Combined continuous nonlinear mathematical and computer models of the Information Warfare

Nugzar Kereselidze

Abstract - In this paper, the first attempt is made to combine existing approaches of mathematical and computer modeling of information warfare. As a result, integration mathematical and computer models of information warfare were created. Until now, in the mathematical modeling of the information warfare, issues of information flows and information dissemination were considered separately. The first direction was initiated by the idea of Professor T. Chilachava, to study the distribution of information flows of the two opposite and third peacekeeping sides by mathematical models. The second direction was laid by Academician A.A. Samarskiy and Professor A.P. Mikhailov, who proposed a mathematical model for the dissemination of information among the population. Both these directions have been intensively developed and many scientific studies have been devoted to them. Several dozens of interesting models were created, which reflect the various nuances of the problem. But it is natural that the information and the information for which it is intended should be studied together. During the implementation of this idea, integrated mathematical and computer models of information warfare were created. Integrated common linear and nonlinear mathematical and computer models of information warfare were created. In this paper, integrated general and particular mathematical and computer models for ignoring the enemy are presented. With the help of computer research, a numerical experiment, the question of the existence of a solution to the problem of the Chilker range is studied, which is equivalent to the task of completing information warfare.

Keywords— information warfare, integration mathematical model, integration computer model, Chilker type task.

I. Introduction

Among the diverse types of information warfare, we are attracted to the confrontation of the opponents in the information field: when the sides use, for example, mass dissemination of information and with it try to misinform the enemy, compromise them, etc. The objectives and means of information warfare in this direction are detailed in the works [1], [3]. This type of information war can be called information confrontation, but for simplicity we will use "information warfare" for this type of information warfare.

Mathematical methods of studying information warfare are quite productive and successfully implemented in different countries of the world. A number of mathematical and computer models have been developed, which takes into account the course of information war in the conditions of different regimes: Restriction of information, continuity, discretion, and more. The models are basically two studying objects. First, the information itself, which is distributed as a stream and is intended to discredit the opposing side. Second - the number of people who have received the information disseminated by the side. It is noteworthy that the models developed so far have mostly discussed only the number of information flows or the number of people who have received the information. In the presented work, an attempt is proposed to unify these two approaches, which results in new types of mathematical and computer models. Before representing the results of combining these two traditional methods together, let's talk about them separately.

II. Mathematical models of flow of the Information Warfare

The mathematical modeling of the streams in the information war starts from year 2009, when Georgian scientist **Temur Chilachava's** original idea was worked out and reported in the same year on the fifth congress of Georgian Mathematicians [14] and it was published in Sokhumi State University works [15],[16]. We will bring you a general linear model, as well as a nonlinear model with restrictions. Let's consider the mathematical and computer models of information warfare, in which the search size is considered as the number of provocative information spread by two antagonists and the number of peacekeeping calls made by the third, peacekeeper side. All three parties involved in the information warfare spread information of the relevant party, and any number of promotional information to achieve its goal. At the moment of time $t \in [0;+\infty)$ the number of information disseminated by each party should be noted as $N_1(t)$, $N_2(t)$, $N_3(t)$. Quantity of information at the moment of time t is calculated as the sum of the relevant party, the number of any provocative information that is distributed by all means of mass information. At the same

This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) under Grant № YS17_78.

Nugzar Kereselidze is with the Department of Mathematics and Computer Science, Sukhumi State University, Tbilisi, Georgia, (phone: 995 577 963 923; e-mail: nkereselidze@sou.edu.ge).

time, the opposing parties are distributing $N_1(t)$ and $N_2(t)$ numbers of information. Third, the Peaceful Party calls on the parties to stop the information war, for which the peacekeeper spreads relevant information. As the opposing side aims to influence the information impact on his rival, he tries to disseminate as much as discreditable information as possible about the opponent. At the same time, the dissemination of previously used information is allowed and new disinformation is added. Thus, the speed of dissemination of information by the opposing party depends on the number of information already distributed: $\frac{dN_1(t)}{dt} \sim \alpha_1 N_1(t)$, $\frac{dN_2(t)}{dt} \sim \beta_2 N_2(t)$. In addition, the opposing side reacts to the number of information disseminated by the rival and the peacekeeping parties. Thus, the speed of dissemination of information by the opposing party depends also on: $\frac{dN_1(t)}{dt} \sim \left(\alpha_2 N_2(t) - \alpha_3 N_3(t)\right)$, $\frac{dN_2(t)}{dt} \sim$ $\sim \left(\beta_1 N_1(t) - \beta_3 N_3(t)\right)$. Third, the intensity of information disseminated by the Peace Party depends not only on the number of information disseminated by him, but also on how aggressively the information warfare is going on, how much number of information is disseminated by opposing sides. Taking into account these considerations, we can discuss general linear continuous mathematical model of information warfare:

$$\begin{cases} \frac{dN_1(t)}{dt} = \alpha_1 N_1(t) + \alpha_2 N_2(t) - \alpha_3 N_3(t), \\ \frac{dN_2(t)}{dt} = \beta_1 N_1(t) + \beta_2 N_2(t) - \beta_3 N_3(t), \\ \frac{dN_3(t)}{dt} = \gamma_1 N_1(t) + \gamma_2 N_2(t) + \gamma_3 N_3(t), \end{cases} \quad (1.1)$$

