A Constrained ILC Method for Signal Intersection's Green Ratio in Fixed Cycle

Taiyuan Ruan, Hao Zhou and Zhiyong Liu

Abstract—Under the urban arterial traffic coordination control, in order to ensure the formation of phase difference between the road intersections, the signal cycle of each intersection must be same. The vehicle queue length in the intersections is determined by the green time of each phase when the cycle is fixed. However, to optimize the green ratio quickly and effectively in each cycle is difficult. In this paper, we proposed a constrained Iterative Learning Control(ILC) method to solve this problem.In order to verify its effectivenes, six experiments are carried out to investigate the efficiency of ILC and constrained ILC for differernt traffic load under the arterial coordination control. The simulation results show: the control efficiency is higher under the constrained ILC than that under the ILC. In each cycle, the constrained ILC can calculate the optimum green ratio according to the vehicle queue length in the last cycle.So the constrained ILC presented in this paper,can optimize the green ratio effectively for different traffic load in a fixed cycle, it can minimize the vehicle queue length.

Key words—arterial coordination control, green ratio, constrained Iterative Learning Control, queue length.

I. INTRODUCTION

In urban roads, traffic trunks usually undertake a large amount of traffic load, so it has a significant effect the situation of traffic congestion when the traffic arterial coordinated control is implemented [1]. Under the traffic arterial coordination control, the signal control of intersections is usually divided into several periods. The length of signal cycle is fixed in every period. So when the cycle and the phase difference are fixed, the vehicle queue length can be reduced by optimizing the green ratio of each phase [2].

Obviously, an intersection which runs repetitively (as a fixed cycle) can be regarded as a subject controlled by the signal in each period, so the ILC method can be used to control the traffic signal. The essence of iterative learning is continuously modifying the learning control algorithm by tracking the error between the ideal output and the actual output, so that the traffic signal control can reach the ideal state [3]. But in practice, the change of traffic flow in each phase is uncertain, so the general ILC method cannot track the change of each phase's traffic flow in time [4].

This paper uses a constrained ILC method, it establishes the objective function and constraints according to the error between the ideal vehicle queue length and the actual vehicle queue length during each iteration process, they can ensure the green ratio scheme is optimal and will be used in the next iterative progress, so that the vehicle queue length is minimum. Finally, we use the simulation software of MATLAB to simulate the algorithm in this paper, simulating the change of vehicle queue length in different traffic load. The results show that the method proposed in this paper is effective.

Taiyuan Ruan is with the Department of Electronic, Jiangmen Polytechnic, Jiangmen, 529000 Jiangmen, China (corresponding author; e-mail: wdl8898@yeah.com).

Hao Zhou is with the Department of Electronic, Jiangmen Polytechnic, Jiangmen, 529000 Jiangmen, China

Zhiyong Liu is with the Department of Electronic, Jiangmen Polytechnic, Jiangmen, 529000 Jiangmen, China

II. URBAN ROAD TRAFFIC MODEL

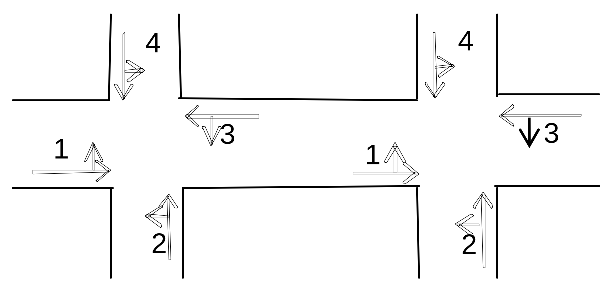

Fig.1 A diagram of phase, two continuous connected intersections on the main road

As shown in **Fig.1**, we establish a diagram of phase in two continuous connected intersections under the arterial coordination control. Assuming that there are exist N traffic flows which influence each other in one intersection. Generally, we need to set up N signal phase. If we set up i as the name of one phase, the traffic parameters about this phase are as follows: q_i as the vehicle arrival rate, s_i as the traffic saturated flow, y_i as the vehicle queue length, $t_i(s)$ as the effective green light time. Generally, the vehicle queue length can reflect the vehicles delay time at the intersection, so we take the vehicle queue length on intersection as the state variable of the control system. Assuming the value of intersection's signal cycle is $C(s)$, we can establish a discrete time state equation of vehicle queue length under the k_{th} cycle:

$$y_i(k+1) = y_i(k) + Cq_i - t_i(k)s_i \tag{1}$$

Define a vector, named $r_i(k)$, the $r_i(k)$ can be written:

$$r_i(k) \overset{\Delta}{=} 1 - \frac{t_i(k)s_i}{Cq_i}\ ,\ i = 1....,\ N \tag{2}$$

