Design of Fractional Order PID Controller for Induction Motor Speed Control System by Cuckoo Search

C. Thammarat and D. Puangdownreong

***Abstract*—**The non-integer (fractional) order PID (FOPID or $PI^{\lambda}D^{\mu}$) controller was introduced almost two decades and demonstrated to perform the better responses in comparison with the conventional integer order PID (IOPID). The design of an optimal FOPID controller for induction motor speed control system by the cuckoo search (CS), one of the most efficient metaheuristic optimization search techniques, is presented in this paper. Based on the modern optimization framework, five parameters of the FOPID controller are optimized by the CS to meet the response specifications of the three-phase induction motor (3ϕ-IM) speed control system defined as particularly constraint functions. Results obtained by the FOPID controller are compared with those obtained by the IOPID designed by the CS. As simulation results, the FOPID can provide superior speed responses to the IOPID, significantly.

***Keywords*—**Fractional order PID controller, Cuckoo Search, metaheuristic optimization.

I. Introduction

The fractional order PID (FOPID) controller (or $PI^{\lambda}D^{\mu}$) was firstly proposed by Podlubny in 1999 [1], as an extended version of the conventional integer order PID (IOPID). Based on the fractional calculus, the FOPID controller is characterized by five parameters: proportional gain (K_p), integration gain (K_i), derivative gain (K_d), integration order (λ) and derivative order (μ). Once the FOPID is compared with the IOPID, there are two extra parameters λ and μ making the FOPID controller more efficient, but more complicate than the IOPID in design and implementation procedures. By literatures, the FOPID has been successfully conducted in many applications, for instance, process control [2], automatic voltage regulator (AVR) [3], DC motor control [4], power electronic control [5], inverted pendulum control [6] and gun control system [7]. Several design and tuning methods for FOPID have been consecutively launched, for example, rule-based methods [8],[9] and analytical methods [10],[11]. Review and tutorial articles of the FOPID controller providing the state-of-the-art and its backgrounds have been completely reported [12],[13].

Recently, control synthesis has been changed from the conventional paradigm to the new framework based on modern optimization using metaheuristics as an optimizer [14],[15]. The cuckoo search (CS), firstly proposed by Yang and Deb in 2009, is one of the most powerful population-based metaheuristic optimization search techniques [16]. The CS was proved for the global convergent property [17] and successfully applied to many real-world engineering problems, such as wind turbine blades [18], antenna arrays [19], power systems [20], travelling salesman problem [21], structural optimization problem [22], wireless sensor network [23], flow shop scheduling problem [24], job shop scheduling problem [25] and control systems [26]. The state-of-the-art and its applications of the CS have been reviewed and reported [27]. In this paper, the CS is applied to optimally design the FOPID controller for the 3ϕ-IM speed control system. The rest of the paper is outlined as follows: fractional calculus and fractional order PID (FOPID) controller are briefly described in section II. Problem formulation of the CS-based FOPID controller design for the 3ϕ-IM speed control system is performed in section III. Results and discussions are illustrated in section IV, while conclusions are summarized in section V.

II. Fractional Calculus and FOPID Controller

A. Fractional Calculus

In fractional calculus, a generalization of integration and differentiation can be represented by the non-integer order fundamental operator ${}_aD_t^{\alpha}$, where a and t are the limits of the operator. The continuous integro-differential operator is defined as expressed in (1), where $\alpha \in \Re$ stands for the order of operation.

There are three definitions used for the generally fractional differintegral. The first definition is Grunwald-Letnikov (GL) as stated in (2), where $[\cdot]$ is integer part and n is an integer satisfying the condition $n-1<\alpha<n$. The binomial coefficient is stated in (3), while the Euler's gamma function $\Gamma(\cdot)$ is defined by (4).

