Analytical Performance Evaluation of Relay Assisted OFDMA Cellular Systems with Various Frequency Reuse Schemes Under Different Propagation Impacts

Osama H. Elgazzar, Imbaby I. Mahmoud, Sherief Hashima, and H. A. Konber

***Abstract*—** This paper addresses the Co-Channel Interference (CCI) mitigation in Relay-Assisted (R-A) cellular systems to improve Cell Edge User's (CEU's) performance. Analytical treatments are conducted. The network performance improvement through reducing CCI effects are evaluated using two proposed interference mitigation models. These models denote the R-A sectored Fractional Frequency Reuse (FFR) and R-A Soft Frequency Reuse (SFR). Each model contains two different scenarios for further network performance improvement. The first scenario considers three Relay Stations (RSs) per cell while the other one proposes six RSs in each cell. The best RS placement is proposed. Moreover, closed form expressions for worst cases CEU's SIR, Cell Centre User's (CCU's) SIR and inner radius are implemented. These expressions are used to compare between the considered models using different performance evaluation metrics. The work outcomes enable the system designer to characterize and optimize the multi-cell network performance without a need to execute complex calculations. Also the obtained results contributes to achieve much higher network performance improvement with a lower cost.

***Keywords*—** Relay assisted systems, Interference mitigation, Cellular systems, FFR schemes.

I. Introduction

Interference causes a performance degradation in wireless communication networks. Therefore, many techniques are studied to reduce this effect [1]. Fractional Frequency Reuse (FFR) technique is one of these techniques. In this technique, the cell area is divided into outer and inner regions. Moreover, each region has its own Bandwidth (BW) to reduce the Co-Channel Interference (CCI) influence [2]. However, this technique is not enough to totally eliminate the CCI problem. So, the relaying methodologies are applied to raise the efficiency of this technique. There are several kinds of Relay Stations (RSs) but Amplify-and-Forward (AF) is the most common in use due to its operation simplicity. In an AF relaying system, RS amplifies the received signal before forwarding it to the destination node [3-4]. Hence, AF-RSs are considered in this paper.

There are many efforts in the literature to demonstrate the effect of relaying process on the network performance. In [5], the authors surveyed cooperation stimulation approaches of wireless multi-hop networks. The authors in [6], compared the performance of Down-Link (DL) cellular system with and without half-duplex RSs. In [7], the authors derived an outage probability expression in relay-assisted Multi-Input Multi-Output (MIMO) cellular networks. In [8], the authors proposed that the RSs lie on the inner region edge. However, this increases the interference between inner and outer frequencies. In [9], the authors proposed a relay positioning at a distance equal to (0.66) of the cell radius from BS. Although this proposed placement scenario gives better results than the proposed scenario in [8], it is not the best solution to increase the coverage probability, especially for worst case Cell Edge User (CEU). This paper extends the framework proposed in [2]. Additionally, two different DL interference mitigation models are analysed. These models represent Relay-Assisted (R-A) sectored FFR networks and R-A Soft Frequency Reuse (SFR). Moreover, a proposed RS location is presented. This location is selected to increase the coverage area and reduce the interference between neighbouring RSs. Also, closed form expressions for Cell Centre User's (CCU's) SIR, CEU's SIR and inner radius are introduced. These expressions are used to compare two models using different performance evaluation metrics. The results confirm the superiority of using three RSs than six RSs per cell for R-A sectored FFR system. However, this superiority is lost for the R-A SFR system. The outcomes of these comparisons contribute to improving the performance of cellular networks while keeping lower network cost.

Paper organization is as follows: Section II discusses the basic assumption and model description. Section III focuses on different performance evaluation metrics used to evaluate different models. The inner radius calculation is introduced in Section IV. Section V previews the results and its discussion. Section VI concludes the work.

II. Basic Assumptions and Model Description

These models consider a two-hop cellular system that uses AF RS. It is assumed that the Mobile Station (MS) located at the

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cell edge. This MS can communicate either directly with a BS by the direct link which represents the first time slot (SIR_F) or by the access link (link between RS and MS) which represents the second time slot (SIR_S) as shown in Fig. 1. These two SIRs are combined using the Maximum Ratio Combined (MRC) technique. Therefore, the SIR at this MS can be evaluated from the following formula [10]

$$MRC = SIR_F + SIR_S \tag{1}$$

where MRC, SIR_F and SIR_S denote the MRC of the two time slots SIRs, CEU's SIR in the first time slot and CEU's SIR in the second time slot, respectively.

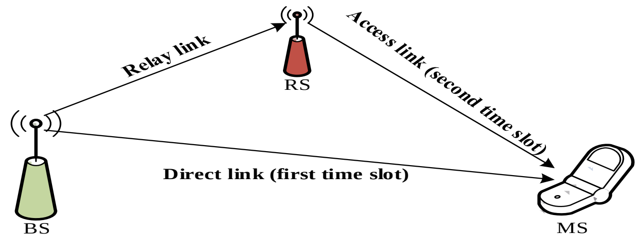


Fig. 1 illustration of MRC concept

Also, it is assumed that, for each cell of radius R_{ce}, all RSs are located symmetrically around the BS at a distance R (R = 0.75 of cell radius) as shown in Fig. 2. There are two scenarios for each model as revealed in this figure. The first scenario supposes that each cell contains BS and three RSs. Moreover, the second scenario considers six RSs and one BS in each cell to serve the cell users.

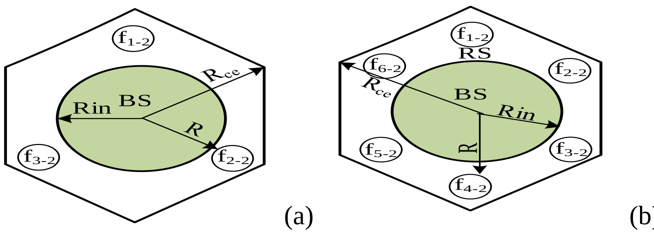


Fig. 2 R-A cellular system layout illustrating (a) first scenario and (b) second scenario

Based on these assumptions the analysis of R-A sectored FFR and R-A SFR networks are performed. It is observed that, the two scenarios of R-A sectored FFR model achieve the same value of CEU's SIR. Therefore, the analysis of the first scenario is introduced for this model. However, in R-A SFR model the obtained SIR of CEU differs from one scenario to the other. So, the analysis of these two scenarios are executed for this model.

A. Analysis of Relay Assisted Sectored FFR Model

Two tier relay assisted sectored FFR system is considered as shown in Fig. 3. In this system, there are 19 cells and each cell is divided into inner and outer regions. Furthermore, it is assumed that the user of interest located at the farthest point in the cell with coordinates (0, 1). Moreover, the outer region is divided into three sectors as shown in Fig. 3(a). Thus, total BW is divided into four sub bands. The frequency band (f_1) is allocated to the inner users. Additionally, the frequency bands (f_2, f_3 and f_4) are assigned to the outer region where each sector has its own frequency band as shown in Fig. 4(a). Furthermore,

