A Quadratic Optimization Model for Dynamic Intensity Modulated Radiotherapy and Volumetric Modulated Arc Therapy with Tongue and Groove Constraints

Yihua Lan, Jinjiang Liu, Yang Wang, Xiao Song and Chih-Cheng Hung

***Abstract*—**The paper provides an improved model for quadratic programming for dynamic intensity modulated radiotherapy (dIMRT) and volumetric modulated arc therapy (VMAT) schemes. This improved model suppresses the total beam-on-time as well as tongue and groove chamfer effects. First, we model the goal of clinical dose with the traditional quadratic programming technique. Then, we describe leaf-moving trajectory matrices and obtain the relationship between trajectory matrices and the fluence map matrix. We establish convex constraints for the leaf collision and tongue and groove chamfer effects based on the relationship. Furthermore, by analyzing the relationship between the leaf movement speed and leaf moving trajectory matrix as well as the relationship between the beam-on time and leaf moving trajectory matrix, we establish the convex constraint of the leaf moving maximum speed and the convex constraint of the total number of monitor units (the beam-on time). Finally, considering the limitation for the unidirectional leaf-moving pattern, we form the bidirectional leaf-moving pattern and embed it into the model. This paper proposes a theoretical model which can meet the majority of the clinical demand. It is also easy for the implementation of hardware and software aspects of constraints by multi-leaf collimator.

***Keywords*—**dynamic intensity modulated radiotherapy, volumetric modulated arc therapy, tongue and groove chamfer effects, total number of monitor units, multi-leaf collimator.

I. INTRODUCTION

At present, the main treatment methods for malignant tumors include surgical treatment, chemical treatment and radiation treatment. Especially for terminal-stage cancer cases, radiation treatment is quite popular [1]. In the radiation treatment, the most widely used treatment is intensity modulated radiation therapy (IMRT) [2], which is the conformal radiotherapy with intensity modulation dimension. By using the multi-leaf collimator (MLC), the IMRT can be implemented by the static (step and shoot) model as well as the dynamic model.

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Comparing with the static IMRT, the volumetric modulated arc therapy (VMAT)[3] needs to deal with the higher dimension data. Therefore, it introduces a huge amount of calculations, much more than that of the static IMRT. For the processing capacity of the general computing devices, it is difficult to find the optimal solution or even the sub-optimal solution of deterministic algorithms in a limited amount of time. Hence, the commonly used algorithms in the current VMAT inverse planning system are heuristic methods. The heuristic algorithm can effectively converge faster, but it may not obtain the optimal solution.

Another widely adopted strategy is to decompose the optimization process into two major steps: the fluence map optimization process and the leaf sequencing optimization process. The advantage of this two-step method for solving the inverse plan is that it can decompose the original complex problem into two independent and relatively easy sub-problems for solutions. Hence, in solving sub-problems, we can always get the optimal solution. On the other hand, the disadvantage of the two-step method is that the solution is indirect optimized results, which may reduce the quality of the final plan, especially if two-step optimizations are not well linked. Therefore, the fluence map smoothing strategy is widely used in the two-step method.

II. RELATED WORKS

Several commercial companies have launched their own IMRT/VMAT reverse planning systems such as Varian, Elekta, Philip and Simens; however, these systems use different models and optimization methods.

Varian developed the Eclipse RapidArc inverse planning system in which the Direct Aperture Optimization (DAO) method was used [4-7]. For the typical cases, it uses single arc and multiple control points, while for special cases it adopts multi-arc to ensure dose covering. The DAO method optimizes the leave position for the multi- leaf collimator as well as the beam intensity calculation by using the simulated annealing (SA) algorithm. After each leaf position changed, the algorithm automatically confirms if dose coverage constraints are in line with the requirements or not. Once the violation of constraints occurs, the algorithm will repeat the optimization calculation again.

Elekta produces the ERGO++/Monaco VMAT inverse planning system [8, 9]. The ERGO++ module in the system uses the beam shape based on the anatomy of the patient, optimizes the leaf position directly, and obtains the intensity map of the sequence beam in each arc. In meeting the objective just defined, it needs further sampling and calculation when the requirement of doses is not satisfied. The Monaco VMAT module accomplishes this calculation by using the simulated annealing (SA) optimization algorithm, and adopting the uniform direction movement patterns for the MLC leaves.

Philip proposes the Pinnacle SmartArc treatment planning system [10]. The planning system is the improved and expanded product on the function of the Pinnacle DMPO planning system, which is developed by RaySearch in Stockholm, Sweden. The workflow of the entire system is as follows: it sets the arc parameters first, and then obtains the fluence map of each beam by the IMRT inverse planning system. After that, it considers the sliding window parameters to realize the leaf sequencing for the map segmentation. Finally, it derives the optimized inverse plan by combining the machine constraint parameters (including leaf-moving speed, the dose modulation rate, and the frame rate, etc.) through the optimization process again.