Initial conditions:

$$N_1(0) = N_{10}, \quad N_2(0) = N_{20}, \quad N_3(0) = N_{30}. \quad (1.2)$$

Where $\alpha_1, \alpha_3, \beta_2, \beta_3 \ge 0$, γ_i $\gamma_i \ge 0$ $i = \overline{1,3}$; α_2, β_1, -are constant sizes. Let's name these constant sizes "Model ratios". The speed of change in the number of information disseminated by the first and second parties in the general linear model (1.1) depends on the number of information disseminated by the parties and the international peace organization. Third - the speed of change of the number of calmative information released by international organizations is linearly increasing and is directly proportionate to the number of information disseminated by all three parties. In the initial (1.2) conditions, N_{10}, N_{20}, N_{30} are non-negative permanent sizes.

At the beginning the third party does not disseminate information ($N_{30} = 0$) or make preventative conciliatory statements ($N_{30} > 0$). Peacekeeping side then begins to react to the provocative information spread by the parties. For the first time mathematical and computer models of information warfare, taking into consideration the possibilities of IT technologies, It was proposed in 2012 [4], [5], and was generalized later [2]:

$$\begin{cases} \frac{d}{dt} x(t) = \alpha_1 x(t)\left(1 - \frac{x(t)}{I_1}\right) - \beta_1 z(t), \\ \frac{d}{dt} y(t) = \alpha_2 y(t)\left(1 - \frac{y(t)}{I_2}\right) - \beta_2 z(t), \\ \frac{d}{dt} z(t) = \left(\gamma_1 x(t) + \gamma_2 y_2(t)\right)\left(1 - \frac{z(t)}{I_3}\right). \end{cases} \quad (1.3)$$

$$x(0) = x_0, \; y(0) = y_0, \; z(0) = z_0. \quad (1.4)$$

Where $x(t)$, $y(t)$ is the number of disseminated information by the antagonistic sides at moment - t. Similarly, the peacekeeping - third side's disseminated amount of information $z(t)$ at the same t time. α_1, α_2 accordingly is aggressive coefficients of the first and second opposing sides, β_1, β_2 - peaceful activity options for the opposing sides, γ_1, γ_2 - index of peacekeeping activity of the third party towards the relevant opposing side. I_j, $j = 1,2,3$ - maximum number of technological capabilities of the first and second sides. $x(t)$, $y(t), z(t)$ functions are defined in the section $[0,T]$.

The opposing side reduces the speed of information dissemination according to how close is the number of information dissemination by him at the time to the maximum number of available of information dissemination.

Besides mentioned mathematical models of information warfare, continuous linear and discrete models of ignorance of the opponent have been developed; non-linear, continuous and linear discrete models taking into consideration authoritative organizations or institutions (Religious, non-governmental, scientific, political and other) [12], [13].

III. Mathematical Models of Dissemination Information Flow

The mathematical model of dissemination of information discussed in the classical work of mathematical modeling by **A.A. Samarskiy** and **A.P. Mikhailov** [11]. The mathematical model of organizing advertising campaigns is offered, based on the principle of model universality, can be used in mathematical modeling of information warfare. However, this model does not directly focus on information streams, information receivers' - the number of informed personnel variability is essentially considered. In particular, the following model is discussed: let's $N(t)$ is a number of informed potential customers about the object, N_0 - the total number of potential users, $\alpha_1(t)$-intensity of advertising campaigns, $\alpha_2(t)$ - advertising intensity by users, who know about the object , $\frac{dN(t)}{dt}$ - the changing speed of users informed (adepts), which naturally depends on the number of uninformed consumers and the intensity of advertising campaigns. That's why we get the mathematical model of the following kind of **Samarskiy-Mikhailov**:

$$\frac{dN(t)}{dt}=\left[\alpha_1(t)+\alpha_2(t)N(t)\right]\left(N_0-N(t)\right). \quad (2.1)$$

Let's consider (2.1) substantiation of the model with more details. Advertising, as information is distributed in two ways: first, directly through the advertising campaign, for example via mass media, when by this way, increasing speed of the informed population is proportionate to the number of uninformed population - $\alpha_1(t)\left(N_0-N(t)\right)$ and second, an informed person about advertising tells his acquaintances about this and thus become information spreaders, increasing speed of the informed population is proportionate to the number of uninformed population - $\alpha_2(t)\left(N_0-N(t)\right)$. In fact, advertising is spread by interpersonal relationships. When we combine the speed of advertising spread in these two ways, we will get the model of the whole distribution speed of advertising. In equation (2.1) starting point is, at the beginning of the time, the number of advertised individuals is equal to zero:

$$N(t)|_{t=0}=0 \quad (2.2)$$

One of the authors, **A.P. Mikhailov**, overtime, changed the name of (2.1) and (2.2) model's. At first in cooperation with academician **A.A. Samarskiy**, the monograph published in 1997 was called model of advertising campaign; then, since 2002 , it got called an information threat model [7]; since 2004 - information dissemination model [8]; since 2009, information confrontation model [6]; in 2011 - the model of information warfare [15]; in2015 - information attack and duel model [10], [17].

With model of (2.1), (2.2), **A.P. Mikhailov** and coauthors create different models of dissemination of information, for example: model with forgetting the information, model of information duel and others.