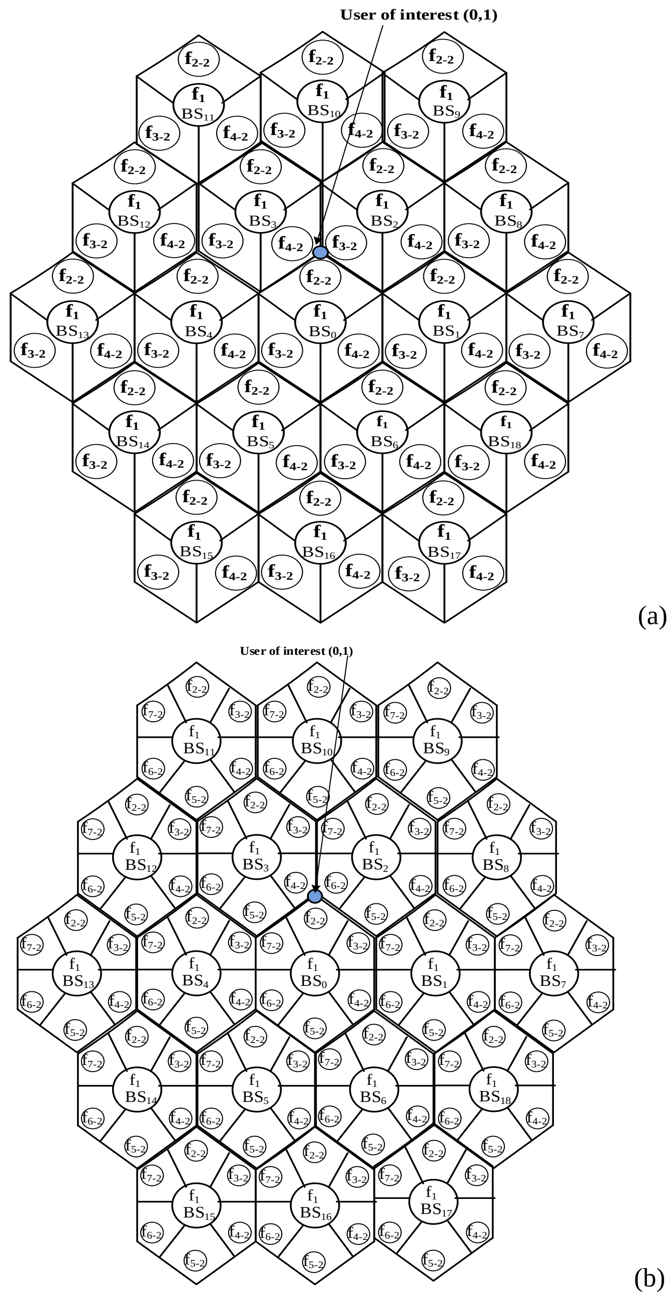


Fig. 3 two tier relay assisted sectored FFR system (a) first scenario (b) second scenario

each frequency band in outer region is further portioned into two different sets of sub-bands to reduce the interference level between the RSs and BSs. The first set represents ($f_{2\text{-}1}$, $f_{3\text{-}1}$ and $f_{4\text{-}1}$) sub-bands which are allocated to BSs. However, the second set denotes ($f_{2\text{-}2}$, $f_{3\text{-}2}$ and $f_{4\text{-}2}$) sub-bands which are assigned to RSs.

A.1 Analysis of first time slot

In cellular networks, the SINR of a user u (X, Y) located at a distance $r = \sqrt{X^2 + Y^2}$ from the serving BS (BS_0) is given by the following equation [11]

$$SINR(X,Y) = \frac{G_{ou} P_{on} h_{oun}}{\sigma_n^2 + \sum_{w=1}^{e} \left(G_{wu} P_{wn} h_{wun} \right)} \tag{2}$$

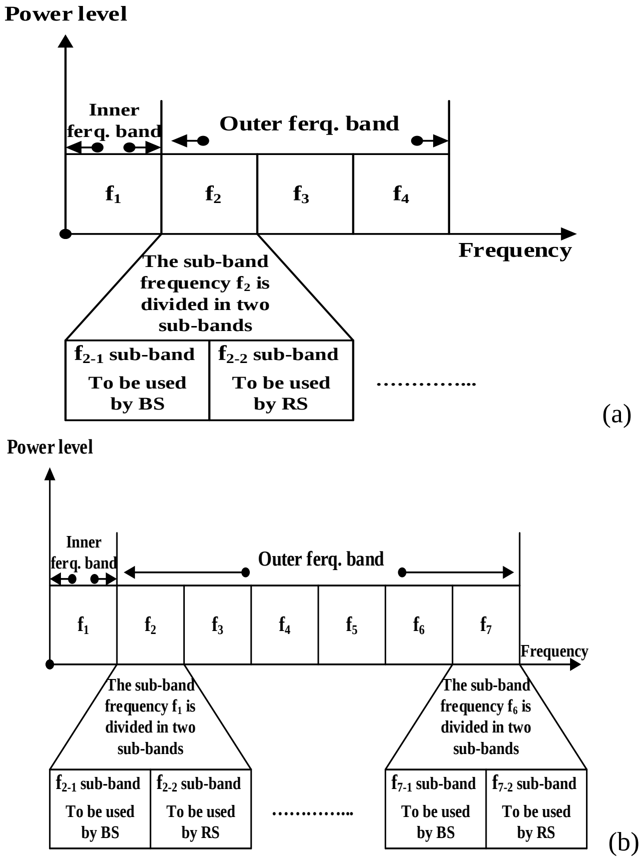


Fig. 4 frequency allocation for (a) first scenario (b) second scenario

where G_{ou}, G_{wu}, P_{on}, P_{wn}, h_{oun}, h_{wun} and σ_n^2 denote the PL associated with the channel between user u and BS_0, PL related to the channel between user u and interfering BSs, transmit power of BS_0 on subcarrier n, transmit power of interfering BSs, the exponentially distributed channel Fast Fading (FF) power between user u and BS_0, the exponentially distributed channel FF power between user u and interfering BSs and the noise power of the Additive White Gaussian Noise (AWGN) channel, respectively. Furthermore, the parameter G_{ou} is directly proportional to $r^{-\eta}$ i.e. (G_{ou} α $r^{-\eta}$) where η and r represent the PL exponent and distance between the BS and user u, respectively. So, this relation can be formulated as $G_{ou} = G_o\, r^{-\eta}$ where G_o is the first proportional constant which can be defined as $G_o = \left({c}/{4\pi f_o} \right)^2$ where f_o and c signify the center frequency of serving BS and speed of light, respectively. Moreover, the parameter G_{wu} is defined as $G_{wu} = G_w\, r_w^{-\eta}$ where G_w and r_w denote the second proportional constant and distance between the interfering BSs and user u, respectively. Furthermore, the second proportional constant is defined as $G_w = \left({c}/{4\pi f_w} \right)^2$ where f_w is the center frequency of interfering BSs. In addition, symbols w and e refer to the set of all interfering BSs (i.e. BSs that are using the same frequency band as BS_0). Therefore, e and w represent the co-channel cells number and cell index, respectively. In this time slot, RSs are idle. Hence, the interfering sources are all neighboring BSs that use the same frequency band as BS_0. Accordingly, CCUs suffer from 18 interfering BSs ($BS_{1:}$ BS_{18}) that use the same frequency band (f_1) as BS_0 in the inner region as depicted in Fig. 2. Moreover, CEUs suffer from 7 interfering BSs which denote cells number (5, 6, 14, 15, 16, 17, and 18) that use sub-band frequency ($f_{2\text{-}1}$) as BS_0 in the outer region. Additionally, it is assumed that channel fading powers are independent with unit mean, i.e. (h_{oun} =1), and the main effect in cellular network comes from the interference than from noise. So, noise term is neglected in our analysis. Moreover, user u can be considered either a CCU or CEU as he moves from the inner region to the cell edge. Thus, Eq. (2) is reformulated to calculate the CCU's SIR and CEU's SIR of the user u as follows [12]

$$SIR\,(X,Y) = \frac{P_0 G_o r^{-\eta}}{\left[Z \sum_{k=1}^{k=18} P_k G_k r_k^{-\eta} \right] + \left[Z' \sum_{j=1}^{j=7} P_j G_j r_j^{-\eta} \right]} \quad (3)$$