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$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & \Re(\alpha) > 0 \\ 1 & \Re(\alpha) = 0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & \Re(\alpha) < 0 \end{cases} \tag{1} $$

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{r=0}^{\left[\frac{t-a}{h}\right]} (-1)^r \binom{n}{r} f(t-rh) \tag{2} $$

$$ \binom{n}{r} = \frac{\Gamma(n+1)}{\Gamma(r+1)\Gamma(n-r+1)} \tag{3} $$

$$ \Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \tag{4} $$

The second definition is Riemann-Liouville (RL) as expressed in (5), for $n-1<\alpha<n$. The third definition is Caputo definition as shown in (6), where n is an integer and $n-1<\alpha<n$. Among those, the Caputo definition is most popular in engineering applications [12].

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{5} $$

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{n}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{6} $$

$$ \begin{aligned} \mathcal{L}\{{}_aD_t^{\alpha} f(t)\} &= \int_0^{\infty} e^{-st} {}_0D_t^{\alpha} f(t) dt \\ &= s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{\alpha-k-1} f(t)\Big|_{t=0} \end{aligned} \tag{7} $$

For solving engineering problems, the Laplace transform is routinely conducted. The formula of the Laplace transform of the RL fractional derivative in (5) is stated in (7), for $n-1<\alpha\le n$, where $s \equiv j\omega$ denotes the Laplace transform (complex) variable. Under zero initial conditions for order α ($0 < \alpha < 1$), the Laplace transform of the RL fractional derivative in (5) can be expressed in (8).

$$ \mathcal{L}\{{}_aD_t^{\pm\alpha} f(t)\} = s^{\pm\alpha} F(s) \tag{8} $$

B. FOPID Controller

The fractional order PID controller (FOPID or $PI^{\lambda}D^{\mu}$) is an extended version of the conventional integer order PID (IOPID). The generalized transfer function of the FOPID is given by the differential equation as stated in (9), where $u(t)$ is the control signal, $e(t)$ is the error signal and λ and $\mu \ge 0$, and by the Laplace transform as expressed in (10).

$$ u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^{\mu} e(t) \tag{9} $$

$$ G_c(s)\big|_{FOPID} = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{10} $$

Relationship between the IOPID and the FOPID can be represented by a graphical way as visualized in Fig. 1. In general, the range of fractional orders (λ and μ) is varied from 0 to 2. However, in most research works, the range of λ and μ is varied from 0 to 1. Referring to Fig. 1, if $\lambda = \mu = 1$, it is the IOPID controller.

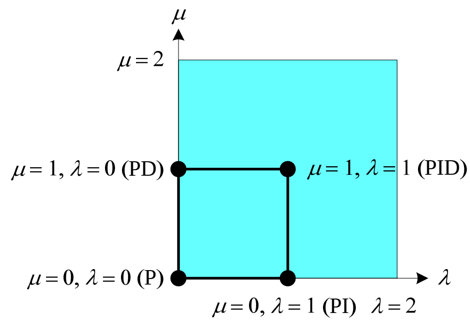

Fig. 1 relationship between IOPID and FOPID controllers

III. Problem Formulation

In this section, the FOPID control loop, the 3ϕ-IM model and the CS-based FOPID controller design are consecutively presented as follows.

A. FOPID Control Loop

The FOPID control loop is represented by the block diagram as shown in Fig. 2, where $G_p(s)$ and $G_c(s)$ are the plant and the FOPID controller models, respectively. The FOPID receives the error signal $E(s)$ and produces the control signal $U(s)$ to control the output signal $C(s)$ and regulate the disturbance signal $D(s)$, referring to the reference input $R(s)$.

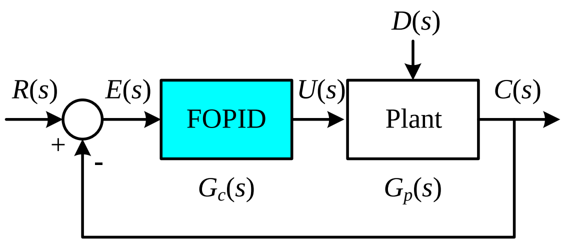

Fig. 2 FOPID control loop

B. Induction Motor Model

In this work, a 0.37 kW, 1400 rpm, 50 Hz, 4-pole 3ϕ-IM was conducted. Such the motor was tested as shown in Fig. 3 to record its speed dynamics. By using MATLAB and system identification toolbox [28], the third-order transfer function was identified as given in (11). Good agreement between the model plot and the experimental speed (sensory data) can be observed in Fig. 4. The plant model in (11) will be used as the plant $G_p(s)$ in Fig. 2.