IV. Combined nonlinear mathematical and computer models of Information flow and dissemination

For the integrated mathematical models of the information warfare, let's bring the relevant indications while taking into account the existing traditions. In particular, at $t\in[0;+\infty)$ moment, the number of information, distributed by each side is to be noted with $N_{10}(t)$, $N_{20}(t)$, $N_3(t)$. Quantity of information at moment t, it is calculated as the sum of the relevant party, the number of any provocative information, that is disseminated by all means of mass information. At the same time, the opposing parties are reporting $N_{10}(t)$ and $N_{20}(t)$ number of information. The third - peaceful side calls on the sides to stop the information warfare, for which it disseminates $N_3(t)$ number of information. Let's name $N_{10}(t)$, $N_{20}(t)$, $N_3(t)$ the number of "officially" disseminated information by the respective parties. Then model (1.3), (1.4) will be:

$$\begin{cases} \frac{d}{dt}N_{10}(t)=\alpha_1 N_{10}(t)\left(1-\frac{N_{10}(t)}{I_1}\right)-\beta_1 N_3(t), \\ \frac{d}{dt}N_{20}(t)=\alpha_2 N_{20}(t)\left(1-\frac{N_{20}(t)}{I_2}\right)-\beta_2 N_3(t), \\ \frac{d}{dt}N_3(t)=\left(\gamma_1 N_{10}(t)+\gamma_2 N_{20}(t)\right)\left(1-\frac{N_3(t)}{I_3}\right). \end{cases} \quad (3.1)$$

With initial conditions:

$$N_{10}(0)=N_{10}, N_{20}(0)=N_{20}, N_3(0)=N_{30}. \quad (3.2)$$

Let's say, the first side represents the population, which has maximum value of x_p, in similar, the second side represents the population, which has maximum value of y_p. $x_1(t)$ - the number of population of the first side, who at the time of $t\in[0;+\infty)$ have "received" information officially reported by the first party, became it's adept and by interconnected relationships themselves disseminate beneficial information for the first side by quantity of $N_{11}(t)$. $x_2(t)$ - the number of population of the first side, who at the time of $t\in[0;+\infty)$ have "received" information officially reported by the second party, became it's adept and by interconnected relationships themselves disseminate beneficial information for the second side by quantity of $N_{12}(t)$. Similarly, we can do this for the population of the second party as well: $y_1(t)$ - the number of the population of the second party, which at the time of $t\in[0;+\infty)$ has "received" information officially reported by the first party, became it's adept and by interconnected relationships themselves disseminate beneficial information for the first side by quantity of $N_{21}(t)$. Also, $y_2(t)$ - the number of the population of the second party, which at the time of $t \in [0;+\infty)$ has "received" information officially reported by the second side, became it's adept and by quantity of $N_{22}(t)$. Speed of $N_{11}(t)$ information dissemination, naturally, depends on the meeting of the $x_1(t)$ adepts and the population free of this information, particularly, on $x_1(t)(x_p - x_1(t))$. In addition, if the adept spreads information without interpersonal communication, let's say social network, or through the adepts own web means, then we should additionally consider the maximum capabilities of adept's I_4 IT technologies. In addition, it is natural that the "official" activity of the party, whose we consider, affects the adept activity. So, we have the following ratio:

$$\frac{dN_{11}(t)}{dt} = \alpha_{11}N_{11}(t)(x_p - x_1(t))x_1(t) + \\ +\alpha_{12}N_{10}(t)(I_4 - N_{11}(t)). \quad (3.3)$$

With initial conditions: $N_{11}(0) = 0.$ (3.4)

In the same way we can make correlations for $N_{12}(t)$, $N_{21}(t)$, $N_{22}(t)$, where we can note I_5, I_6, I_7, with maximum possibility of the information technologies of the adepts $x_2(t)$, $y_1(t)$, $y_2(t)$. As for adepts $x_1(t)$, $x_2(t)$, $y_1(t)$, $y_2(t)$, for them correlations are derived from **Samarskiy-Mikhailov** model, where an intensive advertising campaign is presented as $\alpha_i N_{j0}(t)$, where $i = 3,5,7$, $j = 1,2$, and the intensity of the dissemination of information by the adept is assessed as $\alpha_k N_{ls}(t)$, where $k = 4,6,8,10$, $l,s = 1,2$. As a result we get the integrated mathematical model information warfare with restrictions:

