Peak Field Approximation of Shock Wave Overpressure Based on Sparse Data

Yongli Zhang, Tailin Han, Yuqun Chen, Enkui Zhang, and Xuan Liu

***Abstract*—**To obtain the shock wave field distribution, two kinds of caliber weapons shock wave overpressure by the electric measuring method are analyzed, and the attenuation formula in far field is corrected, and the approximation method of overpressure peak field with cubic spline interpolation is put forward. The sparse array data of small caliber muzzle shock wave field measured by sensor array is first analyzed and the curve fitting was carried out by using the correction formula in the radial direction; secondly, cubic spline interpolation was used in the circumferential direction; and lastly, Delaunay triangulation method was used to divide the mesh to approximate the overpressure peak surface. Compared with the cubic spline interpolation results obtained by this method, the approximate isobars showed obvious advantages. Further, the large-diameter muzzle shock wave using polar coordinate sensors were tested, and the approximate method of sparse matrix overpressure data processing field effect is good. The approximate method provides reference for the drawing of the shock wave field by polar coordinate.

***Keywords*—**peak value of overpressure; curve fitting; cubic spline function; surface approximation.

I. INTRODUCTION

NUMERICAL simulation of the initial flow field of muzzle shock wave is used to obtain the peak value of overpressure, etc. With the propagation of shock wave (especially in the far-field) and the enlargement of the calculation area, the calculation amount increases rapidly and the calculation accuracy decreases [1]-[2]. The decrease of the simulation precision of the far field of the shock wave caused the error of overpressure peak to increase. The shock wave test of conventional weapon shock wave is mostly carried out by electrometric method to ensure the accuracy of the data, and the shock wave data is obtained through sensor array calibrated dynamically. The overpressure peak surface and its contour are interpolated by mathematical algorithm [3]-[4]. The reasonable sensor placement method in order to reconstruct explosion overpressure field at the highest precision is very necessary, especially Interpolation algorithm methods [5]-[6].

Because of restrictions on the launching conditions of weapons and space areas, there is a shortage of standards of test method of the safety and the serviceability of gun for placing one hundred sensors that are placed in the 4.5m range of the X axis and the Y axis by using Cartesian coordinate on the one side of the muzzle. Polar coordinate is used to reduce the number of sensors for a naval gun test that the number of sensors is twenty-five [7]. Based on the advantages of polar coordinate [8], the experiment of muzzle flow field was carried out in high altitude environment to obtain shock wave overpressure data [9].

Based on the above analysis, this study interpolates the sparse data obtained through the polar coordinate to obtain the high accuracy of the shockwave field contour. The overpressure surface is approximated by curve fitting on the radial direction of propagation to modify mathematical model and increasing the data through the cubic spline function interpolation on the circumferential direction. In order to verify the effectiveness of the algorithm, the error was compared with the cubic spline function interpolation.

II. PROPAGATION LAW OF SHOCK WAVE OVERPRESSURE

In the existing literatures, the shock wave overpressure decays exponentially in the far field propagation process for the far-field propagation of shock wave [10], and the mathematical model is simply expressed as follows:

$$\Delta P = K(\theta)\gamma^{-\alpha(\theta)} \tag{1}$$

Where, the coefficient K represents the overpressure value of the shock wave at the beginning, γ represents the distance along the ray, and α represents the attenuation rate of the overpressure value of the shock wave.

In polar coordinate, the radial direction of the fixed angle shows the change of power function. Equation (1) is logarithmically processed as follows:

$$\log \Delta P = \log K(\theta) - \alpha(\theta)\log \gamma \tag{2}$$

The numerical change of the corresponding formula (2) after the logarithmic process has a linear relationship with the independent variable.