One of the key steps in the IMRT planning system is the inverse planning calculation. The inverse planning calculation input includes the clinical dose demands for the target area, the protection dose constraints for normal tissues as well as the organ at risk, and hardware constraints output X-rays modulation device, etc. While the inverse planning calculation output is the available hardware implementation files for radiation therapy, which include the leaf sequencing location and speed, as well as the number of monitor units.

The mathematical model of the inverse planning calculation is established by the large-scale variable along with complex objectives and constraints description; many manufacturers use the heuristic algorithm to recognize the whole optimization process. For example, the Varian treatment planning system uses the DAO to achieving the optimal solution of multi-leaf collimator leaves for each beam. The Elekta treatment planning system uses the SA optimization algorithm to obtaining the fluence map by solving the optimization of the beamlet leaf sequencing directly. Philip obtains the fluence map of each beam by the IMRT inverse planning system and performs the map segmentation by sliding window parameters.

Although the heuristic algorithm can quickly converge to a solution, the result may not be the optimal. Unlike the heuristic algorithm, many non-heuristic optimization methods can obtain the global optimal solution [11]. However, the mathematical model that is applicable to the non-heuristic optimization algorithm is very limited. The convex programming mathematical model is suitable in this aspect.

III. Our Proposed Methodology

The traditional non-convex mathematical programming model does not always provide the global optimal solution..

This paper provides a kind of the optimization method for the dynamic intensity modulated radiation therapy (dIMRT) and volumetric modulated arc therapy (VMAT) which is based on the quadratic optimization model to reduce the total number of monitor units (beam-on-time).

A. Quadratic programming model meets the demand of clinical treatment

We assume that the treatment target area is T which can be segmented into several parts according to the different dose demand. First, we obtain the Gross Tumor Volume (GTV), which is the tumor area confirmed by the medical doctor using some examination methods such as pathological examination or histopathological examination. The tumor area and the potential subclinical tumor will then form the Clinical Target Volume (CTV). Finally, the Planning Target Volume (PTV) can be obtained by expanding the CTV according to some indefinite factors (e.g. the possible organ movements, the daily set-up errors, the location of target area and the possible varieties of the shape in radiation exposing [12].

The Planning Target Volume (PTV) is set to u parts. Other non-diseased organs along with the key protection organ are assumed to be the non-target area N, which is set to v parts. The model will formulate the target cell irradiation as the objective with respect to the non-target cell protection as constraints. The constraint conditions for ordinary parallel organs adopt average dose constraint, and for the protection of parallel organs, the constraint uses the maximum dose. We assume that the clinical treatment dose for the target is denoted by $\boldsymbol{d}^{TP}=\left\{d_1^{TP},d_2^{TP},\cdots,d_u^{TP}\right\}$; the minimum dose constraint by $\boldsymbol{d}^{TL}=\left\{d_1^{TL},d_2^{TL},\cdots,d_u^{TL}\right\}$, the importance of the corresponding weight coefficient by $\boldsymbol{p}^{T}=\left\{p_1^{T},p_2^{T},\cdots,p_u^{T}\right\}$, the maximum dose constraints of the non-target organ by $\boldsymbol{d}^{NU}=\left\{d_1^{NU},d_2^{NU},\cdots,d_v^{NU}\right\}$, the average dose constraint by $\boldsymbol{d}^{A}$, and the importance of the corresponding weight coefficient by $\boldsymbol{p}^{N}=\left\{p_1^{N},p_2^{N},\cdots,p_v^{N}\right\}$. We represent the ith radiation field discretization fluence map variable matrix by $\overline{\boldsymbol{X}}_i$, and it can be described as in (1).

$$\overline{\boldsymbol{X}}_i=\begin{bmatrix} x_{i,1,1} & x_{i,1,2} & & x_{i,1,n} \\ x_{i,2,1} & x_{i,2,2} & \cdots & x_{i,2,n} \\ & \vdots & \ddots & \\ x_{i,m,1} & x_{i,m,2} & & x_{i,m,n} \end{bmatrix},\ 1\le i\le l \tag{1}$$

In order to facilitate the subsequent modeling, we transfer those l fluence maps into a fluence map column vector in (2):

$$\boldsymbol{x}=\left(x_{1,1,1},x_{1,1,2},\cdots,x_{1,1,n},x_{1,2,1},\cdots,x_{1,m,n},x_{2,1,1}\cdots x_{l,m,n}\right)^T \tag{2}$$

We define a given exposure of radiation effect of human body model irradiation dose of transfer matrix by $\boldsymbol{F}$ .The dose effect matrix is $\boldsymbol{F}^T = \left(\boldsymbol{F}_1^T;\boldsymbol{F}_2^T;\cdots;\boldsymbol{F}_u^T\right)$, then the dose value for each voxel in target matrix can be signified as $\boldsymbol{d}^T = \boldsymbol{F}^T \cdot \boldsymbol{x} = \left(\boldsymbol{F}_1^T;\boldsymbol{F}_2^T;\cdots;\boldsymbol{F}_u^T\right)\cdot \boldsymbol{x} = \left(\boldsymbol{d}_1^T;\boldsymbol{d}_2^T;\cdots;\boldsymbol{d}_u^T\right)$, while for non-target area, $\boldsymbol{F}^N = \left(\boldsymbol{F}_1^N;\boldsymbol{F}_2^N;\cdots;\boldsymbol{F}_v^N\right)$, and the dose value is $\boldsymbol{d}^N = \boldsymbol{F}^N \cdot \boldsymbol{x} = \left(\boldsymbol{F}_1^N;\boldsymbol{F}_2^N;\cdots;\boldsymbol{F}_v^N\right)\cdot \boldsymbol{x} = \left(\boldsymbol{d}_1^N;\boldsymbol{d}_2^N;\cdots;\boldsymbol{d}_v^N\right)$.