$$\begin{cases} \frac{d}{dt}N_{10}(t) = \alpha_1 N_{10}(t)\left(1 - \frac{N_{10}(t)}{I_1}\right) - \\ -\beta_1 N_3(t), \\ \frac{d}{dt}N_{20}(t) = \alpha_2 N_{20}(t)\left(1 - \frac{N_{20}(t)}{I_2}\right) - \\ -\beta_2 N_3(t), \\ \frac{d}{dt}N_3(t) = (\gamma_1 N_{10}(t) + \gamma_2 N_{20}(t)) \times \\ \times\left(1 - \frac{N_3(t)}{I_3}\right), \\ \frac{dx_1(t)}{dt} = (\alpha_3 N_{10}(t) + \alpha_4 N_{11}(t)x_1(t)) \times \\ \times(x_p - x_1(t)), \\ \frac{dx_2(t)}{dt} = (\alpha_5 N_{20}(t) + \alpha_6 N_{12}(t)x_2(t)) \times \\ \times(x_p - x_2(t)), \\ \frac{dy_1(t)}{dt} = (\alpha_7 N_{10}(t) + \alpha_8 N_{21}(t)y_1(t)) \times \\ \times(y_p - y_1(t)), \\ \frac{dy_2(t)}{dt} = (\alpha_9 N_{20}(t) + \alpha_{10} N_{22}(t)y_2(t)) \times \\ \times(y_p - y_2(t)), \\ \frac{dN_{11}(t)}{dt} = \alpha_{11}N_{11}(t)(x_p - x_1(t))x_1(t) + \\ +\alpha_{12}N_{10}(t)(I_4 - N_{11}(t)), \\ \frac{dN_{12}(t)}{dt} = \alpha_{13}N_{12}(t)(x_p - x_2(t))x_2(t) + \\ +\alpha_{14}N_{20}(t)(I_5 - N_{12}(t)), \\ \frac{dN_{21}(t)}{dt} = \alpha_{15}N_{21}(t)(y_p - y_1(t))y_1(t) + \\ +\alpha_{16}N_{10}(t)(I_6 - N_{21}(t)), \\ \frac{dN_{22}(t)}{dt} = \alpha_{17}N_{22}(t)(y_p - y_2(t))y_2(t) + \\ +\alpha_{18}N_{20}(t)(I_7 - N_{22}(t)). \end{cases} \quad (3.5)$$

With initial conditions:

$$\begin{cases} N_{10}(0)=n_{10}, N_{20}(0)=n_{20}, N_3(0)=n_{30}, \\ N_{11}, \quad N_{12}, \quad N_{21}, \quad N_{22}=0, \\ x_1(0), \quad x_2(0), \quad y_1(0), \quad y_2(0)=0. \end{cases} \tag{3.6}$$

Thus, we have acquired an integrated mathematical model of information warfare with restrictions, which is described by the ordinary differential equation system (3.5), in which there are eleven functional searches and eleven initial conditions (3.6). Note that in (3.5), (3.6) – Cauchy task, right side of (3.5) system have features and it dive us the basis to conclude that this Cauchy task has the one solution for the time segment $t \in [0;+\infty)$.

Thus the number of useful information for the first party $N_1(t)$ is the sum of the "official" information of the first party, information of first side's adepts in the first party population and information spread by adepts in the second party's population which is beneficial for the first party –

$$N_1(t)=N_{10}(t)+N_{11}(t)+N_{21}(t). \tag{3.7}$$

And the number of useful information for the second party - $N_2(t)$, represents the following sum:

$$N_2(t)=N_{20}(t)+N_{12}(t)+N_{22}(t). \tag{3.8}$$

Third side impacts are clearly demonstrated in $N_{10}(t)$, $N_{20}(t)$ information flows and by their means this impact is realized in $N_{11}(t)$, $N_{12}(t)$, $N_{21}(t)$, $N_{22}(t)$ flows. So, it is natural to raise the question – whether it is possible to put out information warfare with the activity of the third side, at any point in time, in different moments, $N_1(t)$, $N_2(t)$ to become zero:

$$N_1(t^*)=0, \ N_2(t^{**})=0. \tag{3.9}$$

Let's call (3.5), (3.6), (3.9) boundary task the Chilker type task, because conditions for the right side are specific, in particular, $N_1(t)$, $N_2(t)$ functions cross zero generally in different conditions and also these times are not fixed. In model (3.5), (3.6), in the information flow, spread by adepts, participating in the "official" information streams of opposing sides. It not happens on the contrary, but it is possible. That's why it's natural, in speed correlations of information spread by the adepts $N_{11}(t)$, $N_{12}(t)$, $N_{21}(t)$, $N_{22}(t)$ were involved $N_{10}(t)$, $N_{20}(t)$ of information streams. This case in considered other models.

V. Combined Mathematical and Computer Models of the Information Warfare for Ignoring Opponent

Let's consider the model task of Integrated mathematical and computer models of the Information Warfare, in other words, let's consider the simplest case of the model. For this, for information flows we need to take the ignorance model of the enemy:

$$\begin{cases} \frac{d}{dt}N_{10}(t)=\alpha_1 N_{10}(t)-\beta_1 N_3(t), \\ \frac{d}{dt}N_{20}(t)=\alpha_2 N_{20}(t)-\beta_2 N_3(t), \\ \frac{d}{dt}N_3(t)=\gamma_1 N_{10}(t)+\gamma_2 N_{20}(t). \end{cases} \tag{4.1}$$

With (3.2) initial conditions. In the model of Samarskiy-Mikhailov, let's represent a linear form for the activity of parties and adepts. In addition we are only considering $x_1(t)$, $y_2(t)$ adepts, who are only involved in correlations of disseminated information by $N_{10}(t)$, $N_{20}(t)$ - sides. Let's say, that $x_1(t)=x(t)$, $y_2(t)=y(t)$. As a result we will get an integrated mathematical model of information warfare, which was formed at Sokhumi State University, faculty of Mathematics and Computer Sciences, after discussion with Professor **T. Chilachava** and which has the form:

$$\begin{cases} \frac{d}{dt}N_{10}(t)=\alpha_1 N_{10}(t)+\nu_1 x(t)-\beta_1 N_3(t), \\ \frac{d}{dt}N_{20}(t)=\alpha_2 N_{20}(t)+\nu_2 y(t)-\beta_2 N_3(t), \\ \frac{d}{dt}N_3(t)=\gamma_1 N_{10}(t)+\gamma_2 N_{20}(t), \\ \frac{dx(t)}{dt}=(\alpha_3+\gamma_3 x(t))(x_p-x(t)), \\ \frac{dy(t)}{dt}=(\alpha_4+\gamma_4 y(t))(y_p-y(t)). \end{cases} \tag{4.2}$$