where $P_{0,}$ k, $r_{k,}$ P_k, G_k, j, r_j, P_j, G_j, Z and $Z^/$ denote the transmitted power by BS_0, set of interfering BSs due to reuse one frequency (f_1) in the inner region, distance between CCU and the 18 interfering BSs that use frequency band (f_1), transmitted power by the interfering BSs that affect CCU, inner proportional constant which is defined as $G_k = \left({c}/{4\pi f_k} \right)^2$ where f_k is the center frequency of interfering BSs which affect the CCU, set of interfering BSs due to reuse sub-band frequency ($f_{2\text{-}1}$) in the outer region, distance between CEU and 7 interfering BSs that use frequency sub-band ($f_{2\text{-}1}$), transmitted power by the interfering BSs which affect the CEU, outer proportional constant which is defined as $G_j = \left({c}/{4\pi f_j} \right)^2$ where f_j is the center frequency of interfering BSs which affect CEU, parameter that defines the user location if CEU (Z = 0) or CCU (Z = 1) and complement of parameter Z, respectively. Furthermore, it is assumed that equal transmit power is applied for all BSs, and the interference of users is negligible. So, the general SIR expression for CCU can be written as follows

$$SIR_{CCU}\,(X,Y) = \frac{G_o r^{-\eta}}{Z \sum_{k=1}^{k=18} G_K r_k^{-\eta}} \quad (4)$$

where SIR_{CCU} is the CCU's SIR. Also, because all BSs in the inner region use the same frequency (f_1), the values of G_o and G_k are equal. Therefore, Eq. (4) can be rewritten as follows

$$SIR_{CCU}\,(X,Y) = \frac{r^{-\eta}}{\sum_{k=1}^{k=18} r_k^{-\eta}} \quad (5)$$

Moreover, the general equation of CEU's SIR under equal transmitted power assumption can be expressed by the following relation

$$SIR_{CEU}\,(X,Y) = \frac{G_o r^{-\eta}}{Z' \sum_{j=1}^{j=7} G_j r_j^{-\eta}} \quad (6)$$

where SIR_{CEU} is the CEU's SIR. In addition, the values of G_o and G_j are identical due to the deployment of the same sub-band frequency ($f_{2\text{-}1}$) by BS_0 and the interfering BSs. Thus, Eq. (6) can be rewritten as follows

$$SIR_{CEU}(X,Y)=\frac{r^{-\eta}}{\sum_{j=1}^{j=7} r_j^{-\eta}} \tag{7}$$

Additionally, it is assumed that (x, y) be the normalized coordinates to R_{Cell} i.e. (x, y) =(X/ R_{Cell}, Y/ R_{Cell}). So, the normalized distance will equal to $R_{Cell}\, r=\sqrt{X^2+Y^2}$. Consequently, Eq. (5) is rewritten to express the CCU's SIR in normalized coordinates (x, y) as follows

$$SIR_{CCU}(x,y)=\frac{\left(x^2+y^2\right)^{\sigma}}{I_K(x,y)} \tag{8}$$

where σ and I_K denote the negative half of the PL exponent (-η/2) and the PL distances summation of 18 interfering BSs due to reuse one frequency in the inner regions, respectively. The normalized coordinates of the all BSs are illustrated in Table II in [2]. So, the parameter I_K is expressed as follows

$$I_K(x,y)=\left(\begin{array}{l}\left[\left(x\pm2\sqrt{3}\right)^2+y^2\right]^{\sigma}+\left[\left(x\pm\frac{3\sqrt{3}}{2}\right)^2+\left(y\pm\frac{3}{2}\right)^2\right]^{\sigma}+\left[\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(y\pm\frac{3}{2}\right)^2\right]^{\sigma}\\ +\left[\left(x\pm\sqrt{3}\right)^2+\left(y\pm3\right)^2\right]^{\sigma}+\left[\left(x\pm\sqrt{3}\right)^2+y^2\right]^{\sigma}+\left[\left(x\right)^2+\left(y\pm3\right)^2\right]^{\sigma}\end{array}\right) \tag{9}$$

Thus, by substituting the value of Eq. (9) into Eq. (8), the worst case CCU's SIR for a CCU located at the inner region edge with coordinates (R_{in}, 0) is formulated as follows

$$SIR_{CCU}=R_{in}^{2\sigma}\left[\left(R_{in}-3\right)^{2\sigma}+\left(R_{in}+3\right)^{2\sigma}+2\left(\begin{array}{l}\left(R_{in}^2+12\right)^{\sigma}+\left(\left(R_{in}-3\right)^2+3\right)^{\sigma}+\left(\left(R_{in}+\frac{3}{2}\right)^2+\frac{27}{4}\right)^{\sigma}\\ +\left(\left(R_{in}+3\right)^2+3\right)^{\sigma}+\left(\left(R_{in}-\frac{3}{2}\right)^2+\frac{3}{4}\right)^{\sigma}+\left(R_{in}^2+3\right)^{\sigma}\\ +\left(\left(R_{in}+\frac{3}{2}\right)^2+\frac{3}{4}\right)^{\sigma}+\left(\left(R_{in}-\frac{3}{2}\right)^2+\frac{27}{4}\right)^{\sigma}\end{array}\right)\right]^{-1} \tag{10}$$

where R_{in} is the inner region radius. On the other hand, in the outer region, there are 7 interfering BSs that use the same sub-band frequency ($f_{2\text{-}1}$) as BS_0. Therefore, Eq. (7) is reformulated to represent the CEU's SIR expression in normalized coordinates as follows

$$SIR_{BS}(x,y)=\frac{\left(x^2+y^2\right)^{\sigma}}{I_{Sec}(x,y)} \tag{11}$$

where I_{Sec} is the PL distances summation of the 7 interfering BSs. So, the parameter I_{Sec} is computed as follows

$$I_{Sec}(x,y)=\left(\begin{array}{l}\left[\left(x\pm\sqrt{3}\right)^2+\left(y+3\right)^2\right]^{\sigma}+\left[\left(x\pm\frac{3\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{2}\right)^2\right]^{\sigma}\\ +\left[x^2+\left(y+3\right)^2\right]^{\sigma}+\left[\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{2}\right)^2\right]^{\sigma}\end{array}\right) \tag{12}$$

where j_{BS} represents the set of interfering BSs due to reuse sub-band frequency ($f_{2\text{-}1}$) in the outer region. Moreover, CEU located at the second corner with coordinates (0, 1) is deduced. Thus, by substituting the value of Eq. (12) into Eq. (11), the worst case CEU's SIR is formed as follows

$$SIR_{BS}=\left[2\left(7^{\sigma}+13^{\sigma}+19^{\sigma}\right)+16^{\sigma}\right]^{-1} \tag{13}$$

where SIR_{BS} is the worst case CEU's SIR closed form expression in the first time slot.