Then, we have the following model in (3).

$$\min\left\{\sum \boldsymbol{p}^T \left\|\boldsymbol{d}^T - \boldsymbol{d}^{TP}\right\|_2^2 + \sum \boldsymbol{p}^N \left\|\boldsymbol{d}^N - 0\right\|_2^2\right\}$$
$$S.t.\begin{cases}\boldsymbol{d}^{TL} \le \boldsymbol{d}^T \\ \boldsymbol{d}^N \le \boldsymbol{d}^{NU} \\ \boldsymbol{Average}\left(\boldsymbol{d}^N\right) \le \boldsymbol{d}^A \\ 0 \le \boldsymbol{x}\end{cases} \tag{3}$$

In the objective function [3], the first term $\sum \boldsymbol{p}^T \left\|\boldsymbol{d}^T - \boldsymbol{d}^{TP}\right\|_2^2$ denotes the Euclid Distance between the actually dose and the prescription dose for target area. It is expected to reduce the disparity between the target dose and the prescription dose by minimizing this term. The objective of minimizing the second term $\sum \boldsymbol{p}^N \left\|\boldsymbol{d}^N - 0\right\|_2^2$ is that the dose for non-targets will be made as small as possible. The term $\boldsymbol{Average}\left(\boldsymbol{d}^N\right)$ represents the average dose for the non-target area.

This model minimizes the objective function in which the first term desires that the target receives the dose approximating the clinical demand dose, and the second term requires that the non-target receives the dose close to zero. There are four constraints in the objective function; the first one requires that the lower bound of the target must be satisfied. The second requires that some non-target organs should satisfy the upper bound of the dose constraint, for the protection of some important organs. The third one requires that some non-target organs should satisfy the average dose constraint, for the protection of the parallel organ. The fourth one is the actual exposure of the dose should be non-negative.

Assuming (4)

$$\boldsymbol{Q} = \sum p^T \left(\boldsymbol{F}^T\right)' \cdot \boldsymbol{F}^T + \sum p^N \left(\boldsymbol{F}^N\right)' \cdot \boldsymbol{F}^N ,$$
$$\boldsymbol{c} = -2\sum p^T \left(\boldsymbol{d}^{TP}\right)' \cdot \boldsymbol{F}^T , \tag{4}$$
$$e = \sum p^T \left(\boldsymbol{d}^{TP}\right)' \cdot \boldsymbol{d}^{TP} .$$

Where the notation (*)’ denotes the transpose of matrix *.

The above model can be transformed into a quadratic programming model as shown in (5).

$$\min\left\{\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{c}\cdot\boldsymbol{x} + e\right\}$$
$$S.t.\begin{cases}\boldsymbol{d}^{TP} \le \boldsymbol{F}^T \cdot \boldsymbol{x} \\ \boldsymbol{F}^N \cdot \boldsymbol{x} \le \boldsymbol{d}^{NU} \\ \boldsymbol{Average}\left(\boldsymbol{F}^N \cdot \boldsymbol{x}\right) \le \boldsymbol{d}^A \\ 0 \le \boldsymbol{x}\end{cases} \tag{5}$$

This model can be solved quickly by using an interior point method.

B. Mathematical matrix description of the leaf-moving process

The Intensity modulated radiation therapy (IMRT) is one of different lead metal compensator as ray intensity adjustment method in early days. It may not be the most widely used method based on the Multi-Leaf Collimator (MLC) ray intensity adjustment method. The MLC is a device used to produce the conformal radiation field, commonly known as multiple leaf apertures. The MLC components consist of two rows of discretization leaves, drive motor for each leaf, the drive circuit controlling motor movement and leaf motion controlling PC program. The device developed in Varian and Elekta, the MLC uses adjacent blade tongue and groove chaffer to prevent ray going through the plane contacting blade gap. The blade width of the multi-leaf collimator and its amount will determine the intensity modulation lateral resolution; the control precision of leaf motion determines the vertical resolution. The intensity modulation process of each beam receives the demand dose distribution by the accelerator continuously at a constant or variable dose rate according to the MLC drive plan to control each leaf blade variable motion. Compared with the traditional step and shoot intensity modulation, dIMRT and VMAT have shorter treatment time, but the verification of the dose is harder and the MLC requirement is higher.

