The conditional connectivity and restricted connectivity of ECQ(s,t)

Chen Guo, Leng Ming, Zhifang Xiao, and Shou Peng

***Abstract*—**The exchanged crossed cube ECQ(s,t) is a novel interconnection network which has better properties than other variations of hypercube, such as crossed cube and exchanged hypercube, in terms of diameter, number of links and cost factor. In order to clarify the fault-tolerant ability of exchanged crossed cube and lay a foundation for the further study, in this paper, we study the conditional connectivity and restricted connectivity of ECQ(s,t). By exploring the topological of ECQ(s,t), we show several topological properties of ECQ(s,t). Based on these properties, we determine that the conditional connectivity and restricted connectivity of ECQ(s,t) are 2s, where t≥s>2. The research results of this paper will provide the key parameters for the reliability evaluation of ECQ(s,t) in the future. So it has important theoretical significance and application value.

***Keywords*—**connectivity, conditional connectivity, exchanged crossed cube, restricted connectivity

I. Introduction

WITH the continuous development of large-scale integration, multiprocessor systems can now consist of hundreds of processors, especially in high-performance parallel computing systems. However, the high complexity of these systems may threaten their reliability. As a result, the study of deterministic measures of reliability has become important. The notions of all kinds of connectivities have been employed to capture the reliability of a given multiprocessor system.

The traditional connectivity of Menger[1], in which some processor subsets of multiprocessor systems can potentially fail at the same time, is not an accurate measure of reliability. Traditional connectivity is an important measure of the fault-tolerance of multiprocessor interconnection networks. However, such a measure underestimates the resilience of larger multiprocessor interconnection networks, it only correctly reflects the fault-tolerance of a network with few processors [2]. To compensate for this shortcoming, Harary introduced the concept of conditional connectivity by requiring some property for disconnected components of *G-F*[3]. If the property is that each vertex has at least one neighbor not in *F*, the conditional connectivity under this property is referred to simply as conditional connectivity. Following this trend, restricted connectivity was proposed in [4,5], which assumes each processor has at least one neighbor not in *F*, even though processors in *F*.

Conditional connectivity and restricted connectivity are usually used in measure the connectivity of a large scale multiprocessor interconnection network, at the expense of small probability events. So far, the conditional connectivity and restricted connectivity of many kinds of interconnection network has been successively solved, including hypercubes[2], folded hypercube[6], crossed cubes[7], möbius cubes[8], twisted cubes[9], locally twisted cubes[9]. There are, however, still some interconnection networks are not involved such as exchanged cube and exchanged crossed cube.

It is well known, conditional connectivity and restricted connectivity represent fault tolerance of interconnection networks and are necessary prerequisites for diagnosability and conditional diagnosability studies. Therefore, The study of conditional connectivity and restricted connectivity is importance and necessary.

In this paper, we determine that the conditional connectivity and restricted connectivity of $ECQ(s,t)$ are both $2s$ where $t \geq s > 2$. In section 2, we recall some definitions and theorems of $ECQ(s,t)$. The proofs of our results are in section 3. A simulation experiments was performed in section 4. The simulation results illustrate that our conclusions are correct.

II. Definitions and theorems

Before defining the exchanged crossed cube,the notion of pair related must first be introduced. Let $T = \{(00,00), (10,10),(01,11),(11,01)\}$. Two binary strings $X = x_1x_0$ and $Y = y_1y_0$ are pair related if $(X,Y) \in T$,denoted by $X \sim Y$ [10].

Definition 1[11]. The exchanged crossed cube $ECQ(s,t)$ is defined as an undirected graph $G(V,E)$, $V = \{a_{s-1}a_{s-2}...a_0b_{t-1}b_{t-2}...b_0c \mid a_i, b_j, c \in \{0,1\}, i \in [o,s), j \in [o,t)\}$ where $s \geq 1$ and $t \geq 1$, $E = \{(u,v) \mid (u,v) \in V \times V\}$,which

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C. Guo is with School of Electronic and Information Engineering, Jinggangshan University, Jiangxi, 343009, China.
M. Leng is with School of Electronic and Information Engineering, Jinggangshan University, Jiangxi, 343009, China.(e-mail: bearr@21cn.com).
Z.F. Xiao is with School of Electronic and Information Engineering, Jinggangshan University, Jiangxi, 343009, China.
S. Peng is with School of Electronic and Information Engineering, Jinggangshan University, Jiangxi, 343009, China.

consists of three types of edges,i.e., E_1, E_2 and E_3, as described below.