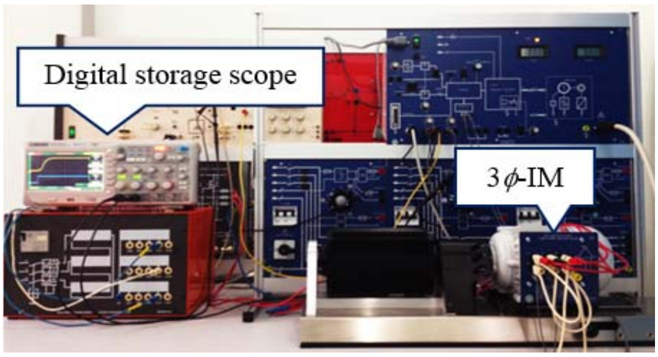

Fig. 3 3ϕ-IM testing rig

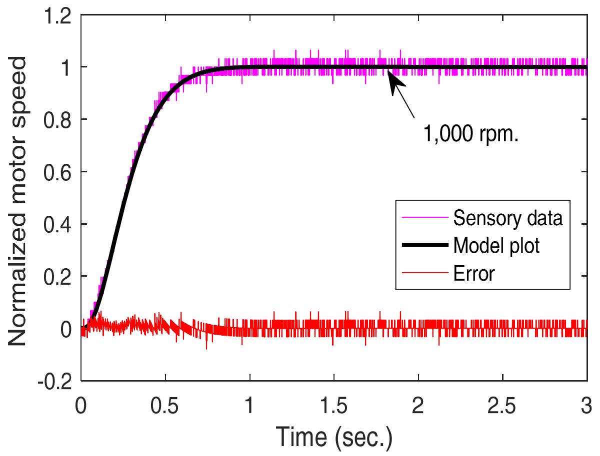

Fig. 4 model plot against sensory data

$$G_p(s) = \frac{1{,}360}{s^3 + 39.16s^2 + 398.14s + 1{,}360} \tag{11}$$

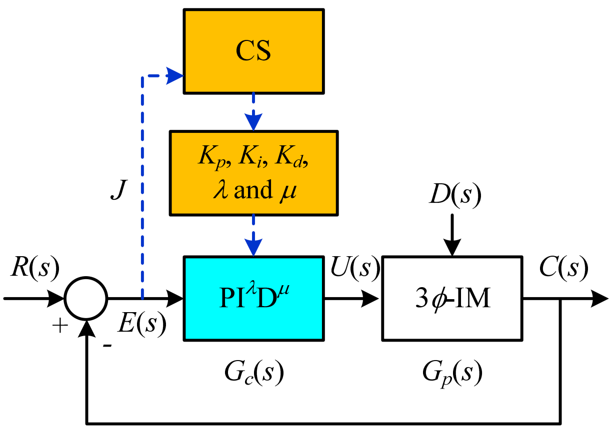

Fig. 5 CS-based FOPID design framework

C. CS-Based FOPID Controller Design

Based on modern optimization framework, application of the CS to design optimal FOPID controller for 3ϕ-IM speed control system can be represented by the block diagram in Fig. 5. The CS algorithms [16] mimic the behaviour of cuckoo species and the Lévy flight behaviour of some birds and fruit flies. Two essential parameters of the CS are a number of cuckoos (n) and a fraction p_a denoting the ability of host birds that can find the cuckoos' eggs. Referring to Fig. 5, the objective function J, sum-squared error between the reference speed $R(s)$ and the actual speed $C(s)$ in (12), will be fed to the CS to be minimized by searching for the appropriate values of the FOPID parameters, i.e. K_p, K_i, K_d, λ and μ subject to inequality constraint functions satisfying to the predefined response specifications as stated in (13).

$$\textbf{Min} \;\; J(\boldsymbol{x}) = \sum_{i=1}^{N}[r_i - c_i]^2 \tag{12}$$

$$\textbf{Subject to} \quad g_j(\boldsymbol{x}) \le 0, \quad j = 1,2,\ldots \tag{13}$$