Both the dIMRT and VMAT can use leaf-moving matrix to describe the dose modulation process of the MLC mathematically [13]. The MLC leaves tend to be one-way movement; namely, on a beam of modulation the leaf-moving from left to right (or from right to left) forms the fluence map. Therefore, the leaf-moving matrix using two matrices $\boldsymbol{I}^L$ and $\boldsymbol{I}^T$ to describe the beamlet opening time (from left to right movement corresponds to the right leaf) and the beamlet closing time (from left to right movement corresponds to the left leaf) of each different location in the fluence map. The coordinate (i, j) in matrix $\boldsymbol{I}^L$ represents the position of leading leaf opening time, and coordinate (i, j) in matrix $\boldsymbol{I}^T$ represents the position of trailing leaf closing time. Therefore, the beam modulated fluence map is represented as $\overline{\boldsymbol{X}} = \boldsymbol{I}^T - \boldsymbol{I}^L$.

Based on the MLC, the leaf-moving trajectory matrix has certain physical constraints (set leaf movement from left to right). The typical constraints are shown in (6),

$$\begin{aligned} &\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^L(j,k+1), \\ &\boldsymbol{I}^T(j,k) \le \boldsymbol{I}^T(j,k+1), \\ &\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^T(j,k), \\ &\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^T(j+1,k), \\ &\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^T(j-1,k). \end{aligned} \tag{6}$$

where j denotes the jth row, k the kth column. The constraints $\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^L(j,k+1)$ and $\boldsymbol{I}^T(j,k) \le \boldsymbol{I}^T(j,k+1)$ respectively means that the opening time of any leading leaf should not occur before its left beam block; to any trailing leaves, the closing time of them should not happen before its left beam block. The constraint $\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^T(j,k)$ represents that the opening time of the leading leaf should occur before the closing time of the trailing leaf to a beam block in any position. Otherwise, a collision will certainly occur on this block between the leading leaf and the trailing leaf. Constraints of $\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^T(j+1,k)$ and $\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^T(j-1,k)$ mean that to a beam block in any position, the opening time of the leading leaf should occur before the closing time of the trailing leaf of the block either above or below this block.

If the leaf movement from right to left, the inequality sign is reversed. Assume (7):

$$\boldsymbol{A} = \overline{\boldsymbol{X}} \cdot \boldsymbol{W} = \left(\boldsymbol{I}^T - \boldsymbol{I}^L\right) \cdot \boldsymbol{W} = \boldsymbol{I}^T \cdot \boldsymbol{W} - \boldsymbol{I}^L \cdot \boldsymbol{W} = \boldsymbol{A}^+ - \boldsymbol{A}^- \tag{7}$$

Then we have (8)

$$\begin{aligned} &\boldsymbol{A}^+ = \frac{1}{2}\left(|\boldsymbol{A}| + \boldsymbol{A}\right) \\ &\boldsymbol{A}^- = \frac{1}{2}\left(|\boldsymbol{A}| - \boldsymbol{A}\right) \\ &\boldsymbol{I}^T = \boldsymbol{A}^+ \cdot \boldsymbol{W}^{-1} \\ &\boldsymbol{I}^L = \boldsymbol{A}^- \cdot \boldsymbol{W}^{-1} \end{aligned} \tag{8}$$

Where $\boldsymbol{A}^+$ and $\boldsymbol{A}^-$ represent the positive and negative parts of matrix $\boldsymbol{A}$ respectively,

$$\boldsymbol{W} = \begin{bmatrix} 1 & -1 & 0 & & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ 0 & 0 & 1 & & \vdots \\ & \vdots & & \ddots & -1 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{n\times n},$$

$$\boldsymbol{W}^{-1} = \begin{bmatrix} 1 & 1 & 1 & & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 1 & & \vdots \\ & \vdots & & \ddots & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{n\times n}.$$

$\boldsymbol{W}$ is the transverse gradient matrix of the fluence map A and $\boldsymbol{W}^{-1}$ is the inverse matrix of the $\boldsymbol{W}$.

Therefore, the leaf-moving trajectory matrix convex programming constraints can be described as follows:

$$\begin{aligned} &\overline{\boldsymbol{X}} \cdot \boldsymbol{W} = \boldsymbol{A}^+ - \boldsymbol{A}^- \\ &0 \le \boldsymbol{A}^+ \\ &0 \le \boldsymbol{A}^- \\ &\boldsymbol{I}^T = \boldsymbol{A}^+ \cdot \boldsymbol{W}^{-1} \\ &\boldsymbol{I}^L = \boldsymbol{A}^- \cdot \boldsymbol{W}^{-1} \\ &\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^T(j+1,k) \\ &\boldsymbol{I}^L(j+1,k) \le \boldsymbol{I}^T(j,k) \end{aligned} \tag{9}$$

Where definitions of $\boldsymbol{A}^+$, $\boldsymbol{A}^-$, $\boldsymbol{W}$, $\boldsymbol{W}^{-1}$, $\boldsymbol{I}^L$, $\boldsymbol{I}^T$ are similarly defined as in (6)-(8). According to two constraints $\boldsymbol{I}^L(j,k) \le \boldsymbol{I}^L(j,k+1)$ and $\boldsymbol{I}^T(j,k) \le \boldsymbol{I}^T(j,k+1)$ in (6), elements in $\boldsymbol{I}^T \cdot \boldsymbol{W}$ and $\boldsymbol{I}^L \cdot \boldsymbol{W}$ should not be lower than 0; we have $0 \le \boldsymbol{A}^+$ and $0 \le \boldsymbol{A}^-$. Other parameters used in (9) are same as those used in Eqs. above.