By processing the formula (1), we can obtain the result:

$$y_i(k+1) = y_i(k) + Cq_i r_i(k) \tag{3}$$

Because under the traffic arterial coordination control, the cycle is fixed at during a certain period of time, and the vehicle arrival rate is also fixed before the next cycle begins. So we can use $r_i(k)$ as the vehicle control variable on the i_{th} phase of the k_{th} cycle of the intersection, use the N as the sum of intersection's phase. Through the above process, we can establish the state equation and output equation for discrete linear constant system:

$$\begin{cases} y_i(k+1) = y_i(k) + Cq_i r_i(k) \\ z_i(k) = y_i(k) \end{cases} \tag{4}$$

where i =1,..... N . The system output, $z_i(k)$ represents the vehicle queue length of i_{th} phase.

Ideally, at the intersection, the vehicle arrival rate is lower than the designed capacity, so the vehicle queue length is 0. However, in fact, at the intersection, the vehicle arrival rate is random, sometimes it will be higher than the designed capacity. Therefore, at the intersection, the vehicle queuing phenomenon is unavoidable.

According to the above description, we construct such a process: in a period of time, the cycle is constant, taking every $K(K>1)$ cycles as an iteration process, rewritten formula (4):

$$\begin{cases} y_{i,j}(k+1) = y_{i,j}(k) + Cq_{i,j} r_{i,j}(k) \\ z_{i,j}(k) = y_{i,j}(k) \end{cases} \tag{5}$$

i is the phase number, j is the index of running iteration, and k is the index of cycle in each iteration period[5].

The ILC law is:

$$r_{i,j+1}(k) = Q_i(q)[r_{i,j+1}(k) + L_i(q)e_{i,j+1}(k+1)] \tag{6}$$

where, the $Q_i(q)$ is the Q filter and the $L_i(q)$ is the learning function. q is the one step forward operator, $qx(k) = x(k+1)$. i is the phase number, j is the index of running iteration, and k is the index of cycle in each iteration period[6].

The error between the ideal vehicle queue length and the actual vehicle queue length is:

$$e_{i,j}(k) = z_d(k) - z_{i,j}(k) \tag{7}$$

where, $z_d(k)$ is the ideal output value of i_{th} phase[7].

In the actual signal control, in order to ensure the road safety, each phase needs to set the minimum green time $t_{\min}(s)$, so the effective green time $t_i(s)$ in each phase should meet the condition[8]:

$$t_i \ge t_{\min} \tag{8}$$

where, $t_{\min}$ is the minimum green time, generally should be set 10~15 (s), i = 1, 2,... N .

On the other hand, the cycle is fixed, if the total lose time of each cycle be set $L(s)$, so the equation can be written:

$$t_1 + t_2 + \cdots + t_N = C - L \tag{9}$$

Use the formula (3) in formula (7) and formula (8), and take the index of iteration to equation, the equation can be written:

$$\begin{cases} a_i(1-r_{i,j}) \ge t_{\min} \\ a_1(1-r_{1,j}) + a_2(1-r_{2,j}) + \cdots + a_N(1-r_{N,j}) = C - L \end{cases} \tag{10}$$

where , $a_i = C\dfrac{q_i}{s_i},\ i = 1, \cdots, N$

III. CONSTRAINED ILC METHOD FOR OPTIMIZING THE INTERSECTION'S GREEN RATIO

A. *THE DESCRIPTION OF ILC PROCESS*

The control process is as follows:

Step 1: initialize the basic traffic parameters, where the initial green time is greater than the minimum green time;

Step 2: use the formula (5) and (7) to get the vehicle queue length and learning error in each cycle of the j_{th} iteration;

Step 3: update the $(j+1)_{th}$ iterative control learning law r_{j+1} by using the formula (6);

Step 4: check whether the sum of green time of every phase of each cycle in the r_{j+1} iteration is over the set value C if over, the green time of each phase should be redistributed in the cycle C according to the proportion in the updated cycle, so that it can ensure the cycle is fixed after each iteration;

Step 5: repeat step 2 ,3 and 4 until the number of iteration reach a set value.