A.2 Analysis of second time slot

In this time slot, each RS forwards its amplified signal to the MS while BSs are idle. Therefore, the worst case CEU's SIR is estimated by the following formula

$$SIR_{R\sec}(x,y)=\frac{\left(x^2+\left(y-0.75\right)^2\right)^{\sigma}}{I_{\mathrm{Rsec}}(x,y)} \tag{14}$$

where I_{Rsec} is the interference factor due to CCI from all RSs in neighboring cells that use the same frequency band ($f_{2\text{-}2}$) as the serving RS. It is assumed that the user of interest is served by RS which deploys sub-band frequency ($f_{2\text{-}2}$) in the central cell. So, there are 7 interfering RSs in cells number (5, 6, 14, 15, 16, 17, and 18) that use the same sub-band frequency ($f_{2\text{-}2}$) as shown in Fig. 3 (a). Furthermore, based on the proposed location of RSs, the coordinates of the 7 interfering RSs are shown in Table 1. Consequently, the parameter I_{Rsec} is evaluated from the following formula

$$I_{\mathrm{Rsec}}(x,y)=\left(\begin{array}{l}\left[\left(x\pm\frac{\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{4}\right)^2\right]^{\sigma}+\left[\left(x\pm\frac{3\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{4}\right)^2\right]^{\sigma}\\ +\left[\left(x\pm\sqrt{3}\right)^2+\left(y+\frac{9}{4}\right)^2\right]^{\sigma}+\left[\left(x\right)^2+\left(y+\frac{9}{4}\right)^2\right]^{\sigma}\end{array}\right) \tag{15}$$

By substituting the value of Eq. (15) into Eq. (14), the worst case SIR of CEU with coordinates (0, 1) is expressed as follows

$$SIR_{\mathrm{Rsec}}=\left(1/16\right)^{\sigma}\left[2\left(\left(61/16\right)^{\sigma}+\left(157/16\right)^{\sigma}+\left(217/16\right)^{\sigma}\right)+\left(169/16\right)^{\sigma}\right]^{-1} \tag{16}$$

Accordingly, by substituting the values of Eqs. (13) and (16) into Eq. (1), the MRC closed-form formula for the worst case CEU's SIR for R-A sectored FFR system is computed as follows

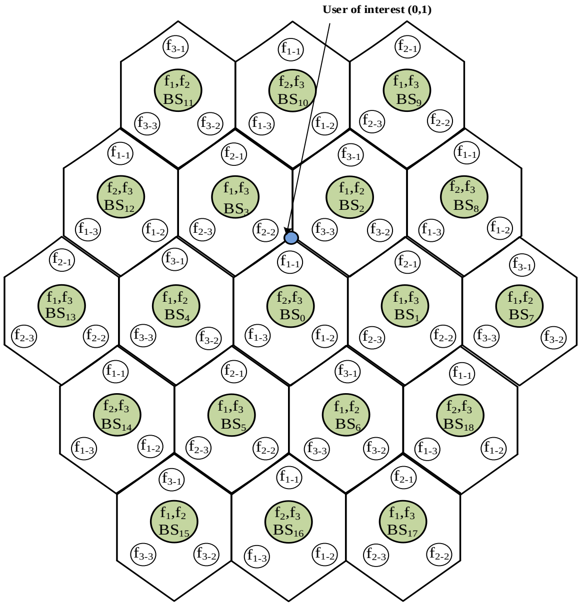
$$MRC_{\sec}=\left(\left[16^{\sigma}+2\left(\begin{array}{l}7^{\sigma}+13^{\sigma}\\ +19^{\sigma}\end{array}\right)\right]^{-1}\right)+\left(\left(1/16\right)^{\sigma}\left[2\left(\begin{array}{l}\left(61/16\right)^{\sigma}+\left(157/16\right)^{\sigma}\\ +\left(217/16\right)^{\sigma}\end{array}\right)+\left(169/16\right)^{\sigma}\right]^{-1}\right) \tag{17}$$

B. *Analysis of Relay Assisted SFR Model*

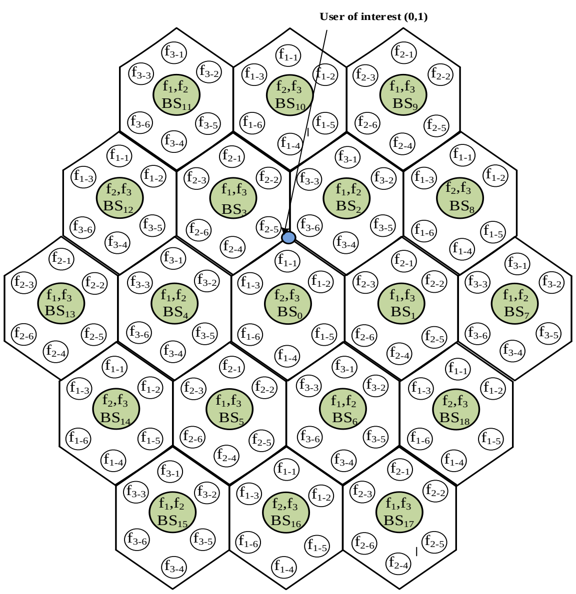
Although R-A sectored FFR improves the CEU's SIR but it is not possible to use full network bandwidth in a cell. So, a reuse 1 pattern with Two Level Power Control (TLCP) mechanism is used as in R-A SFR technique. The total BW in

Table 1 Interfering RSs' coordinates for the R-A sectored

Interfering RS coordinates in cell no.	X	Y
5	-√3/2	-3/4
6	√3/2	-3/4
14	-3√3/2	-3/4
15	-√3	-9/4
16	0	-9/4
17	√3	-9/4
18	3√3/2	-3/4



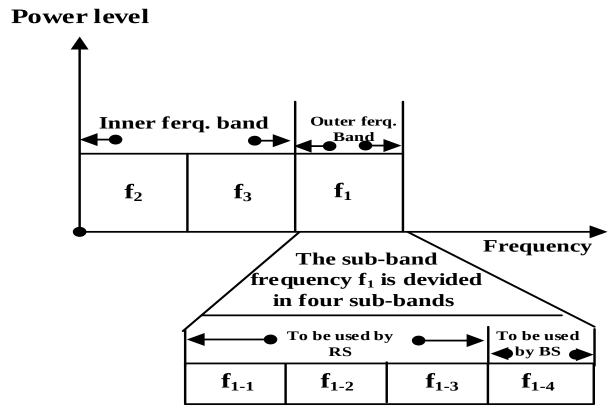
(a)



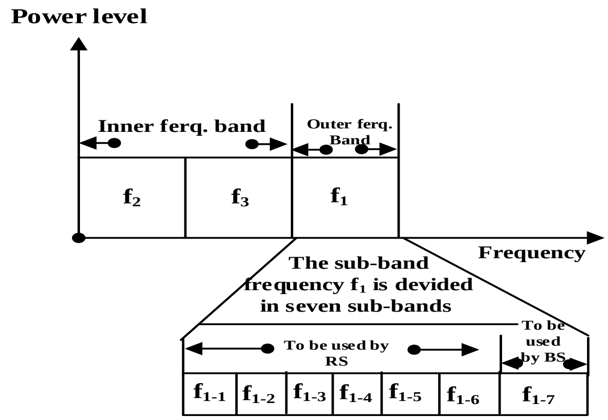
(b)

Fig. 5 Two tier R-A SFR system (a) first scenario (b) second scenario

this technique is divided into three equal parts. Two parts allocated to inner region and one to the outer region. It is assumed that, the output power per subcarrier in the inner region is P_{in}. Moreover, the output power per subcarrier in the outer region is P_o. These two values of power are related to each other by the relation $P_o = \beta P_{in}$ where β represents power control factor ($\beta \geq 1$). The FFR allocation of three BW parts (f_1, f_2 and f_3) differs from one cell to another in such a way that there is a pseudo-reuse 3 scheme between outer regions. Thus, total network BW is used in every cell [13]. Furthermore, each frequency band in outer region is further portioned into two different groups of sub-bands. The first group represents (f_{1-1}, f_{1-2} and f_{1-3}) sub-bands which are allocated to RSs. However, the second group denotes (f_{1-4}) sub-band which is assigned to BS as depicted in Fig. 6(a). Furthermore, the sub-band frequency (f_{1-7}) is allocated to BS and the rest of all sub-bands are assigned to RSs as shown Fig. 6(b). The analysis of two time slots is introduced in the



(a)



(b)

Fig. 6 Frequency allocation for (a) first scenario (b) second scenario

following two subsections, to calculate the MRC closed form expression for R-A SFR model.