III. DATA SOURCES

In this chapter, two different weapons of a minor-caliber gun (7.62mm) [5] and a large-diameter gun [8] are arranged in polar coordinate according to the position of the test

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points. Considering the environmental interference factors, the simulation is carried out under the atmospheric pressure environment. 7.62mm caliber gun is placed in the direction of 0 degrees and the test point is at the origin of the muzzle. The interval between adjacent points is 22.5 degrees and the distance between adjacent measuring points in the radial direction is 0.25m, that is to say, 4 points are placed on the same radial direction, and 28 points are placed in total, and the test results are shown in **Table I.**

Table I Peak overpressure data of a 7.62-mm gun shockwave (*kPa*)

r/θ	0.25 m	0.5 m	0.75 m	1.0 m
22.5°	47.41	18.25	10.73	6.96
45°	41.86	14.69	9.10	5.67
67.5°	29.91	10.32	5.81	3.76
90°	16.76	7.30	4.55	3.06
112.5°	10.66	4.34	2.88	2.36
135°	5.76	3.37	2.10	1.56
157.5°	4.01	2.27	1.26	0.87

And the large-dianmeter gun is placed in the direction of 0 degrees and the test point is at the origin of the muzzle. The interval between adjacent points is 15 degrees and the distance between adjacent measuring points in the radial direction is 1m, that is to say, 5 points are placed on the same radial direction, and 40 points are placed in total, and the test results are shown in **Table II.**

Table II Peak overpressure data of a large-dianmeter gun shockwave (kPa)

r/θ	4 m	5 m	6 m	7 m	9m
15°	21.7	17.3	14	11.9	9.2
30°	22.1	17.6	14.2	10.6	9.4
45°	21.2	13.9	11.1	8.8	8.7
60°	28.1	16.8	12.3	10.1	8.9
75°	26	19.7	14	10.7	8.9
90°	33.2	21.9	17.7	14.3	12.1
105°	30.9	25.9	19	15.4	12.6
120°	40.3	36.9	22.3	16.4	12.9
135°	41.3	37.1	22.6	14.8	15.8

IV. OVERPRESSURE PEAK FIELD APPROXIMATION

Overpressure peak field approximation belongs to surface modeling. By fitting some discrete points in a certain range around any point (x_i, y_i) and its corresponding discrete point $f(x_i, y_i)$, a mathematical model of curved surface is established to solve the corresponding values $f'(x_i', y_i')$ of the interpolation point (x_i', y_i').

According to the general procedure of the algorithm, firstly, the independent curve approximation of the axis x and the axis y is carried out to obtain the single direction more value points. Secondly, the mesh is encrypted and finally the surface is approximated to obtain the surface and contour plot. In the study, the data of the propagation direction (radial direction) are fitted to ensure the accuracy of the data. The cubic spline function interpolation is used to guarantee the first order and second order continuity of the fitting curve. The surface of the pressure peak field is approached by grid encryption.

A. Radial Direction for Fitting

1) Fitting index formula

The three indexes of the curve fitting evaluation index are the sum of squares due to error (SSE), Coefficient of determination(R-square), and Root mean squared error (RMSE) [11]-[12]. The error statistics are analyzed and the formula is given as follows:

$$SSE = \sum_{i=1}^{n} \left| Z_i - Z_i' \right| \tag{3}$$

$$R - square = 1 - \frac{SSE}{\sum_{i=1}^{n} (Z_i - \bar{Z})^2} \tag{4}$$

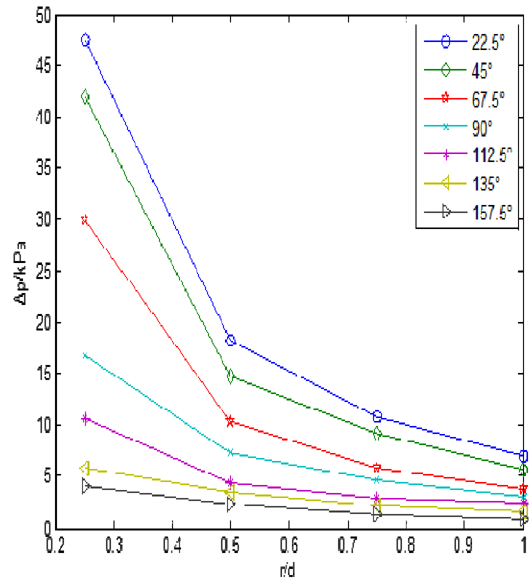
$$RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left(Z_i - Z_i' \right)^2} \tag{5}$$

In the formula, Z_i represents the actual measured values, 、 Z_i' represents the predicted values, $\bar{Z}$ represents the mean values of the first point, n represents the number of points.

