# Face classification based on PCA by using the center and foci of an ellipse

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***Abstract*****—** We introduce an improved method for face classification based on the nearest and the center of ellipse (NCE). The NCE is a method for face classification by using an ellipse. Before face classification, we use two-dimensional principal component analysis (PCA) for feature extraction, then a three points from the same class produce an ellipse together with inside three points, the foci and the center.

The classification rules are as follows: 1) if the tested image is inside the ellipse, we calculate the average distance between the tested image and each of the inside three points; 2) otherwise, we find the minimum distance. The distance between the tested image and each of the three points which produce the ellipse. From both cases, we obtain the distance corresponding to the ellipse. Then we conclude that the tested image is in the same class with the ellipse which gives the minimum distance. A large number of experiments were investigated on the Faces94 and the Grimace database. Meanwhile, we compare our method with the shortest feature line segment (SFLS), the nearest and the center of ellipse (NCE). The proposed algorithm shows high performance and it has the average recognition rate over 96.88 %.

***Keywords*****—**face classification, ellipse, focus of ellipse, center of ellipse.

## I. Introduction

A face recognition system can be divided into three steps: face detection, feature extraction and face recognition. The first step is detecting the existence of a face image also called face detection. Second, if there are the faces in an image, we proceed the process of extracting faces from backgrounds. So, the system identifies a certain image region as a face. Finally, identification individuals by comparing the tested face image against a database of known faces also called the face recognition.

Sirovich and Kirby [1] presented a method for the representation of face images which use eigenfaces, derived from the covariance matrix of face images, for the recognition. Turk and Pentland [2] proposed principal component analysis (PCA) which is a method to the detection and identification of faces. Face images are projected onto a feature space that best encodes the variation among know face images. Belhumeur, Hespanha and Kriegman [3] proposed the face recognition algorithm which is insensitive to large variation in lighting direction and facial expression. Taking a pattern classification approach, they considered each pixel in an image as a coordinate in a high-dimensional space. Karande and Talbar [4] presented a problem of face recognition under variation of illumination and poses with large rotation angles using the Independent Component Analysis (ICA) and PCA before applying the ICA. Face recognition using ICA which extracting the most relevant information contained in that face. The independent components obtained by the ICA algorithm are used as feature vectors for classification. The Euclidian distance classifier is used for testing of images. Zhang, Zhou and Chen [5] proposed a novel subspace method which is called diagonal principal component analysis (DiaPCA). DiaPCA directly attempted to find the optimal projective vector from diagonal face images without image to vector transformation, DiaPCA is much more accurate than PCA.

In this paper, we focus on the last step of the face recognition system that is face recognition. There are several methods for face classification that involve geometry shape. Normally, the face classification is done by checking the minimum distance between an input image and the training images. The distance is calculated form various methods. An example algorithms for face classification are nearest neighbour (NN) [6], the nearest feature line (NFL) [7], the nearest feature centre (NFC) [8], extended nearest feature line (ENFL) [9], the shortest feature line segment (SFLS) [10], the restricted nearest feature line with ellipse RNFLE [11], the nearest feature midpoint (NFM) [12], The nearest and the center of ellipse (NCE) [13] and others methods [15-17].

Some face classifications in this paper are follow: Han, Han and Yang investigated the shortest feature line segment (SFLS): the method used a circle and tried to find the shortest feature line segment. Ieosanurak and Klongdee present the nearest and the center of ellipse (NCE) for classification, this method used the center of ellipse to find the distance between the tested image and the center. The ellipse is created from 3 points of the same class, then class that the minimum of the distance is class of the tested image.

In this paper, we proposed a novel modified the nearest and the center of ellipse (NCE). Instead of calculating the distance between the tested image the center of an ellipse, the proposed method attempts to find the average distance between the tested image and each point in the three points of the foci and the center of the ellipse.

