The BER and P_{out} performances for WP-MC\MCD-CDMA system employing MCR and SF in the presence of a narrow-band BPSK has been investigated. The results presented in this paper are consistent with other published results. It has been shown that high rate service outperforms the other service rates. For each service rate, by increasing the number of codes the symbol duration increases and thus the percentage of distorted information gets smaller. Accordingly, the system performance improved. Also, it is found that the two performances are improved by using suppression filter, furthermore improvement can be noticed when increasing the number of taps in the filter provided that $\Im/S'$ >30 dB. The system performance depends on the cross-correlation properties of the wavelet family and the filter length of it. It is found that *Symmlets* wavelet packets have better performance than *Daubechies* wavelet packets and *Coiflet*s wavelet packets, but the differences in their performance are marginal. As the filter length of WPs increases, the system characteristics approach the ideal filter conditions, thus the cross-correlation decreases and better system performance can be noticed.

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