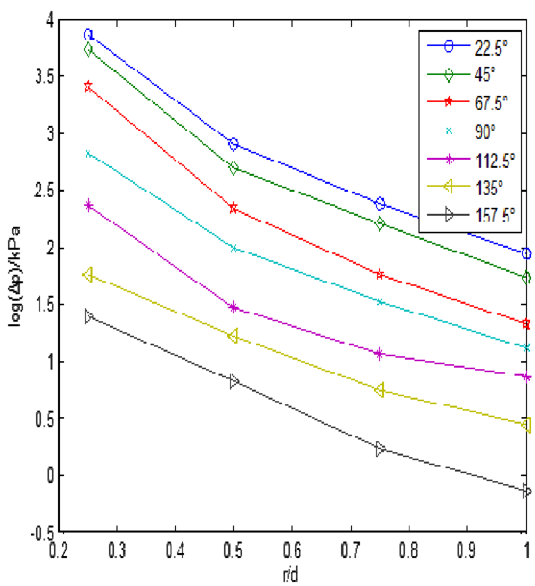
The closer the SSE is to 0, the better the fitting is. The closer R-square is to 1, the stronger the interpretation ability of the fitting data. RMSE reflects the sensitivity and extreme value of fitting values.

2) Date analysis

Peak overpressure data of a 7.62-mm gun shockwave was compared in seven radial directions, as shown in Fig. 1 (a). The consistency of the change of data that The greater the overpressure peak near the original point, the amplitude attenuation in the radius 0.5m range is larger, and the outside change is relatively gentle, which is consistent with the empirical formula (1). The data in different radial directions are logarithmically processed, as shown in Fig. 1 (b). The curve change is not a linear relationship, and the curve fitting error is larger.



(a)Variation of peak value of shock wave with radius at different angles



(b)The peak value of shock wave varies with radius at different angles Logarithmically

Fig. 1 Variation of peak value of shock wave at different angles with radius

The fitting error of the data is larger by using the formula (1), so as to eliminate the error, the correction term is added, and The modified formula is given as follows:

$$\Delta P = K(\theta)\gamma^{-\alpha(\theta)} + f(\gamma) \qquad (6)$$

In the formula, f represents the correction term is linear with the radius γ.

The modified formula (6) is used to curve the data of seven radial directions, and the fitting error is shown in **Table III**. The error data of the 6 directions are ideal, but the error in the direction of 45 degrees is slightly larger. In the radial direction, the variation of the error is consistent, the maximum value of SSE is 0.0679, the value of r-square is higher than 0.99, and the maximum value of RMSE is 0.26. The fitting results of the curve are shown in Fig. 2.

Table III Precision index of curve fitting

error /θ	SSE	R-square	RMSE
22.5°	0.0679	0.9999	0.2606
45°	0.5053	0.9994	0.7109
67.5°	0.0190	1	0.1380
90°	0.0093	0.9999	0.0962
112.5°	0.0002	1	0.0129
135°	0.0184	0.9982	0.1357
17.5°	0.0189	0.9968	0.1375