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## II. LITERATURE REVIEW

### *A. Principal Component Analysis (PCA)*

The principle component analysis approach was described by Turk and Pentland [12] in 1991. The training database contains M images which are represented as the same size of matrix. Each image matrix is normalized by converting to the equivalent image vector (column matrix) $z_i$. The training matrix $X$ contains the image vectors as

$$X = [z_1 \;\; z_2 \;\; \ldots \; z_M].$$

The process of PCA as show in in Fig 1.is described as follows:

Step 1. Set $z_i^c$ be the image vector of $i^{th}$ image class $c$.
Step 2. Calculate the mean face image which is defined by

$$\overline{z} = \frac{1}{M}\sum_{i=1}^{M} z_i.$$

Step 3. Calculate the covariance matrix C of the training image matrix by

$$C = \frac{1}{M} AA^T,$$

where

$$A = \; [\,(z_1 - \bar{z}) \;\; (z_2 - \bar{z}) \ldots \; (x_M - \bar{z})\,].$$

Step 4. Since the matrix $C$ is high dimension, the eigenvectors of $C$ are considered by the matrix $L = \frac{1}{M} A^T A$ of size M× M (if λ is eigenvalue of $L$, then λ is also eigenvalue of $C$).
Let $V_i$ be the eigenvector of matrix $L$ corresponding to the eigenvalue $\lambda_i$ where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$. Thus,

$$U_i = AV_i,$$

is eigenvector of $C$ corresponding to the eigenvalue $\lambda_i$. The eigenfaces is defined by $U = [U_1 \;\; U_2 \;\; U_3 \ldots \; U_M]$.

Step 5. (Return to Vector Conversion) The weight of each eigenvector $z_i$ represents the image in the eigenface space as given by

$$z_i = U^T(x_i - \overline{x}),$$

where U is the eigenfaces.

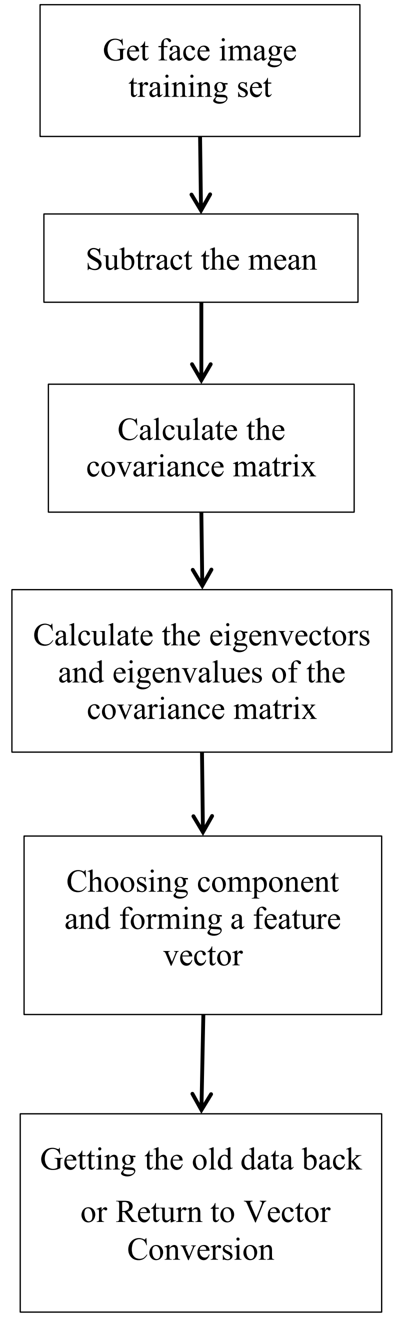


Fig. 1 Step of PCA

### *B. The nearest feature line (NFL)*

$\overline{x_i^c x_j^c}$ represents the line which is passing through $x_i^c$ and $x_j^c$ called a feature line (FL) of the class $C_{\mathrm{NFL}}$, as shown in Fig. 2. Define $x_p$ as the projection point of $x$ which can be calculated by

$$x_p = x_i^c + t\left(x_j^c - x_i^c\right),$$

where $t = \frac{(x_p - x_i^c)^T (x_j^c - x_i^c)}{(x_j^c - x_i^c)^T (x_j^c - x_i^c)}$.

The distance between the test image $x$ and the feature line $\overline{x_i^c x_j^c}$ can be calculated by

$$d(x, \overline{x_i^c x_j^c}) = \left\| x - x_p \right\|_2,$$

where $\|\cdot\|_2$ is the Euclidean distance.

The classification decision called the nearest feature distance can be defined as follow:

$$C_{\mathrm{NFL}} = \arg\min_c \left\{ \min_{1 \le i < j \le m} d(x, \overline{x_j^c x_i^c}) \right\}$$

for $c = 1,2,3, \ldots, N_c$ and $C_{\mathrm{NFL}}$ is the class of the tested image.