B.1 First time slot

In this time slot, BS transmits the signal to both RS and Mobile Station (MS) while RS remains idle. Thus, the interfering sources are all neighboring BSs that use the same frequency band as the serving BS. Therefore, the CCUs in the central cell suffer from 18 interfering BSs (12 BSs in the inner region and 6 BSs in the outer region) due to reuse (f_2) frequency as depicted in Fig. 5. Furthermore, the CEU suffers from 18 interfering cells (12 BSs in the inner region and 6 BSs in the outer region) that use the sub-band frequency (f_{1-4}). The CEU's SIR expression for R-A SFR network is formulated as follows due to the power level difference [2]

$$SIR_{SFR} = \frac{P_o G_{ou}}{\left[P_{in} \sum_k G_{ku}\right] + \left[P_o \sum_j G_{ju}\right]} \tag{18}$$

where G_{ku} and G_{ju} denote the PL associated with the channel between MS_u and its interfering BS_k in the inner region and PL associated with the channel between MS_u and its interfering BS_j in the outer region, respectively. Moreover, the parameter G_{ku} is defined as $G_{ku} = G_k r_k^{-\eta}$. Additionally, the parameter G_{ju} is defined as $G_{ju} = G_j r_j^{-\eta}$. Therefore, the above equation can be rewritten as follows

$$SIR_{SFR} = \frac{\beta G_o r^{-\eta}}{\sum_k G_k r_k^{-\eta} + \beta \sum_j G_j r_j^{-\eta}} \tag{19}$$

In addition, the values of the center frequency used in the calculation of the two parameters G_k and G_j are the same as that used in the calculation of G_o. Thus, the parameters G_o, G_k and G_j have equal values. Consequently, Eq. (19) can be reformulated as follows

$$SIR_{SFR} = \frac{\beta r^{-\eta}}{\sum_k r_k^{-\eta} + \beta \sum_j r_j^{-\eta}} \tag{20}$$

Accordingly, Eq. (20) is rewritten to express the CEU's SIR in normalized coordinates (x, y) as follows

$$SIR_{SFR_{CEU}}(x,y) = \frac{\beta\left(x^2+y^2\right)^{\sigma}}{I_{iF}(x,y)+\beta I_{oF}(x,y)} \tag{21}$$

where I_{iF} and I_{oF} are the inner and outer interference factors, respectively. The inner CCI comes from BSs (1, 4, 5, 2, 6, 13, 3, 7, 15, 11, 17, and 9) while the outer CCI comes from BSs (8, 10, 12, 14, 16 and 18). Then, the final expression of inner interference factor for a CEU located at x=0 and y=1is expressed by the following equation

$$I_{iF} = 2\left[4^{\sigma} + 2.7^{\sigma} + 13^{\sigma} + 19^{\sigma} + 1\right] \tag{22}$$

Moreover, the final expression of outer interference factor for the same user can be formulated as follows

$$I_{oF} = 2\left[7^{\sigma} + 13^{\sigma}\right] + 4^{\sigma} + 16^{\sigma} \tag{23}$$

By substituting the values of Eqs. (22) and (23) into Eq. (21), the worst case CEU's SIR of the R-A SFR model in first time slot can be characterized as follows

$$SIR_{CEUSFR} = \left[3\times 4^{\sigma} + 16^{\sigma} + 2\left[3\times 7^{\sigma} + 2\times 13^{\sigma} + 19^{\sigma} + 1\right]\right]^{-1} \tag{24}$$

On the other hand, to calculate the CCU's SIR for a user located at (x, y) and using the frequency band (f_2) the following equation is used

$$SIR_{SFRCCU} = \frac{P_{in} G_{ou}}{\left[P_{in} \sum_k G_{ku}\right] + \left[P_o \sum_j G_{ju}\right]} \tag{25}$$

By substituting the relations of G_{ou}, G_{ku}, G_{ju} and P_o in the above equation, The CCU's SIR can be reformulated as follows

$$SIR_{SFRCCU} = \frac{G_o r^{-\eta}}{\sum_k G_k r_k^{-\eta} + \beta \sum_j G_j r_j^{-\eta}} \tag{26}$$

In addition, the values of the center frequency used in the calculation of the two parameters G_k and G_j are the same as that used in the calculation of G_o. Thus, the values of G_o, G_k and G_j are equal. Accordingly, Eq. (26) is expressed as follows

$$SIR_{SFRCCU} = \frac{r^{-\eta}}{\sum_k r_k^{-\eta} + \beta \sum_j r_j^{-\eta}} \tag{27}$$

Additionally, the CCU's SIR which is represented by Eq. (27) can be expressed in normalized coordinates (x, y) as follows

$$SIR_{CCUSFR}(x,y) = \frac{\left(x^2+y^2\right)^{\sigma}}{I_{icF}(x,y)+\beta I_{ocF}(x,y)} \tag{28}$$

where I_{icF} and I_{ocF} are the inner and outer interference factors, respectively. The inner CCI comes from BSs (4, 7, 10, 12, 14, 16, 11, 8, 15, 18, 2 and 6). So, the final inner interference factor expression for a user located at x= 0 and y=R_{in} can be formulated as follows

$$I_{icF} = \left(\begin{aligned} &\left(R_{in}^{2}+3\right)^{\sigma} + \left(\left(R_{in}-3\right)^{2}\right)^{\sigma} + \left(\left(R_{in}+3\right)^{2}\right)^{\sigma} + \left(R_{in}^{2}+12\right)^{\sigma} + \left(\left(R_{in}-3\right)^{2}+3\right)^{\sigma} + \left(\left(R_{in}+3\right)^{2}+3\right)^{\sigma} \\ &+\left(\left(R_{in}+\frac{3}{2}\right)^{2}+\frac{3}{4}\right)^{\sigma} + 2\left[\left(\left(R_{in}-\frac{3}{2}\right)^{2}+\frac{27}{4}\right)^{\sigma} + \left(\left(R_{in}+\frac{3}{2}\right)^{2}+\frac{27}{4}\right)^{\sigma}\right] + \left(\left(R_{in}-\frac{3}{2}\right)^{2}+\frac{3}{4}\right)^{\sigma} \end{aligned}\right) \tag{29}$$

Additionally, the outer CCI comes from BSs (1, 5, 13, 3, 9 and 17). Thus, the final expression of outer interference factor for the same user can be evaluated as follows

$$I_{ocF} = \left(R_{in}^{2}+3\right)^{\sigma} + \left(R_{in}^{2}+12\right)^{\sigma} + \left(\left(R_{in}-3\right)^{2}+3\right)^{\sigma} + \left(\left(R_{in}+3\right)^{2}+3\right)^{\sigma} + \left(\left(R_{in}-\frac{3}{2}\right)^{2}+\frac{3}{4}\right)^{\sigma} + \left(\left(R_{in}+\frac{3}{2}\right)^{2}+\frac{3}{4}\right)^{\sigma} \tag{30}$$

Consequently, by substituting the values of Eqs. (29) and (30) into Eq. (28), the final closed form expression for CCU's SIR is obtained as follows

$$SIR_{SCCU} = R_{in}^{2\sigma}\left[2\left(\begin{aligned} &\left(R_{in}^{2}+12\right)^{\sigma} + \left(\left(R_{in}-3\right)^{2}+3\right)^{\sigma} + \left(\left(R_{in}+\frac{3}{2}\right)^{2}+\frac{27}{4}\right)^{\sigma} \\ &+\left(\left(R_{in}+3\right)^{2}+3\right)^{\sigma} + \left(\left(R_{in}-\frac{3}{2}\right)^{2}+\frac{3}{4}\right)^{\sigma} + \left(R_{in}^{2}+3\right)^{\sigma} \\ &+\left(\left(R_{in}+\frac{3}{2}\right)^{2}+\frac{3}{4}\right)^{\sigma} + \left(\left(R_{in}-\frac{3}{2}\right)^{2}+\frac{27}{4}\right)^{\sigma} \end{aligned}\right) + \left(\left(R_{in}-3\right)^{2}\right)^{\sigma} + \left(\left(R_{in}+3\right)^{2}\right)^{\sigma}\right]^{-1} \tag{31}$$

The above equation represents the worst case CCU's SIR closed-form formula for β =1. The other closed-form formulas for different β values are performed, but not included in this section.