With initial conditions:

$$\begin{cases} N_{10}(0)=n_{10}, N_{20}(0)=n_{20}, N_3(0)=n_{30}, \\ x(0)=y(0)=0. \end{cases} \tag{4.3}$$

If we say that, $\alpha_1=\alpha_2=\alpha$, $\beta_1=\beta_2=\beta$, $\gamma_1=\gamma_2=\gamma$, $\alpha_3=\alpha_4=\delta$, $\nu_1=\nu_2=\nu$.

Then (4.2) will get the form:

$$\begin{cases} \frac{d}{dt}N_{10}(t) = \alpha N_{10}(t) + \nu x(t) - \beta N_3(t), \\ \frac{d}{dt}N_{20}(t) = \alpha N_{20}(t) + \nu y(t) - \beta N_3(t), \\ \frac{d}{dt}N_3(t) = \gamma N_{10}(t) + \gamma N_{20}(t), \\ \frac{dx(t)}{dt} = (\delta + \mu x(t))(x_p - x(t)), \\ \frac{dy(t)}{dt} = (\delta + \mu y(t))(y_p - y(t)). \end{cases} \tag{4.4}$$

Initial conditions are given in (4.3). There are five unknown functions in the system (4.4), but the last two equations of this system, which results from **Samarskiy-Mikhailov** model, will be analytically solved, when the equation in the second string of (4.3) is completed for the initial conditions, as a result we have:

$$x(t) = \frac{\delta x_p\left[\exp((\delta + \mu x_p)t) - 1\right]}{\delta \exp((\delta + \mu x_p)t) + \mu x_p}, \tag{4.5}$$

$$y(t) = \frac{\delta y_p\left[\exp((\delta + \mu y_p)t) - 1\right]}{\delta \exp((\delta + \mu y_p)t) + \mu y_p}, \tag{4.6}$$

By adding (4.4) into two equations, (4.5) and (4.6), we get a system with tree equations:

$$\begin{cases} \frac{d}{dt}N_{10}(t) = \alpha N_{10}(t) + \nu x(t) - \beta N_3(t), \\ \frac{d}{dt}N_{20}(t) = \alpha N_{20}(t) + \nu y(t) - \beta N_3(t), \\ \frac{d}{dt}N_3(t) = \gamma N_{10}(t) + \gamma N_{20}(t). \end{cases} \tag{4.7}$$

with initial conditions:

$$N_{10}(0) = n_{10}, N_{20}(0) = n_{20}, N_3(0) = n_{30}, \tag{4.8}$$

Notice, that it is possible to find analytical solutions of the **Cauchy** task, because with transformations in (4.7) we will get a linear no homogeneous ordinary differential equation of the second order with constant coefficients towards $N_3(t)$, and placing the solution system (4.7) - homogeneous ordinary differential equation of first order with constant coefficient towards $N_{10}(t)$, $N_{20}(t)$. But after adding the **Cauchy** task (4.7), (4.8) conditions to the right end:

$$N_{10}(t^*) = 0, \; N_{20}(t^{**}) = 0. \tag{4.9}$$

Where t^*, t^{**} are non-fixed, in general, different time moments on time $[0, T > 0]$ period, we will get a **Chilker** type task, which we will investigate via computer modeling and we will conduct a computer experiments. Let's say that we have initial conditions:

$$\begin{cases} N_{10}(0) = 0.1, N_{20}(0) = 0.001, N_3(0) = 3, \\ x(0) = y(0) = 0. \end{cases} \tag{4.10}$$

Possibility of solving **Chilker** type (4.7)-(4.7) task, according to computer experiments, depends on the aggressiveness of sides, in particular on $D = \alpha^2 - 8\beta\gamma$. For example, if the aggression of antagonistic sides is large, which cause D to be nonnegative: $D \geq 0$, then there is no solution to the **Chilker** type problem, which means that the third side cannot stop the information warfare by its actions. For example, if $\alpha = 1,8$; $\beta = 0,05$; $\nu = 0,05$; $\gamma = 0,05$; $\delta = 0,3$; $\mu = 0,2$; $x_p = 155$; $y_p = 150$; which means that $D = 3,04$, as we see on Figure 1_a, both antagonist forces strengthen the information warfare and third party's impact on them is unsuccessful. In the conditions of high aggression by the parties, if $D = 0$, what happens for this meaning of parameters: $\alpha = 0,08$; $\beta = 0,08$; $\nu = 0,05$; $\gamma = 1$; $\delta = 0,3$; $\mu = 0,2$; $x_p = 155$; $y_p = 150$; then it is possible for one side to go to zero and third party can influence one of the antagonistic sides. See Figure 1_b.