C. Convex programming constraints for tongue and groove chaffer effects

Early MLC adjacent leaf adopts the flat extrusion contact. However, there is always a flaw in the contact plane. The X-rays can go through the gap to formulate the non-ideal dose distribution. Recently, the MLC leaf selects the tongue and groove chaffer effects design to fit adjacent leaf. But in the dose modulation regions of adjacent leaves, if adjacent parts modulation dose is lower than the lower limit of the up-and-down leaf modulation area, then the so-called tongue and groove chaffer effects will occur [14,15].

Assume (10)

$$\begin{aligned} &x(j,k) = \boldsymbol{I}^T(j,k) - \boldsymbol{I}^L(j,k), \\ &x(j+1,k) = \boldsymbol{I}^T(j+1,k) - \boldsymbol{I}^L(j+1,k) \end{aligned} \tag{10}$$

Where the up-and-down beamlet dose requirement is $x(j,k)$ and $x(j+1,k)$, the tongue and groove chaffer effects adjacent area dose should be greater than the minimum of these two values, i.e. $\min\left(x(j,k), x(j+1,k)\right)$.

The actual dose of the adjacent areas can be expressed as follows (11),

$$\begin{aligned}&\min\left(\boldsymbol{I}^T(j,k),\boldsymbol{I}^T(j+1,k)\right)\\&-\max\left(\boldsymbol{I}^L(j,k),\boldsymbol{I}^L(j+1,k)\right)\end{aligned} \tag{11}$$

If $\boldsymbol{I}^T(j,k)\geq \boldsymbol{I}^T(j+1,k)$, $\boldsymbol{I}^L(j,k)\leq \boldsymbol{I}^L(j+1,k)$, the above dose is $x(j+1,k)$,

If $\boldsymbol{I}^T(j,k)\leq \boldsymbol{I}^T(j+1,k)$, $\boldsymbol{I}^L(j,k)\geq \boldsymbol{I}^L(j+1,k)$, the above dose is $x(j,k)$,

If $\boldsymbol{I}^T(j,k)> \boldsymbol{I}^T(j+1,k)$, $\boldsymbol{I}^L(j,k)> \boldsymbol{I}^L(j+1,k)$, the above dose is

$$\begin{aligned}&\boldsymbol{I}^T(j+1,k)-\boldsymbol{I}^L(j,k)\\&<\boldsymbol{I}^T(j+1,k)-\boldsymbol{I}^L(j+1,k)=x(j+1,k)\end{aligned},$$

If $\boldsymbol{I}^T(j,k)< \boldsymbol{I}^T(j+1,k)$, $\boldsymbol{I}^L(j,k)< \boldsymbol{I}^L(j+1,k)$, the above dose is

$$\begin{aligned}&\boldsymbol{I}^T(j,k)-\boldsymbol{I}^L(j+1,k)\\&<\boldsymbol{I}^T(j,k)-\boldsymbol{I}^L(j,k)=x(j,k)\end{aligned}.$$

The latter two conditions above produce the tongue and groove chaffer effects. The necessary and sufficient condition for the tongue and groove chaffer effects not to be occurred can be expressed in (12).

$$\left(\boldsymbol{I}^T(j,k)-\boldsymbol{I}^T(j+1,k)\right)\cdot\left(\boldsymbol{I}^L(j,k)-\boldsymbol{I}^L(j+1,k)\right)\leq 0 \tag{12}$$

However, the above necessary and sufficient condition produces the non-convex constraint conditions. Therefore, the degradation convex can be expressed as follows.

$$\begin{aligned}&\frac{1}{2}\left(\boldsymbol{I}^T(1,k)+\boldsymbol{I}^L(1,k)\right)\\&=\frac{1}{2}\left(\boldsymbol{I}^T(2,k)+\boldsymbol{I}^L(2,k)\right)\\&=\cdots=\frac{1}{2}\left(\boldsymbol{I}^T(j,k)+\boldsymbol{I}^L(j,k)\right)\\&=\cdots=\frac{1}{2}\left(\boldsymbol{I}^T(m,k)+\boldsymbol{I}^L(m,k)\right)\end{aligned} \tag{13}$$

Rearranging (13), we have

$$\boldsymbol{I}^T(j,k)+\boldsymbol{I}^L(j,k)=\boldsymbol{I}^T(j+1,k)+\boldsymbol{I}^L(j+1,k) \tag{14}$$