E_1: $u[0] \neq v[0], u \oplus v = 1$,where $u[i]$ denotes the ith bit of vertex u and $\oplus$ is the exclusive-OR operator.

E_2: $u[0] = v[0] = 0, u[t:1] = v[t:1]$,where $u[x:y]$ denotes the bit pattern of u between dimensions x and y, inclusive. For all $s \geq 1$, if and only if there exists a positive integer l, $s+t \geq l > t$, such that $u[s+t:l] = v[s+t:l]$, $u[l-1] \neq v[l-1], u[l-2] = v[l-2]$ if $l-t$ is even,and $u[t+2i+2:t+2i+1] \sim v[t+2i+2:t+2i+1]$ for $\lfloor (l-t-1)/2 \rfloor > i \geq 0$.

E_3: $u[0] = v[0] = 1, u[s+t:t+1] = v[s+t:t+1]$. For all $t \geq 1$, if and only if there exists a positive integer l, $t \geq l \geq 1$, such that $u[t:l] = v[t:l]$, $u[l-1] \neq v[l-1]$, $u[l-2] = v[l-2]$ if l is even, and $u[2i+2:2i+1] \sim v[2i+2:2i+1]$ for $\lfloor (l-1)/2 \rfloor > i \geq 0$.

Fig. 1 shows an illustration of $ECQ(s,t)$ with $s = 2$ and $t = 2$, where the dashed links,solid heavy links and solid thin links correspond to E_1, E_2 and E_3, respectively.

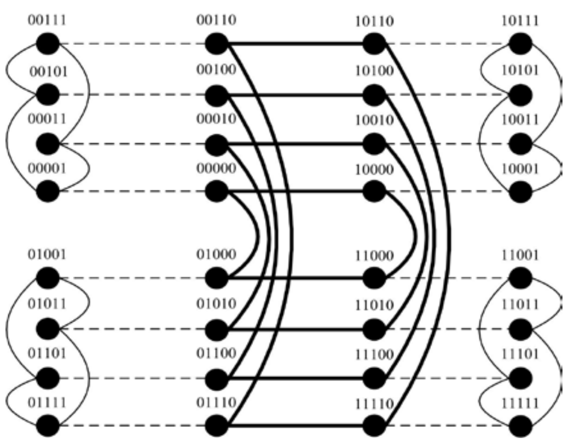

Fig. 1 $ECQ(2,2)$

Theorem 1[11] **.**The degree of $V(ECQ(s,t))$ whose bit addresses end in 0 is $s+1$, while the degree of $V(ECQ(s,t))$ whose bit addresses end in 1 is $t+1$.

By theorem 1, we can show the minimum degree of $ECQ(s,t)$, denoted by $\delta(ECQ(s,t))$, to be $s+1$, where $t \geq s \geq 1$.

Some basic properties and characteristics of $ECQ(s,t)$ were studied[11,12,13,14,15].

Theorem 2[11]**.**An $ECQ(s,t)$ can be decomposed into two $ECQ(s-1,t)$ subgraphs or two $ECQ(s,t-1)$ subgraphs.

By theorem 2, an $ECQ(s,t)$ can be partitioned into two subgraphs L and R, where

$V(L) = \{0a_{s-2}...a_0b_{t-1}...b_0c \mid a_i, b_j, c \in \{0,1\}, i \in [0,s-2], j \in [0,t-1]\}$,

$V(R) = \{1a_{s-2}...a_0b_{t-1}...b_0c \mid a_i, b_j, c \in \{0,1\}, i \in [0,s-2], j \in [0,t-1]\}$,

$L \cong ECQ(s-1,t)$ and $R \cong ECQ(s-1,t)$. Then, $V(L)$ can be divided into A and B ,and $V(R)$ can be divided into C and D, where

$A = \{0a_{s-2}...a_0b_{t-1}...b_0 0 \mid a_i, b_j \in \{0,1\}, i \in [0,s-2], j \in [0,t-1]\}$,

$B = \{0a_{s-2}...a_0b_{t-1}...b_0 1 \mid a_i, b_j \in \{0,1\}, i \in [0,s-2], j \in [0,t-1]\}$,

$C = \{1a_{s-2}...a_0b_{t-1}...b_0 0 \mid a_i, b_j \in \{0,1\}, i \in [0,s-2], j \in [0,t-1]\}$,

$D = \{1a_{s-2}...a_0b_{t-1}...b_0 1 \mid a_i, b_j \in \{0,1\}, i \in [0,s-2], j \in [0,t-1]\}$ [12].