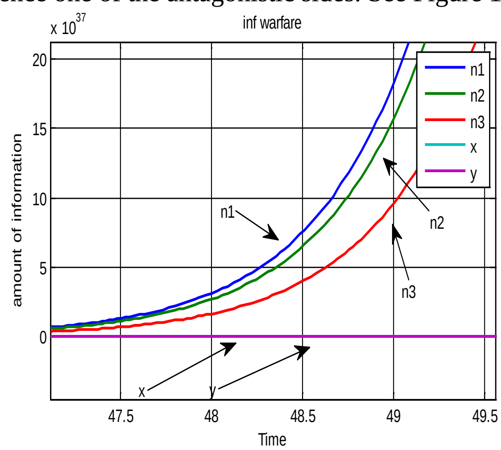

Figure 1_a. The parties involved in the IW, when $D > 0$

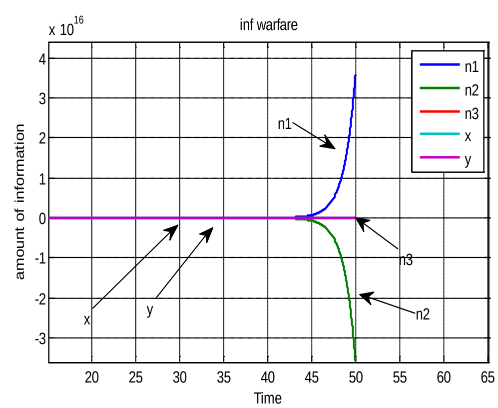

Figure 1_b. The parties involved in the IW, when $D = 0$

As for the case of low aggression, when $D<0$, in particular for parameters: $\alpha=0,08$; $\beta=0,5$; $\nu=0,05$; $\gamma=1$; $\delta=0,3$; $\mu=0,2$; $x_p=155$; $y_p=150$; then third party impacts are effective on antagonistic sides, they go to zero, which means that the Chilker type problem has a solution.

Based on analysis of results of computer experiments, we can conclude, that solution behavior of integrated mathematical model of ignoring the enemy is similar to relevant non integrated mathematical model solutions [16], [17], in particular, dependence of solutions on D-on the level of aggressiveness of parties. At the same time, it should be noted that the number of adepts in the model has made some corrections, for example, under conditions of low aggression, D is negative and increases in module, then for little time value the antagonistic side for parameters $\alpha=0,08$; $\beta=15$; $\nu=0,05$; $\gamma=3,5$; $\delta=0,3$; $\mu=0,2$; $x_p=155$; $y_p=150$, $D=-419,9$, go to zero, see Figure 2_a. but, for big t, one of the antagonistic sides (depends on correlations of the initial values of the parties) doesn't go to zero, see Figure 2_b.

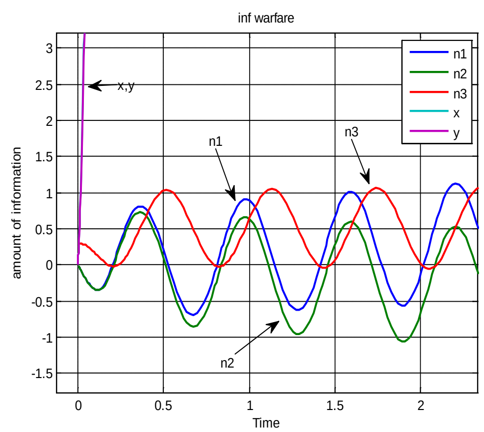

Figure 2_a. The parties involved in the IW, when $D<0$

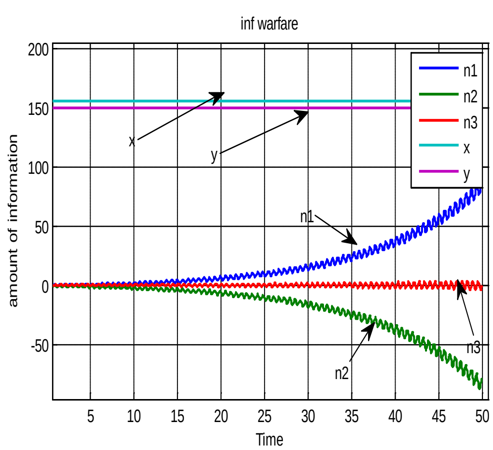

Figure 2_b. The parties involved in the IW, when $D<0$

The computer experiment was also conducted (4.7)-(4.7) in the general case of the **Chilker** type problem. In particular, when the antagonistic parties have different aggressiveness (α_1,α_2) and peace readiness (β_1,β_2) indicators, also different values of peacekeeping activity towards the parties $(\gamma_1,\gamma_2,\gamma_3)$, in the equations of adept's quantity the number of information streams of relevant opposing parties is included:

$$\begin{cases} \frac{d}{dt}N_{10}(t)=\alpha_1 N_{10}(t)+\nu_1 x(t)-\beta_1 N_3(t), \\ \frac{d}{dt}N_{20}(t)=\alpha_2 N_{20}(t)+\nu_2 y(t)-\beta_2 N_3(t), \\ \frac{d}{dt}N_3(t)=\gamma_1 N_{10}(t)+\gamma_2 N_{20}(t)+\gamma_3 N_3(t), \\ \frac{dx(t)}{dt}=\left(\alpha_3 N_{10}(t)+\gamma_4 x(t)\right)\left(x_p-x(t)\right), \\ \frac{dy(t)}{dt}=\left(\alpha_4 N_{20}(t)+\gamma_5 y(t)\right)\left(y_p-y(t)\right). \end{cases} \quad (4.11)$$