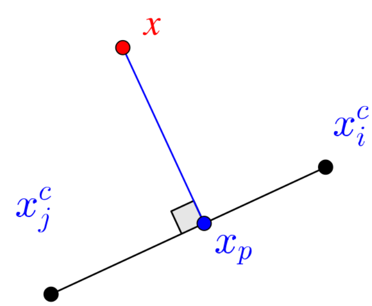


Fig. 2 The nearest feature line

### C. The shortest feature line segment (SFLS)

This method produces the shortest feature line segment which is classified by the tested image inside a circle, created by two training points $x_i^c$ and $x_j^c$ of the class $C_{\mathrm{SFLS}}$ as shown in Fig. 3. The distance metric of SFLS is calculated by

$$d(x, x_i^c x_j^c) = \left\| x_i^c - x_j^c \right\|.$$

The classification decision can be defined as follow:

$$C_{\mathrm{SFLS}} = \arg\min_c \left\{ \min_{i,j} d(x, x_i^c x_j^c) \right\},$$

for $c = 1,2,3, \ldots, N_c$, where $C_{\mathrm{SFLS}}$ is the class of the tested image.

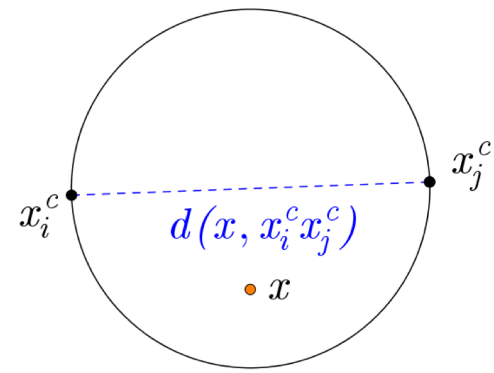


Fig. 3 The metric of the shortest feature line segment

### D. The restricted nearest feature line with ellipse (RNFLE)

The main idea of this method, which uses ellipse to restrict the feature line. Define $x_i^c, x_j^c$ as foci of any ellipse like Fig. 4, and $a_0$ as the ratio between the length of ellipse major axis and the length of the center to either focus.

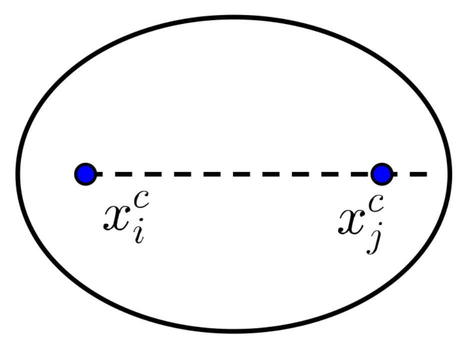


Fig. 4 $x_i^c, x_j^c$ be foci of the ellipse

The classification decision can be defined as follows:

1. If the test $(x)$ is inside the ellipse which is shown in Fig. 5 , the distance between the tested image and the feature line $\overline{z_i^c z_j^c}$ is as follow:

$$\mathrm{d}(\mathrm{x}, \overline{\mathrm{x_i^c x_j^c}}) = \left\| \mathrm{x} - \mathrm{x_{ijp}^c} \right\|,$$

where $\mathrm{x_{ijp}^c}$ represents the projection of x on the feature line $\overline{\mathrm{x_i^c x_j^c}}$.

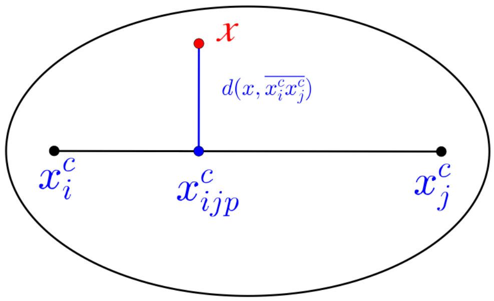


Fig. 5 The tested image inside the ellipse

2. If the test (x) is outside the ellipse which is shown in Fig. 6, the distance between the test image and the feature line is as follow:

$$d(x, \overline{x_i^c x_j^c}) = \min\{||x - x_i^c||, ||x - x_j^c||\},$$

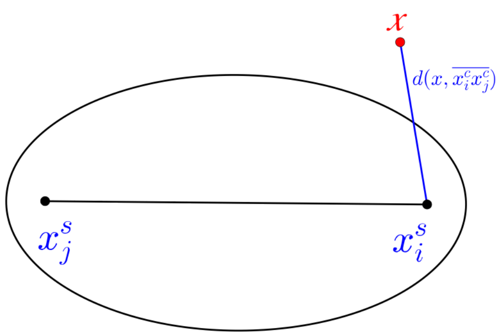


Fig. 6 The tested image is outside the ellipse

The test image is classified into class $C_{RNFLE}$

$$C_{RNFLE} = \arg\min_{c}\left\{\min_{1\le i<j\le m} d(x, x_i^c x_j^c)\right\},$$

where $c = 1,2,3, \dots, N_c$.