A shown in Fig. 2, the edges between A and B and the edges between C and D lie in E_1. The edges between A and C lie in E_2. By the definition of A, B, C and D, there consists three perfect matchings of subgraphs induced by $A \cup B$, $A \cup C$ and $C \cup D$ [10]. Any edge between two distinct vertices of B (or D) lies in E_3. Similarly, any edge between two distinct vertices of A (or C) lies in E_2.

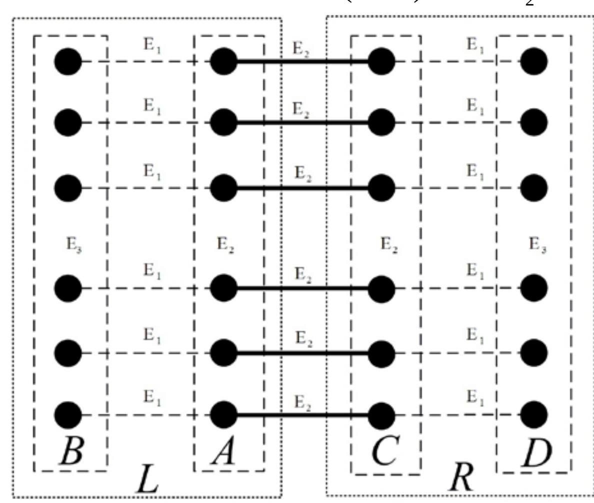

Fig. 2 A, B, C and D in $ECQ(s,t)$

Theorem 3[11]. $ECQ(s,t) \cong ECQ(t,s)$.

Theorem 4[12]. $k(ECQ(s,t)) = s+1$, where $t \geq s \geq 1$.

According to the definition of exchanged crossed cube, it is easy to determine that $ECQ(s,t)$ is triangle-free [12].

Theorem 5. Let a, b, c and d be four arbitrary vertices of $ECQ(s,t)$ where $a \in A, b \in B, c \in B$ and $d \in A$. Then, a - b - c - d - a is not a cycle of length four.

Proof. We prove this theorem by contradiction. As shown in Fig.3, we assume a - b - c - d - a is a cycle of length four. Let $a = \{0a_{s-2}...a_0b_{t-1}...b_0 0\}$. By the definition of E_1 we have $b = \{0a_{s-2}...a_0b_{t-1}...b_0 1\}$. By the definition of E_3, we have

$c=\{0a_{s-2}...a_0d_{t-1}...d_0 1\}$ and $d_{t-1}...d_0$ exists at least one bit different from $b_{t-1}...b_0$. By the definition of E_1, we have $d=\{0a_{s-2}...a_0d_{t-1}...d_0 0\}$. Thus, a and d cannot be connected by a edge because $d_{t-1}...d_0$ exists at least one bit different from $b_{t-1}...b_0$, which contradicts the assumption. □

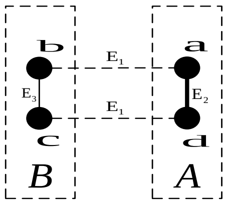

Fig.3 A cycle of length four where $a,d\in A$ and $b,c\in B$

Theorem 6. For any two distinct vertices u and v of $ECQ(s,t)$, they share at most 2 common neighbors,denoted by $|N(u)\cap N(v)|\le 2$.

Proof. By induction. Clearly, the theorem holds for $ECQ(1,1)$. Assume true for $ECQ(s-1,t)$ (or $ECQ(s,t-1)$). According to theorem 2,we partition $ECQ(s,t)$ into L and R, L and R are both isomorphic to $ECQ(s-1,t)$ (or $ECQ(s,t-1)$). Without loss of generality, we assume $L\cong ECQ(s-1,t)$ and $R\cong ECQ(s-1,t)$. When $u,v\in V(L)$ (or $u,v\in V(R)$) we have $N(u)\cap N(v)\subset V(L)$ (or $N(u)\cap N(v)\subset V(R)$) (see fig.2). By the induction hypothesis, we have $|N(u)\cap N(v)|\le 2$. When $u\in L$ and $v\in R$ (or $u\in R$ and $v\in L$), by the fact that $A\cup C$ contains a perfect matching, we have $|N(u)\cap N(v)|\le 2$.