B.2 Second time slot

The MRC closed-form formulas of the two considered scenarios are obtained in this section. In this time slot, there are two interference factors (the inner and outer interference factors) due to TLCP mechanism utilization. The coordinates of the interfering RSs change from one scenario to the other. Therefore, the outer interference factor is not the same in the two scenarios. On the other hand, the inner interference value has the same value in these two scenarios. So, the analysis is done separately for the first and the second scenarios in this time slot.

B.2.1 Analysis of the first scenario

In this scenario, each cell contains 3 RSs as shown in Fig. 5(a) and the communication is done via the access link. Therefore, Eq. (21) can be reformulated according to the proposed location of RSs to calculate the CEU's SIR in second time slot as follows

$$SIR_3(x,y) = \frac{\beta\left(x^2+(y-0.75)^2\right)^{\sigma}}{I_{i3}(x,y)+\beta I_{o3}(x,y)} \tag{32}$$

where I_{i3} and I_{o3} denote the inner and outer interference factors, respectively. Furthermore, the normalized coordinates of the 12 interfering BSs that use the frequency band (f_1) in the inner region are shown in Table II in [2]. Consequently, the parameter I_{i3} is expressed as follows

$$I_{i3}(x,y)=\frac{1}{4}\left(\begin{array}{l}\left[\left(x\pm2\sqrt{3}\right)^2+y^2\right]^{\sigma}+\left[\left(x+\frac{\sqrt{3}}{2}\right)^2+\left(y\pm\frac{3}{2}\right)^2\right]^{\sigma}+\left[\left(x\pm\sqrt{3}\right)^2+\left(y\mp3\right)^2\right]^{\sigma}\\+\left[\left(x\pm\sqrt{3}\right)^2+y^2\right]^{\sigma}+\left[\left(x\pm\sqrt{3}\right)^2+\left(y\pm3\right)^2\right]^{\sigma}+\left[\left(x-\frac{\sqrt{3}}{2}\right)^2+\left(y\pm\frac{3}{2}\right)^2\right]^{\sigma}\end{array}\right) \quad (33)$$

Therefore, the final expression of the inner interference factor for a CEU located at x= 0 and y= 1 is formulated as follows

$$I_{i3}=\frac{1}{2}\left(4^{\sigma}+13^{\sigma}+19^{\sigma}+1\right)+7^{\sigma} \quad (34)$$

Moreover, there are 6 interfering RSs use the frequency band ($f_{1\text{-}1}$) and affecting the user of interest. The coordinates of these RSs based on the proposed locations of RSs are shown in Table 2.

Table 2 Interfering RSs' coordinates for R-A SFR system

Interfering RS' coordinates in cell no.	sub-band freq. $f_{1\text{-}1}$		sub-band freq. $f_{1\text{-}2}$		sub-band freq. $f_{1\text{-}3}$	
	X	Y	X	Y	X	Y
8	3√3/2	9/4	15√3/8	4/4	9√3/8	4/4
10	0	15/4	3√3/8	10/4	-3√3/8	10/4
12	-3√3/2	9/4	-9√3/8	4/4	-15√3/8	4/4
14	-3√3/2	-3/4	-9√3/8	-8/4	-15√3/8	-8/4
16	0	-9/4	3√3/8	-14/4	-3√3/8	-14/4
18	3√3/2	-3/4	15√3/8	-8/4	9√3/8	-8/4

Consequently, the parameter I_{o3} is computed from the following formula

$$I_{o3}(x,y)=\left(\begin{array}{l}\left[(x)^2+\left(y-\frac{15}{4}\right)^2\right]^{\sigma}+\left[(x)^2+\left(y+\frac{9}{4}\right)^2\right]^{\sigma}+\left[\left(x-\frac{3\sqrt{3}}{2}\right)^2+\left(y-\frac{9}{4}\right)^2\right]^{\sigma}\\+\left[\left(x\pm\frac{3\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{4}\right)^2\right]^{\sigma}+\left[\left(x+\frac{3\sqrt{3}}{2}\right)^2+\left(y-\frac{9}{4}\right)^2\right]^{\sigma}\end{array}\right) \quad (35)$$

Thus, the final form of the outer interference factor for CEU located at x= 0 and y= 1 is formulated as follows

$$I_{o3}=\left[\frac{121}{16}\right]^{\sigma}+2\left(\left[\frac{133}{16}\right]^{\sigma}+\left[\frac{157}{16}\right]^{\sigma}\right)+\left[\frac{169}{16}\right]^{\sigma} \quad (36)$$

Accordingly, by substituting the values of Eqs. (34) and (36) into Eq. (32), the worst case SIR of CEU is expressed as follows

$$SIR_3=\left(\tfrac{1}{16}\right)^{\sigma}\left[\begin{array}{l}\frac{1}{2}\left(4^{\sigma}+13^{\sigma}+19^{\sigma}+1\right)+\left[\frac{121}{16}\right]^{\sigma}+7^{\sigma}\\+2\left(\left[\frac{133}{16}\right]^{\sigma}+\left[\frac{157}{16}\right]^{\sigma}\right)+\left[\frac{169}{16}\right]^{\sigma}\end{array}\right]^{-1} \quad (37)$$

By substituting the values of Eqs. (24) and (37) into Eq. (1), the MRC closed form formula for the worst case CEU's SIR with β=1 is computed as follows

$$MRC_3=\left[\begin{array}{l}2\left(3.7^{\sigma}+2.13^{\sigma}+19^{\sigma}+1\right)\\+3.4^{\sigma}+16^{\sigma}\end{array}\right]^{-1}+\left(\tfrac{1}{16}\right)^{\sigma}\left[\begin{array}{l}\frac{1}{2}\left(4^{\sigma}+13^{\sigma}+19^{\sigma}+1\right)+\left[\frac{121}{16}\right]^{\sigma}+7^{\sigma}\\+2\left(\left[\frac{133}{16}\right]^{\sigma}+\left[\frac{157}{16}\right]^{\sigma}\right)+\left[\frac{169}{16}\right]^{\sigma}\end{array}\right]^{-1} \quad (38)$$

where MRC_3 denote the worst case MRC for CEU's SIR when the first scenario is used.