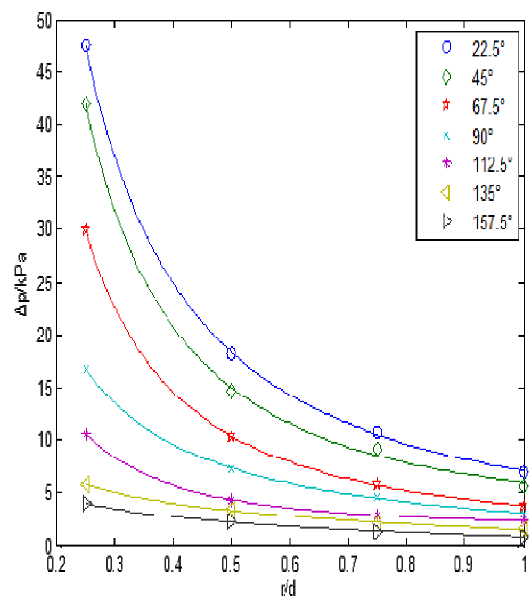


Fig. 2 Curve fitting of peak value of shock wave overpressure with radius

B. *Interpolation Algorithm in Circumferential Direction*

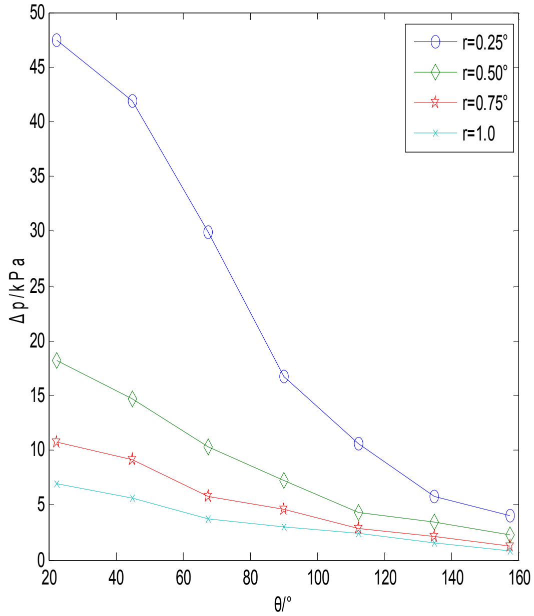
For a given set of known points $(x_i, y_i), i = 1, 2, \cdots, n$, m_j is the slope of type (x_i, y_i), and the expression of cubic spline interpolation function of j section is as follows:

$$y_j(x) = a_j x^3 + b_j x^2 + c_j x + d_j \qquad j = 1, 2, \cdots, n \quad (7)$$

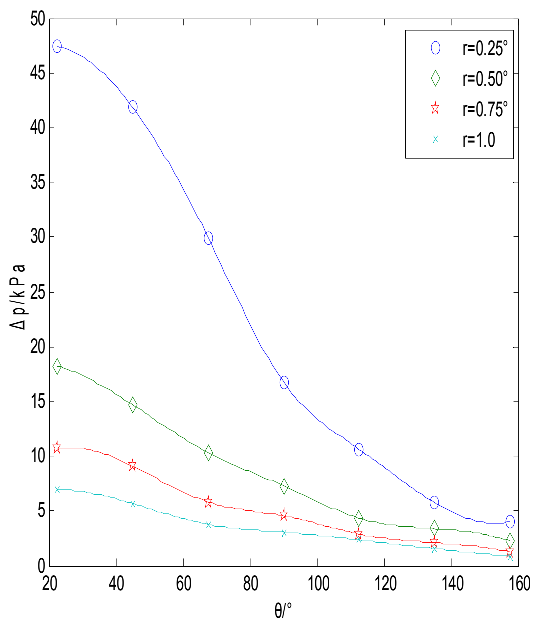
The coefficients of interpolation function can be solved [11].

A comparative analysis on the circumferential direction, the curve in Fig. 3 (a) shows variation of the data is not obvious, it is difficult to use the function fitting uniform, using three order spline function interpolation to ensure smooth curve, interpolation results shown in Fig. 3 (b) shows.

The data were compared in the circumferential direction, as shown in Fig. 3 (a). It is not obvious that the data characteristic changes, and it is difficult to fit in the uniform function. In order to ensure the smoothness of the curve, interpolation is carried out with cubic spline function. And the results of the interpolation are shown in Fig. 3 (b).



(a) Variation of peak value of shock wave overpressure with radian



(b) Peak value of shock wave overpressure varies with radian Three spline interpolation

Fig. 3 Variation and interpolation of different peak radius of shock wave with radian

C. Field Surface Approximation

After the curve fitting of the data points in the radial direction, the interval of 0.01m in the range of 0.25m-1.0m was calculated, and 72 data points were added, and the original data point (4) was increased to a precise 19 times. The cubic spline function is used to interpolate in the circumference direction, and the same interval is 0.01m. Data extraction is carried out in two directions of radial and circumferential directions respectively. Data extraction is carried out in two directions of radial and circumferential directions. The data is meshed by Delaunay triangulation method, and the shock wave overpressure peak surface (Fig. 4) and the overpressure contour are plotted.