*E. The nearest and the center of ellipse (NCE)*

This method uses the center of ellipse to find the distance between the tested image and the center.

The ellipse is produced by the three points, $z_i^c, z_j^c$ and $z_l^c$, which are obtained from the combinations of m distinct points taken from the same class. Without loss of generality, set

$$d(z_i^c, z_j^c) = \operatorname{diam}(\{z_i^c, z_j^c, z_l^c\}),$$

and

$$\operatorname{diam}(A) = \max\{d(u,v) | u, v \in A\}.$$

The center $(h, k)$ is the midpoint of the $z_i^c$ and $z_j^c$,

$$a_{c_{ijl}} = \frac{1}{2} d(z_i^c, z_j^c)$$

and

$$b_{c_{ijl}} = \sqrt{\frac{(-(x_l - h)\sin\theta + (y_l - k)\cos\theta)^2}{1 - \frac{\left((x_l - h)\cos\theta + (y_j - k)\sin\theta\right)^2}{a_{c_{ijl}}^2}}},$$

where $z_l = (x_l, y_l)$, $d(u,v) = ||u - v||_2$, and $||\ \ ||_2$ is Euclidean distance.

The classification rule can be defined as follows:
If the tested image (z) is inside an ellipse which is shown in Fig. 7 and $z_c = [h\ k]^T$ is the center of the ellipse, the distance between the tested image and the center of the $c_{ijk}$ ellipse is as follows:

$$d(z, \overline{z_i^c z_j^c z_k^c}) = ||z - z_c||_2.$$

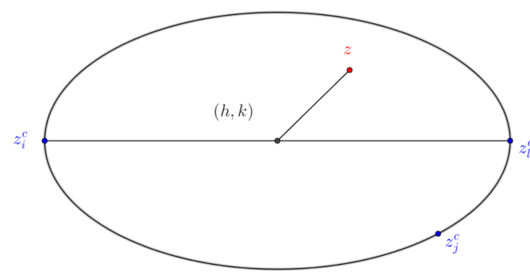


Fig. 7 The tested image inside the ellipse

If the tested image (z) is outside any ellipse which is shown in Fig. 8, or three points lie on the same line, the distance between the tested image and each point in the three points of the ellipse is as follows:

$$d(z, \overline{z_i^c z_j^c z_k^c}) = \min\{||z - z_i^c||_2, ||z - z_j^c||_2, ||z - z_k^c||_2\}.$$

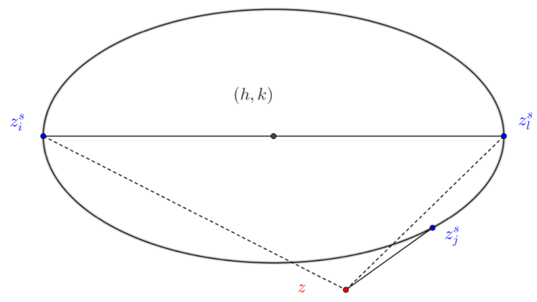


Fig. 8 The tested image outside the ellipse

The tested image is classified into the class $C_{NCE}$,

$$C_{NCE} = \arg\min_{c}\left\{\min_{1\le i<j<k\le m} d\left(z, \overline{z_i^c z_j^c z_k^c}\right) \mid c = 1,2,3,\ldots,N_c\right\}.$$

## III. The Proposed Algorithm

In this section, we improve a classification based on NCE. Firstly, we use the principal component analysis (PCA) [17] to find a subset of the principle component in a set of training faces. Then we project the faces into the principal components space which can be gathered the feature vectors ($z_i$). Finally, we use the classification rule for face classification as follows:

*The center and foci of an ellipse (CFE)*

For this algorithm, each class needs at least three points. The $z_i^c, z_j^c$ and $z_l^c$ are the feature vectors from PCA, which are also the three training points from the same class $c$, $1 \le i < j < l < m, 1 \le c \le N_c$: $N_c$ and $m$ are the number of the class and the number of images per class, respectively.