B.2.2 Analysis of the second scenario

In this scenario, each cell has its serving BS and 6 RSs located symmetrically around the BS. Therefore, Eq. (32) can be reformulated to express the equation of CEU's SIR as follows

$$SIR_6(x,y)=\frac{\beta\left(x^2+(y-0.75)^2\right)^{\sigma}}{I_{i6}(x,y)+\beta I_{o6}(x,y)} \quad (39)$$

where I_{i6} and I_{o6} denote the inner and outer interference factors, respectively. Furthermore, the normalized coordinates of the 12 interfering BSs that use the frequency band (f_1) in the inner region are shown in Table II in [2]. Accordingly, the parameter I_{i6} is evaluated from the following formula

$$I_{i6}(x,y)=\frac{1}{7}\left[\begin{array}{l}\left[\left(x\pm\sqrt{3}\right)^2+y^2\right]^{\sigma}+\left[\left(x+\frac{\sqrt{3}}{2}\right)^2+\left(y\pm\frac{3}{2}\right)^2\right]^{\sigma}+\left[\left(x-\frac{\sqrt{3}}{2}\right)^2+\left(y\pm\frac{3}{2}\right)^2\right]^{\sigma}\\+\left[\left(x\pm2\sqrt{3}\right)^2+y^2\right]^{\sigma}+\left[\left(x\pm\sqrt{3}\right)^2+\left(y\pm3\right)^2\right]^{\sigma}+\left[\left(x\pm\sqrt{3}\right)^2+\left(y\mp3\right)^2\right]^{\sigma}\end{array}\right] \quad (40)$$

Thus, the final expression of the inner interference factor for a CEU located at x= 0 and y= 1 is expressed as follows

$$I_{i6}=\tfrac{2}{7}\left(4^{\sigma}+2.7^{\sigma}+13^{\sigma}+19^{\sigma}+1\right) \quad (41)$$

Additionally, there are 6 interfering RSs use the frequency band ($f_{1\text{-}1}$) and affecting the user of interest. The coordinates of these RSs are shown in Table 3. Consequently, the parameter I_{o6} is computed from the following formula

$$I_{o6}(x,y)=\left(\begin{array}{l}\left[(x)^2+\left(y-\frac{15}{4}\right)^2\right]^{\sigma}+\left[(x)^2+\left(y+\frac{9}{4}\right)^2\right]^{\sigma}+\left[\left(x+\frac{3\sqrt{3}}{2}\right)^2+\left(y-\frac{9}{4}\right)^2\right]^{\sigma}\\+\left[\left(x\pm\frac{3\sqrt{3}}{2}\right)^2+\left(y+\frac{3}{4}\right)^2\right]^{\sigma}+\left[\left(x-\frac{3\sqrt{3}}{2}\right)^2+\left(y-\frac{9}{4}\right)^2\right]^{\sigma}\end{array}\right) \quad (42)$$

Furthermore, the final expression of the outer interference factor for the same CEU is formulated as follows

$$I_{o6}=\left[\frac{121}{16}\right]^{\sigma}+2\left(\left[\frac{133}{16}\right]^{\sigma}+\left[\frac{157}{16}\right]^{\sigma}\right)+\left[\frac{169}{16}\right]^{\sigma} \quad (43)$$

Thus, by substituting the value of Eqs. (41) and (43) into Eq. (39), the worst case SIR of CEU with β=1 is expressed as follows

Table 3 Interfering RSs' coordinates for R-A SFR technique

Interfering RS' coor. in cell no.	sub-band frequency f_{1-1}		sub-band frequency f_{1-2}		sub-band frequency f_{1-3}		sub-band frequency f_{1-4}		sub-band frequency f_{1-5}		sub-band frequency f_{1-6}	
	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
8	3√3/2	9/4	15√3/8	8/4	9√3/8	8/4	3√3/2	3/4	15√3/8	4/4	9√3/8	4/4
10	0	15/4	3√3/8	14/4	-3√3/8	14/4	0	9/4	3√3/8	10/4	-3√3/8	10/4
12	-3√3/2	9/4	-9√3/8	8/4	-15√3/8	8/4	-3√3/2	3/4	-9√3/8	4/4	-15√3/8	4/4
14	-3√3/2	-3/4	-9√3/8	-4/4	-15√3/8	-4/4	-3√3/2	-9/4	-9√3/8	-8/4	-15√3/8	-8/4
16	0	-9/4	3√3/8	-10/4	-3√3/8	-10/4	0	-15/4	3√3/8	-14/4	-3√3/8	-14/4
18	3√3/2	-3/4	15√3/8	-4/4	9√3/8	-4/4	3√3/2	-9/4	15√3/8	-8/4	9√3/8	-8/4

$$SIR_6 = \left[\frac{1}{16}\right]^{\sigma} \left[\frac{2}{7}\left(4^{\sigma}+2.7^{\sigma}+13^{\sigma}+19^{\sigma}+1\right)+\left[\frac{121}{16}\right]^{\sigma} +2\left(\left[\frac{133}{16}\right]^{\sigma}+\left[\frac{157}{16}\right]^{\sigma}\right)+\left[\frac{169}{16}\right]^{\sigma} \right]^{-1} \quad (44)$$

Accordingly, by substituting the values of Eqs. (24) and (44) into Eq. (1), the MRC closed form formula for the worst case CEU's SIR with β=1 is computed as follows

$$MRC_6 = \left[2\left(3.7^{\sigma}+2.13^{\sigma}+19^{\sigma}+1\right) +3.4^{\sigma}+16^{\sigma} \right]^{-1} + \left[\frac{1}{16}\right]^{\sigma} \left[\left[\frac{121}{16}\right]^{\sigma} +2\left(\left[\frac{133}{16}\right]^{\sigma}+\left[\frac{157}{16}\right]^{\sigma}\right)+\left[\frac{169}{16}\right]^{\sigma} +\frac{2}{7}\left(4^{\sigma}+2.7^{\sigma}+13^{\sigma}+19^{\sigma}+1\right) \right]^{-1} \quad (45)$$

where MRC_6 denote the worst case MRC for CEU's SIR when the second scenario is deployed.

I. Performance Evaluation of Different Models

In this section, the obtained worst case SIR closed-form formulas are used to calculate different performance evaluation metrics such as the Spectral Efficiency (SE), throughput, capacity, and outage probability for the different models. Then these metrics are used in the performance comparison between the considered models to enhance the network performance, taking into account the total network cost.

A. *Spectral Efficiency Determination for Relay-Aided Cellular Networks*

The SE is an important metric in any cellular network, where it refers to the information rate that can be transmitted over a given bandwidth in a specific communication system. The SE is calculated from the following equation [14]

$$SE_R = \frac{\log_2\left(1+SIR_{MRC}\right)}{K_{in}} \quad (46)$$

where K_{in} is the reciprocal of the reuse factor (i.e. K_{in} =1/ FRF).

B. *Relay-Aided Cellular Networks Capacity Evaluation*

The potentially achievable link rates in the communication networks are calculated by using the Shannon capacity formula [12]. The capacity for R-A cellular systems are deployed is expressed as follows

$$C_R = BW \log_2\left(1+MRC_{SIR}\right) \quad (47)$$

where MRC_{SIR} is the worst case MRC for CEU's SIR.

C. *Outage Probability Computation*

Outage probability is an essential performance metric in cellular systems. It is defined as the probability that the achievable rate falls below a target transmission rate δ. The outage probability under collective influence of PL, shadowing and FF is studied. Therefore, the different models is compared under worst case propagation condition. The outage probability in this case is expressed as follows [15]

$$\mathrm{P}\left(MRC_{SIR} \prec \delta\right) = \int_0^{\infty} Q\left[\frac{10\log_{10}\left(\frac{x}{\delta}\right)-m_f}{s_f}\right] e^{-x}\,dx \quad (48)$$

where Q, m_f and S_f denote the error function that defined as $Q(u) = 0.5erfc(1.414u)$, the mean and standard deviation of factor y_F that defined as $y_F = 1/SIR_{MRC}$, respectively. The mean and standard deviation can be calculated as follows

$$m_f = \frac{1}{a}\ln\left(y_f(d,\eta)H(d,\tau)\right) \quad (49)$$

$$S_f^2 = 2\left(\tau^2 - \frac{1}{a^2}\ln H(d,\tau)\right) \quad (50)$$

where $H(d,\tau) = e^{a^2\tau^2/2}\left(G(d,\eta)\left(e^{a^2\tau^2}-1\right)+1\right)^{\frac{-1}{2}}$,

$G(d,\eta) = \sum_j d_j^{-2\eta} \Big/ \left(\sum_j d_j^{-\eta}\right)^2$, $y_f(d,\eta) = \sum_j d_j^{-\eta} \Big/ d^{-\eta}$, $a = 0.1\times\ln(10)$ and τ is the (logarithmic) standard deviation of the mean received signal due to shadowing effect.