In CS algorithms, new solutions $\boldsymbol{x}^{(t+1)}$ for cuckoo i can be generated by Lévy distribution as expressed in (14), where a symbol Lévy(λ) represents the Lévy distribution having an infinite variance with an infinite mean as expressed in (15). The step length s of cuckoo flight can be calculated by (16), where u and v stand for normal distribution as stated in (17). Standard deviations of u and v are also expressed in (18). The CS algorithm is applied for the FOPID design application as follows:

$$\boldsymbol{x}_i^{(t+1)} = \boldsymbol{x}_i^{(t)} + \alpha \oplus \text{Lévy}(\lambda) \tag{14}$$

$$\text{Lévy} \approx u = t^{-\lambda}, \quad (1 < \lambda \le 3) \tag{15}$$

$$s = \frac{u}{|v|^{1/\beta}} \tag{16}$$

$$u \approx N(0.\sigma_u^2), \quad v \approx N(0.\sigma_v^2) \tag{17}$$

$$\sigma_u = \sqrt[\beta]{\frac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\Gamma[(1+\beta)/2]\beta 2^{(\beta-1)/2}}}, \quad \sigma_v = 1 \tag{18}$$

Step-1 Perform objective function $J(\boldsymbol{x})$, $\boldsymbol{x} = (K_p, K_i, K_d, \lambda, \mu)^T$ in (12), initial number of cuckoos (n), fraction p_a and search spaces, randomly generate $\boldsymbol{x}$ as initial solutions and set Max_Gen and Gen = 1.

Step-2 If Gen ≤ Max_Gen, n cuckoo find the new nests by Levy flight and lay their eggs in the random nests according to (14) – (18) to create new solutions $\boldsymbol{x}^*$. Otherwise, go to Step-7.

Step-3 If $p_a \le$ rand, m (m <= n) cuckoo's egg is found by host birds. m cuckoo find the new nests by Levy flight again and lay their eggs in the random nests according to (14) – (18) to create new solutions $\boldsymbol{x}^*$.

Step-4 Evaluate all cuckoos' eggs via $J(\boldsymbol{x})$.

Step-5 If $J(\boldsymbol{x}) < J(\boldsymbol{x}^*)$ in (12) and satisfy to (13), update $\boldsymbol{x} = \boldsymbol{x}^*$.

Step-6 Update Gen = Gen+1, and go back to Step-2.

Step-7 Terminate the search process, and report best solution $\boldsymbol{x}$ found.

IV. Results and Discussions

To optimal FOPID design for the 3ϕ-IM speed control system, the CS algorithms were coded by MATLAB version 2017b (License No.#40637337) run on Intel(R) Core(TM) i5-3470 CPU@3.60GHz, 4.0GB-RAM. The FOPID is implemented by MATLAB with FOMCON toolbox [29],[30] where Oustaloup's approximation is realized for fractional order numerical simulation. Number of cuckoos n = 40 and fraction p_a = 0.2 are set according to recommendations of Yang and Deb [16],[17]. Search spaces and constraint functions in (13) are then performed as stated in (19), where t_r is rise time, M_p is percent maximum overshoot, t_s is settling time and E_{ss} is steady-state error, respectively. The maximum generation Max_Gen = 100 is then set as the termination criteria (TC). 50 trials are conducted to find the best solution (optimal FOPID controller for the 3ϕ-IM speed control system). For comparison with the IOPID, λ and μ in (19) will be set as one ($\lambda = \mu = 1$).

$$\textbf{Subject to}\quad \left.\begin{aligned} & t_r \le 0.2 \text{ sec.},\\ & M_p \le 10.0\ \%,\\ & E_{ss} \le 0.01\ \%,\\ & 0 < K_p \le 10,\\ & 0 < K_i \le 20,\\ & 0 < K_d \le 1.0,\\ & 0 < \mu < 1.0,\\ & 0 < \lambda < 1.0 \end{aligned}\right\} \tag{19}$$

$$G_c(s)\big|_{IOPID} = 7.007 + \frac{18.882}{s} + 0.471s \tag{20}$$

$$G_c(s)\big|_{FOPID} = 5.598 + \frac{19.975}{s^{0.911}} + 0.650\,s^{0.994} \tag{21}$$