B. *THE OPTIMIZATION OF LEARNING LAW*

If we use the ILC method, it not only should ensure the learning law in the control is convergent, but also the cycle of each iteration is inconsistent with the initial cycle, so it need post-processing[9]. Therefore, the There is room for improvement on controlling efficiency of the ILC method.

In order to improve the control efficiency of ILC method, the optimal method is used to find the learning law, the objective function and constraints can be written:

$$J(\mathbf{r}_j) = \min_{\mathbf{r}_j} \left\| \mathbf{e}_j(k+1) \right\|^2 \tag{11}$$

$$s.t. \begin{cases} a_i(r_{i,j} - 1) \le -t_{\min} \\ a_1(1 - r_{1,j}) + a_2(1 - r_{2,j}) + \ldots + a_N(1 - r_{N,j}) = C - L \end{cases}$$

where $\mathbf{r}_j = [r_{1,j}, r_{2,j}, \cdots, r_{M,j}]^\tau$,

$\mathbf{e}_j(k+1) = [e_{1,j}(k+1), e_{2,j}(k+1), \cdots, e_{i,j}(k+1)]^\tau$, $a_i = C\dfrac{q_i}{s_i}$, where, i =1,2,3,... N, C is the signal cycle, L is the lose time, and $t_{\min}$ is the minimum green time[10].

The meaning of the above equation, is very clear: finding the r_j that is the vehicle pass control vector, to make the variance between the output of the controlled object and the ideal value be minimum, that is the vehicle queue length of each phase is the shortest.

C. THE BASIC DESCRIPTION OF CONSTRAINED ILC METHOD

The method steps are as follows:

Step 1: initialize the basic traffic parameters, where the initial green time is greater than the minimum green time;

Step 2: use the formulas (5) and (7) to get the vehicle queue length and learning error of each cycle in the j_{th} iteration;

Step 3: use the formula (11) to obtain the $(j+1)_{th}$ control learning law;

Step 4: repeat step 2 and 3 until the number of iteration reach the set value.

IV. SIMULATION RESULTS

Using the simulation software of MATLAB to simulate the control effect of intersection, which use ILC method and constrained ILC method to optimize the green ratio. Two methods use the same traffic conditions, as follows:

Situation 1: the density of traffic flow is not large, and the interaction among vehicles is weak and random.

Situation 2: traffic is crowded, little chance to free driving.

Situation 3: the fluctuation of traffic flow is very large, or observe the number of vehicle in a certain time interval which is classified into peak period and non-peak period.

ILC method and constrained ILC method are used in the same conditions except using different methods to optimize the green ratio. Here we divide a time segment into 60 cycles and 20 iterations, each iteration consists of 3 cycles. The constrained ILC method is simulated by the MATALB. After each iteration process, the vehicle queue length is taken as the initial value of the next iteration process.