II. Inner Cell Radius Determination

The key aspect of the R-A FFR network design is the accurate calculation of its inner radius that defines the CCUs and CEUs, which in turn improves the SIR as well as the SE. Furthermore, in the R-A cellular systems, the required RSs number of the network is depending on the inner radius computation. Hence, for the calculation of the inner region radius, the same authors of this paper proposed a Correction Factor (CF) to calculate the inner radius under practical conditions for the traditional FFR systems in [2]. The CF equation is extended to calculate the CF in R-A FFR systems as follows

$$SIR_{CEU} = CF_R \times SIR_{CCU} \quad (51)$$

where CF_R is a CF used to obtain an accurate inner radius calculation. This parameter is calculated as follows

$$CF_R = \frac{SIR_{RBF}}{SIR_{BSBF}} = \frac{P_{RS} - 10\log(R_R)}{P_{BS} - 10\log(R_{in})} \quad (52)$$

where R_R, P_{RS} and P_{BS} denote the distance between RS and MS, RS transmitted power and BS transmitted power, respectively.

III. Results and Discussion

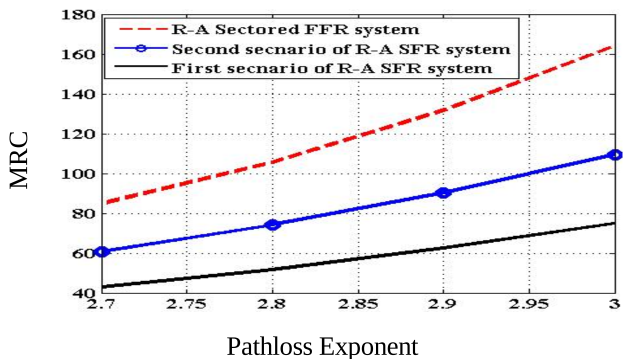


Fig. 7 MRC versus PL exponent for different interference mitigation systems

The variation of MRC against PL exponent is shown in Fig. 7. From this figure it is observed that PL gain increases due to PL exponent increase that in turn compensates the signal attenuation. Consequently, the SIR increases with PL exponent increase. Furthermore, it is noted that the R-A sectored FFR model is superior to the other models. The reason of this result can be attributed to, the increased interference in R-A SFR system due to reuse 1 pattern. Thus, the resulted CEU's SIR of the R-A SFR model is much lower than that obtained from the R-A sectored FFR system model, especially for high PL exponent values. The second point of observation is that the second scenario has the preference in use than first one for R-A SFR model. This result can be credited to the increment rate of SIR value with PL exponent in second time slot of second scenario is much higher than it in first one. So, the resulted MRC of the second scenario exceeds greatly the obtained MRC from the first scenario. The distinction of the SE against the PL exponent for different interference mitigation systems is depicted in Fig. 8. It is observed that data rate which can be transmitted over the system BW increases with SIR increase. So, SE increase with

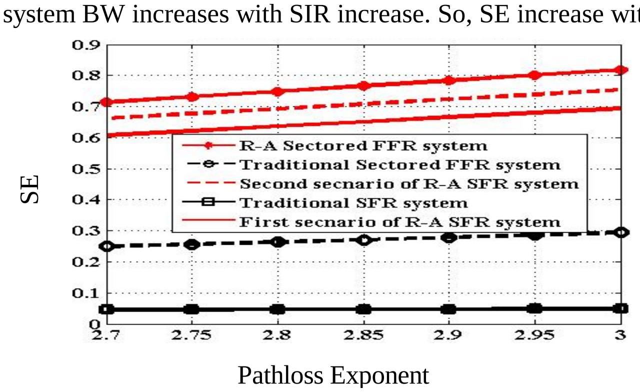


Fig. 8 SE variation against PL exponent for different interference mitigation systems

PL exponent. Also, it is noted that R-A sectored FFR system provides a good solution to increase rate of data transmission.

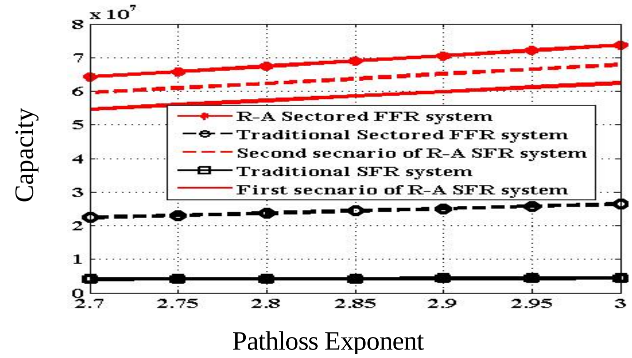


Fig. 9 Capacity variation against PL exponent for different interference mitigation systems

The capacity variation of different interference mitigation systems against PL exponent is revealed in Fig. 9. It discussed above that SIR increases with PL exponent. So, the highest information rate that can be achieved with small error probability increases. Consequently, the channel capacity increases with PL exponent. Moreover, it is noted that the R-A sectored FFR model achieves higher capacity value in high PL exponent range. Therefore, it is preferred in use when high channel capacity is needed. Also, it is observed that, traditional sectored FFR system achieves higher rate at which data can be reliably transmitted. Therefore, this system is superior to traditional SFR system.

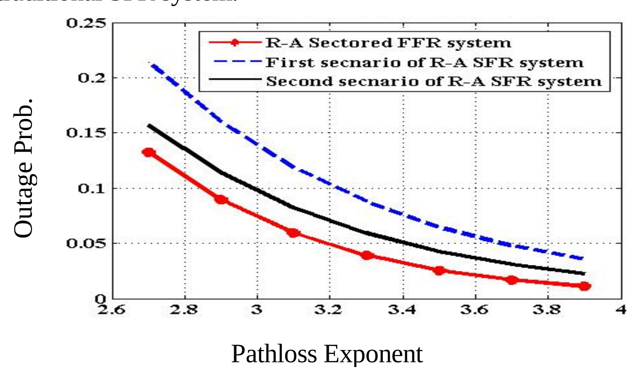


Fig. 10 Variations of the outage probability against PL exponent for different interference mitigation systems

The outage probability variation of different interference mitigation systems against PL exponent is shown in Fig. 10. It is noticed that coverage probability increases as the value of CEU's SIR increase. So, increasing PL exponent value results in increasing the probability at which network can cover CEUs. Consequently, the outage probability decrease.

IV. Conclusion

In this paper, the coordinates of RSs placement are determined in such a way that it increases the coverage probability for the CEUs and in the same time decreases the interference level. Considering the worst cases of CEU's SIR and CCU's SIR, the performance effects of CCI on the CEUs are evaluated through two R-A cellular interference mitigation models. These models represent the R-A sectored FFR system and R-A SFR system. Based on the deployed number of RSs, each model contains two different scenarios. Analytical treatments are conducted. Closed form expressions of the worst cases CEU's SIR, CCU's SIR and inner radius are provided. These expressions are used to compare different models using various performance evaluation metrics. It is concluded that the first scenario is better than the second scenario for R-A sectored FFR system because it achieves the same CEU's SIR with lower cost. Furthermore, it is observed that, the second scenario is much superior to first one for R-A SFR system. The work outcomes contribute to network performance improvement due to enhancing CEU's SIR. Consequently, more users can be served by the network and the total network cost is reduced.

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