The computer experiment was conducted for the system (4.11) with different initial conditions (4.10) and parameters. We can conclude from the results obtained, that the relevant Chilker type task - (4.11), (4.6), (4.9) has a solution, in case of nigh peace readiness and activity. In particular, if we have the data of parameters, where aggressiveness prevails over peace readiness and activity: $\alpha_1=4,8$; $\beta_1=,5$; $\nu_1=1,5$; $\alpha_2=5,6$; $\beta_2=,7$; $\gamma_1=,05$; $\gamma_2=,3$; $\gamma_3=,07$; $x_p=155$; $y_p=150$; $\alpha_3=2,3$; $\alpha_4=2,2$; $\gamma_4=,2$; $\gamma_5=,3$, then the antagonistic parties develop information warfare and the third party's impact is unsuccessful, see Figure 3_a, which is derived for initial conditions -

$$\begin{cases} N_{10}(0)=0.2, N_{20}(0)=0.01, N_3(0)=.03, \\ x(0)=y(0)=0. \end{cases} \quad (4.12)$$

The **Chilker** type problem (4.11), (4.6), (4.9) has a solution when the aggressiveness of opposing parties is relatively low, for example, for parameters: $\alpha_1=,08$; $\beta_1=1,5$; $\nu_1=,05$; $\alpha_2=,06$; $\beta_2=1,7$; $\gamma_1=,05$; $\gamma_2=,03$; $\gamma_3=,07$; $x_p=155$; $y_p=150$; $\alpha_3=,3$; $\alpha_4=,2$; $\gamma_4=,2$; $\gamma_5=,3$, see Figure 3_b.

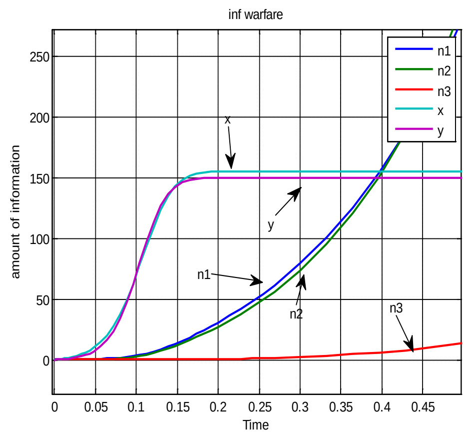

Figure 3_a. The parties involved in the IW for Chilker type task - (4.11), (4.12), (4.9), with high aggression

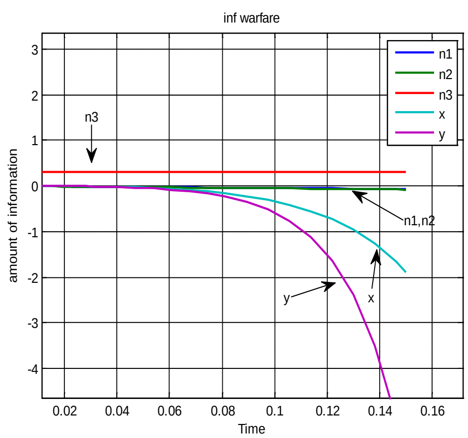

Figure 3_b. The parties involved in the IW for Chilker type task - (4.11), (4.12), (4.9), with low aggression

Computer experiments were conducted in the Mat Lab environment, m-file were created

VI. Conclusion

In the present work there is offered integrated mathematical and computer models of information warfare in which independently expressed approaches till now are reflected. In particular, academician **A.A. Samarskiy** and his coauthor's, Professor **A.P. Mikhailov**'s advertising campaign and professor **T. Chilachava**'s tree sides information stream (flow) models are united. It provides new general linear model of information warfare; model with restrictions on information technologies, private and advanced integrated mathematical and computer models of ignore enemy. For last two models, computer experiments have been conducted, which helped to identify solving possibility of **Chilker** type task for information warfare.

VII. Acknowledgment

I would to thank the Vice-President of the Academy of Sciences of Tskhum-Abkhazia, Sukhumi State university's faculty of applied mathematics and computer science, Professor in direction of mathematical modeling T. Chilachava, for held discussion about information warfare's integrated models and useful advices, which significantly helped research in the direction.

I express my gratitude to the Shota Rustaveli National Science Foundation of Georgia, for financial support in preparing the article through the grant YS17_78

References

[1] Kereselidze N. Mathematical modeling of information warfare. Monogram. (In Georgian). National Parliamentary Library of Georgia ISBN: 978-9941-0-4892-0. 2012 year 176 p. http://kereseli.besaba.com/monografia%20formatizaziiT.pdf.

[2] Chilachava T., Kereselidze N. Mathematical Model of Information Warfare. (In Georgian). Georgian Electronic Scientific Journal: Computer Science and Telecommunications. 2010, no. 1 (24). p. 78 - 105.

[3] Chilachava T., Kereselidze N. About one mathematical model of the information warfare. Fifth congress of mathematicians of Georgia. Abstracts of contributed talks. Batumi/Kutaisi, 9-12 October. 2009. p. 85.