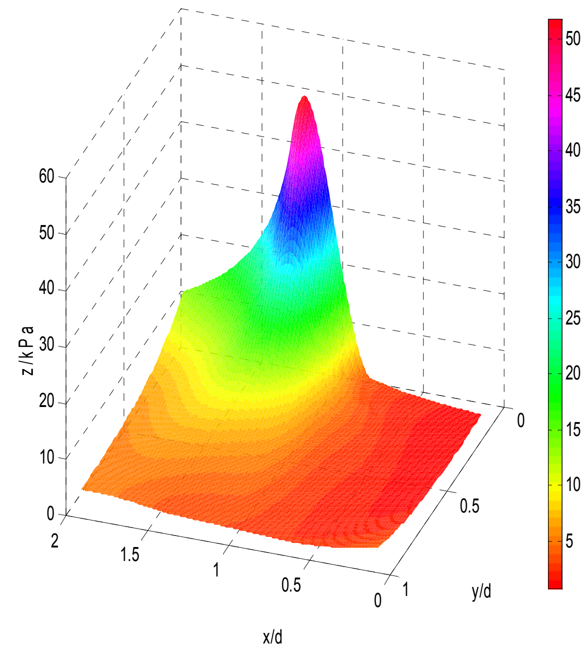
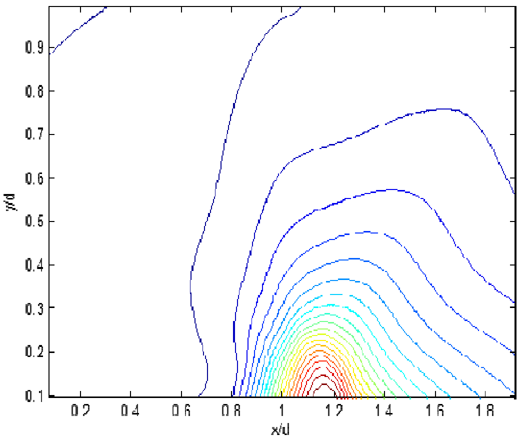


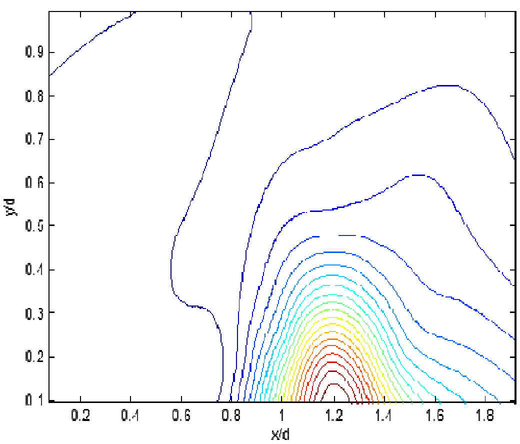
Fig. 4 Peak surface of shock wave overpressure

From the peak overpressure of shock wave surface in Fig. 4, it can be clearly seen that the shock wave of the chamber has the movement rules, the intensity attenuation and the directivity.

In order to verify the effectiveness of the method, the interpolation of the cubic spline function is compared, and the results are shown in Fig.5. The following conclusions can be seen from figure 5. The macroscopic analysis shows that the two methods all reflect the characteristic of peach shape distribution, and the intensity of the overpressure peak is greater than that of the x axis, that is, the blast of blast chamber is directional. But the peak value of radial overpressure has a significant change. The attenuation of cubic spline interpolation is not consistent in the radial direction of about 67.5 degrees, which is not consistent with the previous data analysis. In about 112.5 ° direction, the curve changes, that is to say cubic spline algorithm ensures the curve smooth, but also cause certain influence to the distribution of original data. The validity of this method can be obtained by comparing the overpressure contour lines of two methods in Figure 5.