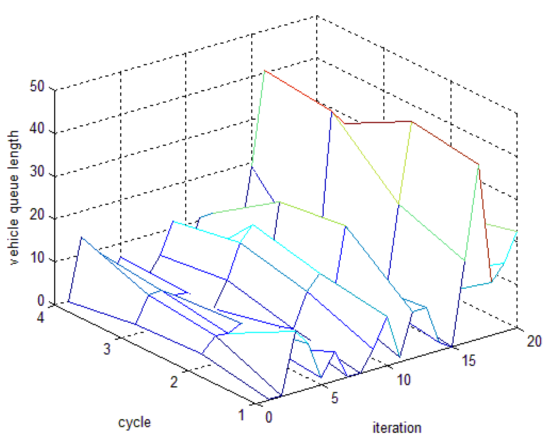

Fig.2.in the situation1, the vehicle queue length with ILC

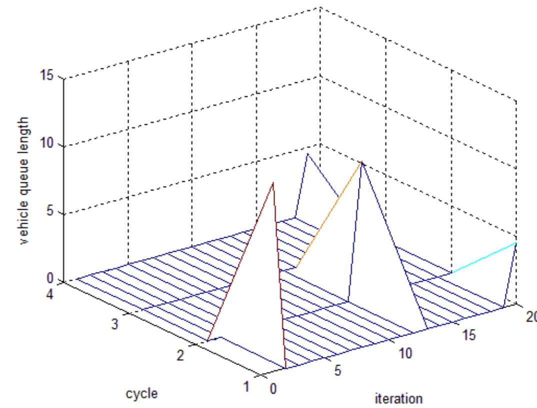

Fig.3.in the situation1, the vehicle queue length with constrained ILC

The simulation results shown in **Fig.2** and **Fig.3** are got under the traffic load of situation 1. In this situation, the traffic flow is small. But because the vehicle arrival rate is random, we can find there are some situations that the vehicle queue length is relatively longer from the **Fig.2** and **Fig.3**. As shown in **Fig.3**, the situations that vehicle queue up in the intersection are few during the 60 cycle. Compared with the control effect of **Fig.2**, the constrained ILC method have higher efficiency in optimizing the green ratio.

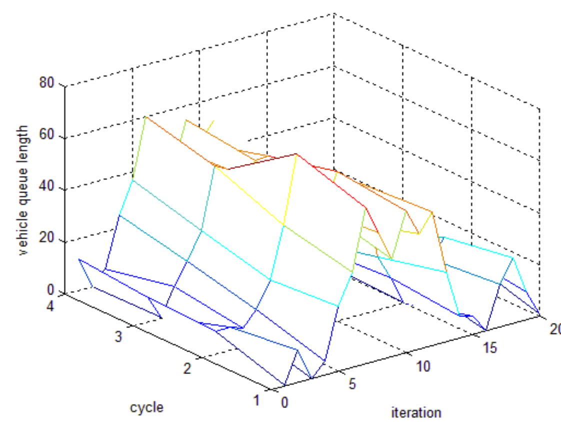

Fig.4.in the situation 2, the vehicle queue length with ILC

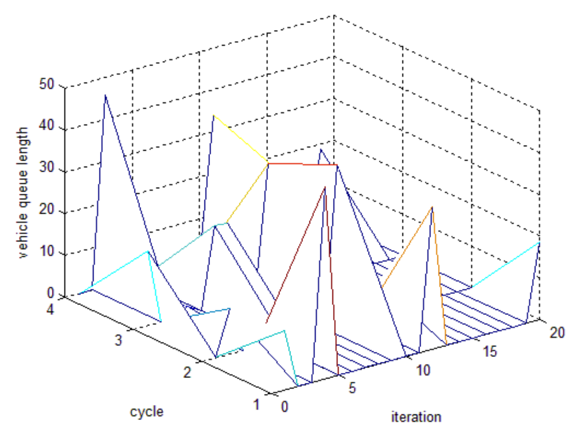

Fig.5.in the situation 2, the vehicle queue length with constrained

ILC

The simulation results shown in **Fig.4** and **Fig.5** are got under the traffic load of situation 2. As the traffic flow is relative higher, so that we can find the vehicle queuing times are relative more and vehicle queue length is relative longer. By compared the **Fig.4** with **Fig.5**, the constrained ILC can get an optimum value for vehicle queue length with a few iterative times. So the constrained ILC is better than the ILC to adjust the green ratio.