The above formula means that the time in the middle of the time interval should be in the strict synchronization of the opening time and the closing time. Obviously, the above expression meets the following constraints:

$$\begin{aligned}&\boldsymbol{I}^L(j,k)\leq \boldsymbol{I}^T(j+1,k),\ \boldsymbol{I}^L(j+1,k)\leq \boldsymbol{I}^T(j,k)\\&\left(\boldsymbol{I}^T(j,k)-\boldsymbol{I}^T(j+1,k)\right)\cdot\left(\boldsymbol{I}^L(j,k)-\boldsymbol{I}^L(j+1,k)\right)\leq 0\end{aligned} \tag{15}$$

D. Leaf-moving speed constraints

The MLC has the strict maximum speed constraints for the leaf movement. Assuming that the output X-ray intensity is constant, and the variable output dose rate modulation process can only be realized through changing the leaf-moving speed. As a result of the limitation of motor horsepower for driving the leaf motion, the movement of the leaves cannot be an unlimited speed. In other words, the opening times of adjacent positions for the leading leaves are not the same, as well as the closing times of adjacent positions for the trailing leaves. Therefore, the adjacent position of differential matrix element must be greater than or equal to a fixed value. The leaf-moving speed constraints can be described as follows.

$$\begin{aligned}&\boldsymbol{I}^T(j,k+1)-\boldsymbol{I}^T(j,k)\geq\tau_0\\&\boldsymbol{I}^L(j,k+1)-\boldsymbol{I}^L(j,k)\geq\tau_0\end{aligned} \tag{16}$$

where τ_0 is a given constant value, $\boldsymbol{I}^T(j,k+1)-\boldsymbol{I}^T(j,k)$ represents the moving time of the trailing leaf from position (j,k) to position $(j,k+1)$, and $\boldsymbol{I}^L(j,k+1)-\boldsymbol{I}^L(j,k)$ represents the moving time of the leading leaf from position (j,k) to position $(j,k+1)$. The constraint of Eq. [16] says that the shortest movement time for leaves in the adjacent position cannot be too short which is larger than τ_0.

E. Total number of monitoring units and gantry motion speed constraints

One of the key indicators of radiotherapy is the X-ray output time, which can be expressed by the total number of monitoring units for dose modulation [16].

The less time the X-ray output takes, the lower of the overall exposure dose rate is for intensity modulation. This means that the whole process is under control. On the other hand, the less the beam-on-time is, the shorter the whole curing process will be. Therefore, the constraint for the total beam-on-time is necessary. However, it needs a careful design as adding the beam-on-time constraints into the planning model which will not produce the non-convex constraints.

Assuming that the leaf movement is on the one-way mode, the initial moment of the leading leaf motion is zero, the end time of the trailing leaf motion is T_0, and the output of the accelerator can be constant or variable dose rate at any leaf motion time. Hence, in the current radiation beam field, the radiation dose can be expressed as $\int_0^{T_0} R(t)dt$, where $R(t)$ is the output dose rate. When the output dose rate is constant, the expression is in the proportion to the beam-on-time (i.e. the total number of monitoring units T_0). Even if the output dose rate is variable, the expression can also be in the proportion to the beam-on-time, i.e. by making the leaf maximum speed with the corresponding change. Therefore, the single radiation field beam-on-time is equal to the maximum of the n columns elements of the trailing leaf motion matrix $\boldsymbol{I}_i^T$,which is shown in (17).

$$T_0^i = MU_i = \max_j \left\{ \boldsymbol{I}_i^T \left(j,n \right) \right\} \tag{17}$$

The total beam-on-time of the whole process of intensity-modulation can be represented as follows (18).

$$TNMU = \sum_{i=1}^{l} MU_i = \sum_{i=1}^{l} \left[\max_j \left\{ \boldsymbol{I}_i^T \left(j,n \right) \right\} \right] \tag{18}$$

If we add $TNMU \le TNMU_0$ as the programming constraint, we have the constraint condition of beam-on-time, which is formulated as follows (19).

$$\sum_{i=1}^{l} \left\{ \max_j \left\{ \boldsymbol{I}_i^T \left(j,n \right) \right\} \right\} \le TNMU_0 \tag{19}$$

Of course, if we consider that the gantry rotation speed is constrained by some maximum number in the VMAT planning, we need to add another beam-on-time constraints as follows (20).

$$\max_j \left\{ \boldsymbol{I}_i^T \left(j,n \right) \right\} \le \zeta_0 \tag{20}$$

F. Bidirectional movement patterns for the MLC leaf

The leaf movement has a variety of patterns based on the multi-leaf collimator the intensity-modulation. One of the most simple movement modes is the leading and trailing leaves which have the reciprocating motion. Although the random reciprocating motion can be very simple to produce a variety of the fluence map, as a result of the leaf reciprocation is too easy to produce the leaf wear. On the other hand, due to the movement of the leaf controlled by the motor, the motor reciprocating motion produced may be the empty back that the movement of the leaf positioning with the specified output location will have a certain error, which leads to the difference between the practical output and the ideal output. Therefore, for the dIMRT or VMAT, the unidirectional movement of the MLC leaf is a good choice for reducing blade wear as well as increasing accuracy of output fluence map. Considering the deficiency for the unidirectional movement patterns for MLC leaf and consistency of each radiation field leaf starting position, we adopt the bidirectional movement patterns. On each of radiation fields, leading and trailing leaves begin from the left side and move to the right side to forming the first modulation. The leaves then start from the right side and move to the left to forming the second modulation.