The propose algorithm or CFE based on PCA can be elucidated as follows:

Step 1: Input the training image set and read the tested image.

Step 2: We use principal component analysis (PCA) to find a subset of the principle component in a set of training faces; then we project faces into the principal components space which can be gathered the feature vectors $z_i^c$.

Step 3: Produce an ellipse from the three points and the same class. Without loss of generality, set

$$\mathrm{d}(\mathrm{z}_\mathrm{i}^\mathrm{c}, \mathrm{z}_\mathrm{j}^\mathrm{c}) = \mathrm{diam}(\{z_i^c, z_j^c, z_l^c\}) \tag{1}$$

and

$$\mathrm{diam}(A) = \max\{d(u,v) \mid u,v \in A\}. \tag{2}$$

The point $(h,k)$ is the center of the $z_i^c$ and $z_j^c$,

$$a_{c_{ijl}} = \frac{1}{2} d(z_i^c, z_j^c) \tag{3}$$

and

$$b_{c_{ijl}} = \sqrt{\frac{(-(x_l-h)\sin\theta + (y_l-k)\cos\theta)^2}{1-\frac{\left((x_l-h)\cos\theta+(y_j-k)\sin\theta\right)^2}{a_{c_{ijl}}^2}}},$$

where $z_l = (x_l, y_l)$, $d(u,v) = \|u-v\|_2$, and $\| \ \|_2$ is Euclidean distance.

The foci of the ellipse are calculated by

$$F_1 = \begin{bmatrix} h \\ k \end{bmatrix} - C\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix},$$

$$F_2 = \begin{bmatrix} h \\ k \end{bmatrix} + C\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, \tag{4}$$

where $C = \sqrt{a_{c_{ijl}}^2 - b_{c_{ijl}}^2}$ and $\theta = \arctan\left(\frac{y_i - y_j}{x_i - x_j}\right)$.

Step 4: Let $z$ be the tested image and $Z_c$ be the center of the ellipse. Without loss of generality, let $a_{c_{ijl}} > b_{c_{ijl}}$ and

$$\mathrm{d}(\mathrm{z}, \text{ellipse ijl}) = d(z, F_1) + d(z, F_2). \tag{5}$$

We define the classification rule as follows:

1). If the tested image is inside the ellipse, that is $\mathrm{d}(\mathrm{z}, \text{ellipse ijl}) \le 2a_{c_{ijl}}$, (as shown in figure 8), the distance between the tested image and the center of the ellipse is as follows:

$$D(z, ellipse\ ijl) = \frac{d(z,F_1) + d(z,F_2) + d(z,Z_c)}{3}. \tag{6}$$

2). If the tested image is outside the ellipse, that is $\mathrm{d}(\mathrm{z}, \text{ellipse ijl}) > 2a_{c_{ijl}}$, (as shown in figure 9), or three points lie on the same line, the distance between the tested image and each point in the three points of the ellipse is as follows:

$$D(z, ellipse\ ijl) = \min\{d(z, z_i^c), d(z, z_j^c), d(z, z_l^c)\}. \tag{7}$$

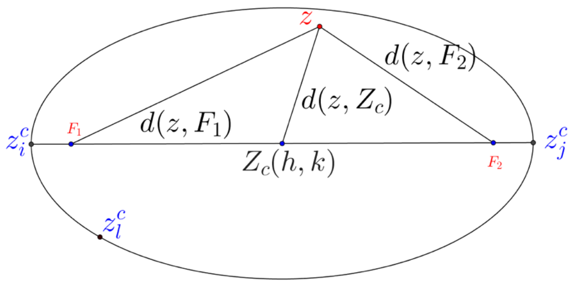


Fig.9 Finding the distance as (6).

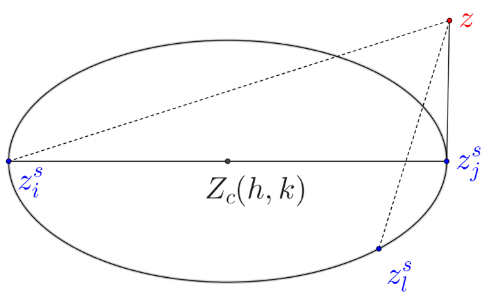


Fig.10 Finding the distance as (7).