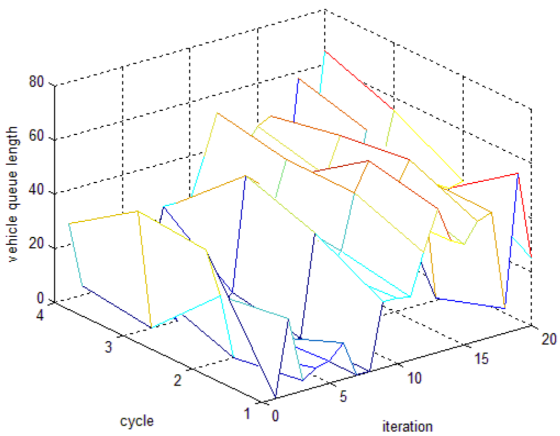

Fig.6. in the situation 3, the vehicle queue length with ILC

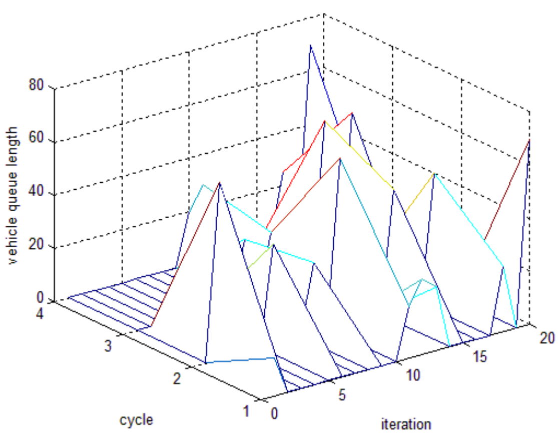

Fig.7.in the situation 3, the vehicle queue length with constrained ILC

The simulation results shown in **Fig.6** and **Fig.7** are got under the traffic load of situation 3. Because the vehicle arrival rate fluctuated greatly, so the vehicle queue length also has a great volatility. But as show from the **Fig.6** and **Fig.7**, constrained ILC have obvious effect in adjust the green ratio on each iteration, and the ILC method have a low control efficiency.

V.CONCLUSION

The control method designed in this paper has been actual test on the street of Fengle, Jiangmen, Guangdong. Under the original model of arterial coordination control, running the constrained ILC method on the traffic signal control system for the situation of morning and evening peak traffic load. The method is run and tested over half year, it can optimize the green ratio in five intersections of the test main road, make the vehicle queue length is minimal.

VI. REFERENCES

[1] Yan Fei, Tian Fuli, Shi Zhongke, "Iterative learning control strategy for traffic signal of urban area", Control and Decision,vol.8, pp.1411-1416, Aug 2015.

[2] Shangtai Jin, Zhongsheng Hou, Ronghu Chi, Danwei Wang, Xuhui Bu,"Iterative learning control based phase splits strategy for oversaturated urban traffic network",Control Conference (CCC),pp.3142-3146, July 2016.

[3] Wei Huang, Francesco Viti & Chris M.J. Tampère,"An iterative learning approach for anticipatory traffic signal control on urban networks", Transportmetrica B,vol.5,no.4,pp.402-425,Sep 2016.

[4] Yan Fei, Tian Fuli, Shi Zhongke, "Robust Iterative Learning Control for Signals at Urban Road Intersections", China Journal of Highway and Transport, vol.29,no.1,pp.120-127, January ,2016.

[5] Jianxin Xu; Xu Jin,"State-Constrained Iterative Learning Control for a ClassOf MIMO Systems",IEEE Transactions on Automatic Control,vol.58, no.5, pp.1322-1327, May 2013.

[6] Xu Jin, Jianxin Xu, "Iterative learning control for output-constrained systems with both parametric and nonparametric uncertainties", Automatica, vol.49, no.8, pp.2508-2516, August 2013.

[7] QZ Yan, MX Sun,"Error-tracking iterative learning control with state constrained for nonparametric uncertain systems",Control Theory & Applications,vol.32, no.7, pp.895-901, Jul 2015.

[8] Hyeong-JunChang, Gwi-TaePark, "A study on traffic signal control at signalized intersections in vehicular ad hoc networks", Ad Hoc Networks, vol.11, no.7, pp. 2115-2124,September 2013.

[9] Ronghu Chi, Xiaohe Liu, Ruikun Zhang, Zhongsheng Hou,Biao Huang,"Constrained data-driven optimal iterative learning control", Journal, vol.55,Pages 10-29, July 2017.

[10] CT Freeman , Y Tan, "Iterative Learning Control With Mixed Constraints for Point-to-Point Tracking", Control Systems Technology IEEE Transactions on,vol.21, no.3, pp.604-616,May 2013.

Taiyuan Ruan was born on Oct.1, 1982, and graduated from the Major of Electric Automation, Wuyi University, Jiangmen, China, in 2004. Currently, He is an Electrical & Automation Engineer and also worked as a teacher in Jiangmen Polytechnic. His research interests focus on ITS, intelligent control system. E-mail:wdl8898@yeah.com

Hao Zhou was born on oct.28,1990. He received the M.Sc. degree in traffic information engineering and control from Wuyi University of China. Currently, he is a teacher at Jiangmen Polytechnic. His major research interests include ITS and intelligent control system. E-mail: wuyizhouhao1990@163.com

Zhiyong Liu was born on 1961.He received his B.Sc., M.Sc., Ph.D. degrees from Xian Jiaotong University in 1983,1986 and 1989. He was visited the Control Engineering Center of London Metropolitan University from Sep,1987 to Feb,1989. His Major research interests include large scale system theory, intelligent control system and ITS. E-mail: zliu@wyu.eclu.cn