After the completion of bidirectional movement modulation, the whole dose modulation for one beam is completed. Then the irradiation head moves to the next beam direction to continue the next modulation round. The advantage of this motion modulation is a single modulation leaf one-way movement, and each beam leaf starting position is fixed. Inspired by this method, if the programming beam number increased to a larger number, such as 36 radiation beams, the VMAT inverse planning model can be obtained as good as dIMRT. If we split the fluence map $\overline{\boldsymbol{X}}$ into two terms, the first term is the first modulation map $\overline{\boldsymbol{Y}}$, and the second term is the second modulation map $\overline{\boldsymbol{Z}}$. We have the following expression (21).

$$\begin{aligned} &\overline{\boldsymbol{X}} = \overline{\boldsymbol{Y}} + \overline{\boldsymbol{Z}} \\ &\overline{\boldsymbol{Y}} \ge 0 \\ &\overline{\boldsymbol{Z}} \ge 0 \end{aligned} \tag{21}$$

IV. OUR PROPOSED MODEL

Based on the above six aspects of requirements and the corresponding constraints, we obtain a general convex programming mathematical model as described below (22):

$$\min\left\{\sum \boldsymbol{p}^T \left\|\boldsymbol{d}^T-\boldsymbol{d}^{TP}\right\|_2^2+\sum \boldsymbol{p}^N \left\|\boldsymbol{d}^N-0\right\|_2^2\right\}$$

$$S.t.\begin{cases}
\boldsymbol{d}^{TL}\le \boldsymbol{d}^T \\
\boldsymbol{d}^N\le \boldsymbol{d}^{NU} \\
\boldsymbol{Average}\left(\boldsymbol{d}^N\right)\le \boldsymbol{d}^A \\
0\le \boldsymbol{x} \\
\\
\overline{\boldsymbol{X}}=\overline{\boldsymbol{Y}}+\overline{\boldsymbol{Z}} \\
\overline{\boldsymbol{Y}}\ge 0 \\
\overline{\boldsymbol{Z}}\ge 0 \\
\\
\overline{\boldsymbol{Y}}\cdot\boldsymbol{W}=\boldsymbol{A}^+-\boldsymbol{A}^- & \overline{\boldsymbol{Z}}\cdot\boldsymbol{V}=\boldsymbol{B}^+-\boldsymbol{B}^- \\
0\le \boldsymbol{A}^+ & 0\le \boldsymbol{B}^+ \\
0\le \boldsymbol{A}^- & 0\le \boldsymbol{B}^- \\
\boldsymbol{I}^T=\boldsymbol{A}^+\cdot\boldsymbol{W}^{-1} & \boldsymbol{J}^T=\boldsymbol{B}^+\cdot\boldsymbol{V}^{-1} \\
\boldsymbol{I}^L=\boldsymbol{A}^-\cdot\boldsymbol{W}^{-1} & \boldsymbol{J}^L=\boldsymbol{B}^-\cdot\boldsymbol{V}^{-1} \\
\boldsymbol{I}^L(j,k)\le \boldsymbol{I}^T(j+1,k) & \boldsymbol{J}^L(j,k)\ge \boldsymbol{J}^T(j+1,k) \\
\boldsymbol{I}^L(j+1,k)\le \boldsymbol{I}^T(j,k) & \boldsymbol{J}^L(j+1,k)\ge \boldsymbol{J}^T(j,k) \\
\boldsymbol{I}^T(j,k)+\boldsymbol{I}^L(j,k)=\boldsymbol{I}^T(j+1,k)+\boldsymbol{I}^L(j+1,k) & \boldsymbol{J}^T(j,k)+\boldsymbol{J}^L(j,k)=\boldsymbol{J}^T(j+1,k)+\boldsymbol{J}^L(j+1,k) \\
\boldsymbol{I}^T(j,k+1)-\boldsymbol{I}^T(j,k)\ge\tau_0 & \boldsymbol{J}^T(j,k)-\boldsymbol{J}^T(j,k+1)\ge\tau_0 \\
\boldsymbol{I}^L(j,k+1)-\boldsymbol{I}^L(j,k)\ge\tau_0 & \boldsymbol{J}^L(j,k)-\boldsymbol{J}^L(j,k+1)\ge\tau_0 \\
\boldsymbol{I}^T(j,k)+\boldsymbol{I}^L(j,k)+\boldsymbol{J}^T(j,k)+\boldsymbol{J}^L(j,k)=\boldsymbol{I}^T(j+1,k)+\boldsymbol{I}^L(j+1,k)+\boldsymbol{J}^T(j+1,k)+\boldsymbol{J}^L(j+1,k) \\
\\
\max\limits_j\left\{\boldsymbol{I}_i^T(j,n)\right\}\le\zeta_0 \ \ \textbf{for } i=0,\cdots,l \\
\max\limits_j\left\{\boldsymbol{J}_i^T(j,0)\right\}\le\zeta_0 \ \ \textbf{for } i=0,\cdots,l \\
\\
\sum\limits_{i=1}^{l}\left\{\max\limits_j\left\{\boldsymbol{I}_i^T(j,n)\right\}\right\}+\sum\limits_{i=1}^{l}\left\{\max\limits_j\left\{\boldsymbol{J}_i^T(j,1)\right\}\right\}\le TNMU_0
\end{cases} \tag{22}$$