[4] Chilachava T., Kereselidze N. Non-preventive continuous linear mathematical model of information warfare. Sokhumi State University Proceedings, Mathematics and Computer Sciences vol. 7. 2009, № 7. p. 91 – 112..

[5] Chilachava T., Kereselidze N. Continuous linear mathematical model of preventive information warfare. Sokhumi State University Proceedings, Mathematics and Computer Sciences vol. 7. 2009, № 7. p. 113 – 141..

[6] Kereselidze N. Mathematical model of information warfare taking into account the capabilities of the information technologies of the opposing sides. (In Russian). Transactions II The International Technical Conference dedicated to the 90th anniversary of the Georgian Technical University "Basic Paradigms in Science and Technology", 21st Century, Tbilisi, Georgia, September 19-21, 2012. Publishing House "Technical University", Tbilisi, 2012, p. 188-190.

[7] Kereselidze N. Mathematical model of information confrontation taking into account the possibilities of Information Technologies of the parties. (In Russian). // Proceedings of the XX International Conference Problems of Security Management of Complex Systems. Moscow, December 2012, art. 175-178.

[8] Kereselidze N. Mathematical and computer models of information warfare. Monograph. (In Georgian). ISBN 978-9941-0-9617-4 (PDF). 2017. http://kereseli.besaba.com/Math_and_comp_models_IW_monograph_2017.pdf.

[9] Chilachava T., Chakhvadze A. Continuous Nonlinear mathematical and computer model of information warfare with participation of interstate authoritative institutes. Georgian Electronic Scientific Journal: Computer Science and Telecommunications 2014| No. 4(44), p. 53 – 74.

[10] Chilachava T., Chakhvadze A. Non llinear Mathematical and Computer Models of Information Warfare with Participation of Autoritative Interstate Institutes. Book of abstracts V Annual International Conference of the Georgian Mathematical Union, Batumi, September 8-12, 2014. p. 76.

[11] Samarskiy AA, Mikhailov AP Mathematical modeling: Ideas. Methods. Examples. 2nd ed. Correction. - M. FIZMATLIT. 2005. 320 pp.

[12] Marevtseva N.A. Mathematical models of information attack and information confrontation. Jour. Sociology. No. 3. 2011 p. 26 -35..

[13] Mikhailov A.P., Izmodenova K.V. On optimal control in the mathematical model of information dissemination. (In Russian).

Proceedings of the seminar "Mathematical modeling of social processes", compilation, issue. 6. - Moscow: MAX Press, 2004.

[14] Marevtseva N.A. The simplest mathematical models of information confrontation. (In Russian). / Series "Mathematical Modeling and Modern Information Technologies", no. 8. // Collection of works of All-Russian scientific youth schools. Rostov-on-Don, the publishing house of the Southern Federal University. 2009. P. 354-363.

[15] Mikhailov A.P., Marevtseva N.A. Models of information struggle. (In Russian). // Math modeling. 2011. P. 23. № 10. P. 19-32.

[16] Mikhailov A.P., Petrov A.P., Proncheva O.G., Marevtseva N.A. Mathematical modeling of information confrontation in society. (In Russian). International Economic Symposium 2015. Materials of the International scientific conferences devoted to the 75th anniversary of the Faculty of Economics of the St. Petersburg State University: a collection of articles. Sheaf Ed. S.A. Belozerov, OOO "Scythia-print", St. Petersburg, 2015. S. 293-303. URL: http://econ-conf.spbu.ru/files/Symposium_Sbornik_Statey.pdf.

[17] Mikhailov A.P., Petrov A.P., Proncheva O.G., Marevtseva N.A. Mathematical Modeling of Information Warfare in a Society // Mediterranean Journal of Social Sciences. Vol. 6. No. 5 S2. pp. 27–35.

Nugzar Kereselidze - was born in Tbilisi, Georgia on January 31, 1955. In 1977 graduated from the Faculty of Cybernetics and Applied Mathematics of the Tbilisi State University, Georgia. Defended doctoral dissertation at the Georgian University named after Andrew the First-Called, and became the academic doctor of Informatics.

Since 2013, he is assistant professor of the Sukhumi State University. 1977-1983 he was the research associate of Institute of Cybernetics of Academy of Sciences of Georgia. 1983-2006 lecturer of the Abkhazian State University and Sukhumi branch of the Tbilisi State University. **N. Kereselidze**, An Optimal Control Problem in Mathematical and Computer Models of the Information Warfare. Differential and Difference Equations with Applications : ICDDEA, Amadora, Portugal, May 2015, Selected Contributions. /Editors: Pinelas, S., **Došlб**, Z., **Došlэ**, O., **Kloeden**, P.E. (Eds.), **Springer** Proceedings in Mathematics & Statistics, 164, DOI 10.1007/978-3-319-32857-7_28, Springer International Publishing Switzerland 2016 http://www.springer.com/gp/book/9783319328553, 2016, p. 303-311. **N. Kereselidze**, Optimizing Problem of the Mathematical Model of Preventive Information Warfare. Informational and Communication Technologies – Theory and Practice: Proceedings of the Inter national Scientific Conference ICTMC-2010 USA, Imprint: Nova https://www.novapublishers.com/catalog/product_info.php?products_id=25352, 2011, p. 525-529.

Dr. Nugzar Kereselidze, member of the Georgian Mathematical Union - GMU, member of presidium of the GMU.