Once the search process stopped, the CS can successfully provide the optimal parameters of the IOPID and FOPID controllers for the 3ϕ-IM speed control system as expressed in (20) and (21), respectively. The convergent rates of the objective functions in (12) associated with inequality constraint functions in (19) proceeded by the CS over 50 trials are depicted in Fig. 6. The step responses of the 3ϕ-IM speed control system without controller, with IOPID controller and with FOPID controller designed by the CS are shown in Fig. 7.

Referring to Fig. 7, the step response of the 3ϕ-IM speed control system without controller provides t_r = 0.65 sec., t_s = 0.72 sec., without M_p and E_{ss}. Once controlled by the IOPID, the step response of the 3ϕ-IM speed control system yields t_r = 0.11 sec., t_s = 0.43 sec., M_p = 9.85% and without E_{ss}. Finally, the step response of the 3ϕ-IM speed controlled system with FOPID controller provided t_r = 0.11 sec., t_s = 0.32 sec., M_p = 3.12% and without E_{ss}. This can be noticed that the FOPID controller designed by the CS can provide very satisfactory speed response of the 3ϕ-IM speed control system superior to the IOPID controller, significantly.

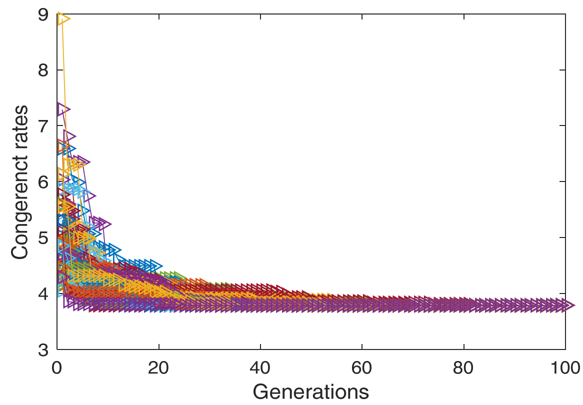

Fig. 6 convergent rates over 50 trials

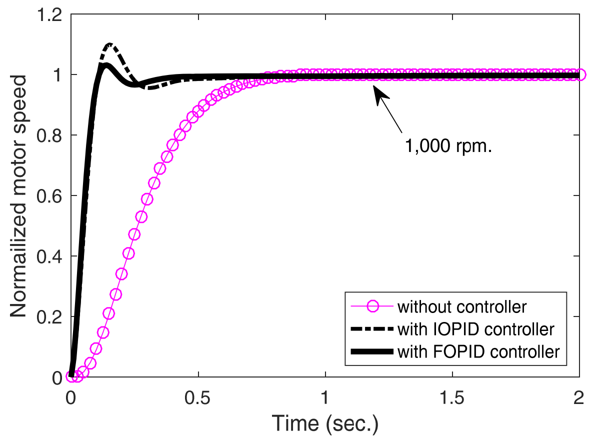

Fig. 7 step responses of 3ϕ-IM speed control

V. Conclusions

An optimal fractional order PID (FOPID or PI$^\lambda$D$^\mu$) controller design for the 3ϕ-IM speed control system by the cuckoo search (CS) has been proposed in this paper. Characterized by five parameters, the FOPID could perform the better responses once compared with the conventional integer order PID (IOPID). In this paper, fractional calculus and FOPID controller have been briefly described. The CS-based FOPID controller design framework has been formulated according to modern optimization context. By numerical simulation with MATLAB and FOMCON toolbox, five parameters of the FOPID controller have been successfully optimized by the CS meeting the predefined response specifications as inequality constraint functions. As simulation results, it was found that the FOPID designed by the CS could yield very satisfactory speed response of the 3ϕ-IM speed control system satisfying to the inequality constraint functions and superior to the IOPID controller. For the future research, the fractional order PIDA (FOPIDA) controller design by the CS will be alternatively conducted to extend the fractional controller applications.

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