Where $\boldsymbol{x}$ is the vector expression of $\overline{\boldsymbol{X}}$,

$$\boldsymbol{W}=\begin{bmatrix}1&-1&0&&0\\0&1&-1&\cdots&0\\0&0&1&&\vdots\\&\vdots&&\ddots&-1\\0&0&\cdots&0&1\end{bmatrix}_{n\times n},\quad \boldsymbol{W}^{-1}=\begin{bmatrix}1&1&1&&1\\0&1&1&\cdots&1\\0&0&1&&\vdots\\&\vdots&&\ddots&1\\0&0&\cdots&0&1\end{bmatrix}_{n\times n},$$

$$\boldsymbol{V}=\begin{bmatrix}1&0&0&&0\\-1&1&0&\cdots&0\\0&-1&1&&\vdots\\&\vdots&&\ddots&0\\0&0&\cdots&-1&1\end{bmatrix}_{n\times n},\quad \boldsymbol{V}^{-1}=\begin{bmatrix}1&0&0&&0\\1&1&0&\cdots&0\\1&1&1&&\vdots\\&\vdots&&\ddots&0\\1&1&\cdots&1&1\end{bmatrix}_{n\times n}.$$

The above mathematical model can be transformed into linear constrained quadratic programming model and the convex programming algorithm can be used to solve the global optimal solution quickly by the interior-point method or conjugate gradient method. The notations $\boldsymbol{I}^T$, $\boldsymbol{I}^L$, $\boldsymbol{J}^T$ and $\boldsymbol{J}^L$ are the MLC trajectory matrices, which can be written directly into driven files to control MLC leaves. The parameters used in Eq. [22] are similarly defined as in above equations except the last three constraint formulae. The constraints

$\max_j \left\{ \boldsymbol{I}_i^T \left(j,n \right) \right\} \le \zeta_0$ **for** $i = 0, \cdots, l$ and $\max_j \left\{ \boldsymbol{J}_i^T \left(j,0 \right) \right\} \le \zeta_0$ **for** $i = 0, \cdots, l$ are described in Eq.[20]. The constraint $\sum_{i=1}^{l} \left\{ \max_j \left\{ \boldsymbol{I}_i^T \left(j,n \right) \right\} \right\} + \sum_{i=1}^{l} \left\{ \max_j \left\{ \boldsymbol{J}_i^T \left(j,1 \right) \right\} \right\} \le TNMU_0$ sets the upper value of the total number of monitor units to be $TNMU_0$. The $\overline{\boldsymbol{Y}}$ and corresponding constraints denote that the first part of fluence map intensity is modulated by the leaves movements from left to right while parameter $\overline{\boldsymbol{Z}}$ and corresponding constraints denote the sinistrad movement of leaves.

Eq. [23] shown below is the constraint for the center based synchronized type. This constraint guarantees that tongue and groove effects will not occur in the ray modulation.

$$\begin{aligned} &\boldsymbol{I}^T \left(j,k \right) + \boldsymbol{I}^L \left(j,k \right) + \boldsymbol{J}^T \left(j,k \right) + \boldsymbol{J}^L \left(j,k \right) \\ &= \boldsymbol{I}^T \left(j+1,k \right) + \boldsymbol{I}^L \left(j+1,k \right) \\ &+ \boldsymbol{J}^T \left(j+1,k \right) + \boldsymbol{J}^L \left(j+1,k \right) \end{aligned} \tag{23}$$

V. Conclusion

This paper discusses the convex programming mathematical model formulation of the X-ray intensity modulation method based on the MLC. By comparing with existing methods, our model benefits from the following: 1) the convex programming mathematical model guarantees the uniqueness of the optimal solution; 2) the model overcomes tongue and groove chamfer effects which are always shown in the dose modulation based on the MLC; 3) the model guarantees the optimal dose distribution by constraining the total number of monitor units; 4) the model makes sure that the fastest leaf-moving speed will not occur; this constraint prevent the modulation failure problem, and 5) the model overcomes the leaf reciprocating motion which always causes the leaf wear and the positioning inaccurate problem. The proposed method can be directly implemented for the dIMRT and VMAT.

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