The tested image is classified into class $C_{CFE}$

$$C_{\text{CFE}} = \arg\min_{c} \left\{ \min_{1 \le i < j \le l \le m} D(z, ellipse\ ijl) \right\}, \quad (8)$$

where $C$ is the class of the training image.

The recognition rate is calculated by

$$\text{The recognition rate} = \frac{N_{cr}}{N_{test}} \times 100\% \quad (9\ )$$

where $N_{cr}$ the is number of correct recognition of the tested face images and $N_{test}$ is the total number of tested images.

## IV. Experimental results

In this section, we present the experimental results of the SFLS, NCE, and the proposed algorithm (CFE). The databases are used by Faces94 (female and male staff) and Grimace as in [18]. The training image and tested image are transformed as column vector and we use PCA for feature extraction. Each database is divided into two sets, the training set and the set of tested image. The training set is divided into 95%, 90%, 85%, …, 60% proportion and the remaining is the set of tested set. The recognition rate is shown in Table 1, 2, and 3.

Table 1 shows the recognition rate of SFLS, NCE, and CFE. The recognition rate of CFE is better than others except 0.60 proportion, NCE is better than the others.

Table 2 shows the recognition rate of SFLS, NCE, and CFE. The recognition rate of SFLS is better than the others except 0.85,0.75,0.70 and 0.65 proportions, NCE and CFE are better than the SFLS.

Table 3 shows the recognition rate of SFLS, NCE and CFE. The recognition rate of CFE algorithm is better than the others except 0.70 proportion, NCE is better than the others.

Table 1. The recognition rate of Faces94 (female) database with various algorithms

| **Faces94 (female)** | **The recognition rate (%)** | | |
|---|---|---|---|
| **Proportion** | **SFLS** | **NCE** | ***CFE*** |
| 0.95 | 94.74 | 94.74 | 94.74 |
| 0.90 | 92.11 | 97.37 | 97.37 |
| 0.85 | 92.98 | 94.74 | 94.74 |
| 0.80 | 94.74 | 94.74 | 96.05 |
| 0.75 | 93.68 | 95.79 | 95.79 |
| 0.70 | 93.86 | 94.74 | 94.74 |
| 0.65 | 93.23 | 92.48 | 94.74 |
| 0.60 | 92.76 | 95.39 | 94.74 |

## V. Conclusions

In this paper, we introduce an improved algorithm for face recognition system based on PCA by using the center and foci of an ellipse which is produced by three points of the same class, then the tested image is in class $C_{CFE}$ when the average distance between the tested image and each point in the three points of the foci and the center of the ellipse of class $C_{CFE}$ is minimum distance. The proposed method (CFE) tests on the faces94 and Grimace databases. The recognition rate is compared with the shortest feature line segment (SFLS), and the nearest and the center of ellipse (NCE). It is evident that the recognition rate of CFE is better than the SFLS and NCE. Moreover, CFE shows high performance and it has average recognition rate over 96.88%, NCE has average recognition rate over 96.61% and the SFLS has average recognition rate over 96.21%.

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Table 2. The recognition rate of Faces94 (male staff) database with various algorithms

| **Faces94 (male staff)** | **The recognition rate (%)** | | |
|---|---|---|---|
| **Proportion** | **SFLS** | **NCE** | ***CFE*** |
| 0.95 | 95.00 | 100 | 100 |
| 0.90 | 97.50 | 97.50 | 97.50 |
| 0.85 | 98.33 | 96.67 | 96.67 |
| 0.80 | 98.75 | 98.75 | 98.75 |
| 0.75 | 99.00 | 97.00 | 98.00 |
| 0.70 | 95.00 | 97.50 | 96.67 |
| 0.65 | 94.29 | 95.71 | 95.00 |
| 0.60 | 95.00 | 97.50 | 97.50 |

Table 3. The recognition rate of Grimace database with various algorithms

| **Grimace** | **The recognition rate** | | |
|---|---|---|---|
| **Proportion** | **SFLS** | **NCE** | ***CFE*** |
| 0.95 | 94.44 | 94.44 | 94.44 |
| 0.90 | 94.44 | 97.22 | 97.22 |
| 0.85 | 92.59 | 94.44 | 96.3 |
| 0.80 | 98.61 | 100 | 100 |
| 0.75 | 96.67 | 98.89 | 98.89 |
| 0.70 | 96.3 | 99.07 | 98.15 |
| 0.65 | 95.24 | 96.03 | 98.41 |
| 0.60 | 95.83 | 97.92 | 98.61 |

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