(a)the corrected mathematical model



(b)cubic spline interpolation

Fig. 5 Isocline contrast map

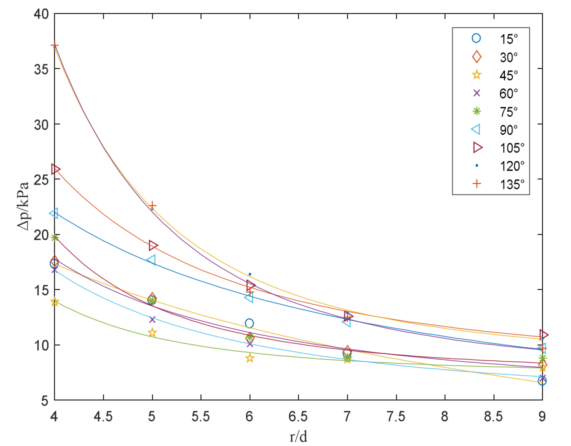
D. Application

The sparse array data of small caliber muzzle shock wave field measured by sensor array is analyzed and the curve fitting was carried out by using the correction formula in the radial direction. On the basis of this, the effect of isocline contrast map is better than the interpolation of cubic spline functions. Then, the method is used to process data of a large-diameter gun shown in **Table II**. The modified formula (6) is used to curve the data of 9 radial directions, and the fitting error is shown in **Table IV**. The error data of the 9 directions is ideal too.

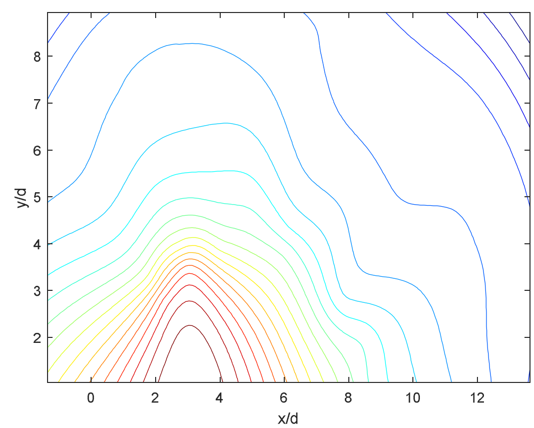
Table IV Precision index II of curve fitting

error /θ	SSE	R-square	RMSE
15°	0.2771	0.9959	0.3722
30°	0.8854	0.9851	0.6653
45°	0.4141	0.9824	0.455
60°	0.08699	0.9985	0.2086
75°	0.8302	0.9901	0.6443
90°	0.1698	0.9982	0.2914
105°	0.2653	0.9981	0.3642
120°	0.1185	0.9997	0.2434
135°	0.9431	0.9981	0.6867

The larger the caliber, the more complex the shock wave. The influence of the bore device and ground reflection on the far-field propagation of the large bore mouth shock wave is different from that of the small diameter, but the radial attenuation rule is consistent. As can be seen from Fig. 6 (a), 90 degrees is the dividing line, and the radial overpressure peak is the strongest between 105~120 degrees, and the 45 degrees is the weakest. It can be seen from Fig. 6 (b) that the distribution of the large-diameter far field is more uniform.



(a)Curve fitting of peak value of shock wave overpressure with radius



(b) The overpressure contour lines

Fig. 6 The results of the large-diameter gun

V. CONCLUSION

To obtain the overpressure peak field approximation method, the peak value of shock wave overpressure by polar coordinates is analyzed, and the mathematical model of attenuation rule is corrected. In the radial direction, the modified mathematical model is adopted to curve fitting, and the overpressure peak surface is approximated, and the contour line is drawn. In order to verify the effect, the interpolation of cubic spline function is compared. The following conclusions could be drawn:

(1) The modified formula curve fitting is suitable for different diameter shock wave fields.

(2) The overpressure value predicted by the modified

propagation formula is more accurate and provides a reference for the prediction of overpressure value in middle and far field.

(3) It is more reasonable for the characteristics of the overpressure field surface with the added point value data interpolation and the contour line precision is high.

Thus, the ground reflection is not considered, and the prediction formula is to be verified in the far field. More influencing factors should be considered in future studies.

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