Thus,we complete the proof.□

Theorem 7. For any edge (u,v) of $ECQ(s,t)$,where $(u,v)\in E_2$, $u\in A$ and $v\in C$, $|N(w)\cap N(u,v)|\le 3$ for any vertex w of $ECQ(s,t)$.

Proof. There are four cases to be considered.

Case 1. $w\in A$.

As can be seen in Fig. 2, $N(w)\cap N(u,v)\subset A\cup C$. By theorem 6, we have $|N(w)\cap N(u)|\le 2$ and $|N(w)\cap N(v)|\le 2$. When $|N(w)\cap N(v)|=2$, we have $u\in N(w)\cap N(v)$ (see Fig.2). Then we also have $|N(w)\cap N(u)|-|N(w)\cap\{u,v\}|\le 1$. Hence,

$|N(w)\cap N(u,v)|=|N(w)\cap N(u)|+|N(w)\cap N(v)|-|N(w)\cap\{u,v\}|$
$\le 2+1=3$.

Case 2. $w\in B$.

w and v have one common neighbor if and only if u, v and w are in a horizontal straight-line of Fig. 2. In this case,because $ECQ(s,t)$ is triangle-free, we have $|N(w)\cap N(u)|=0$. When u, v and w are not in a horizontal straight-line of Fig. 2, we have $|N(w)\cap N(v)|=0$. In this case,by theorem 6,we have $|N(w)\cap N(u)|\le 2$. Thus, $|N(w)\cap N(u,v)|\le 2$.

Case 3. $w\in C$.

The proof procedure is similar to that of case 1.

Case 4. $w\in D$.

The proof procedure is similar to that of case 2.

The proof is complete.

III. The conditional connectivity and the restricted connectivity of $ECQ(s,t)$

We now use the above theorems to determine the conditional connectivity and the restricted connectivity of $ECQ(s,t)$, denoted by $k_C(ECQ(s,t))$ and $k_R(ECQ(s,t))$, as follows.

Theorem 8. $k_c(ECQ(s,t))\le 2s$, $t\ge s>2$.

Proof. Consider an arbitrary edge (u,v) of E_2 with $u\in A$ and $v\in C$. Since $ECQ(s,t)$ is triangle-free, we have $|N(u)\cap N(v)|=0$. Let vertex subset $F=N(u,v)=N(u)\cup N(v)-\{u,v\}$. As we can see in Fig.4, we have $|F|=|N(u)|+|N(v)|-|\{u,v\}|=s+1+s+1-2=2s$. It is easy to see that F is a vertex cut of $ECQ(s,t)$. Next, we will prove F is a conditional vertex cut of $ECQ(s,t)$ (i.e.,every vertex of $V(ECQ(s,t))-F$ has at least one neighbor which is not in F).

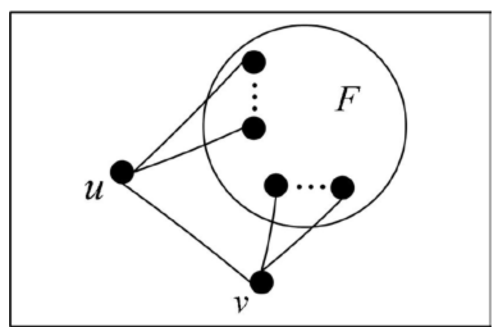

Fig. 4 Illustration for theorem 9

Let w be an arbitrary vertex of $V(ECQ(s,t))-F$. $ECQ(s,t)-F$ has two connected components,one is $\{u,v\}$ and the other is $V(ECQ(s,t))-F-\{u,v\}$. Therefore, there are two cases to be considered.

Case1. $w\in\{u,v\}$.

It is easy to see u is a neighbor of v and v is a neighbor of u with $u,v\notin F$. Hence, w has at least one neighbor which is not in F.

Case2. $w\in V(ECQ(s,t))-F-\{u,v\}$.

By theorem 7, we have $|N(w)\cap N(u,v)|\le 3$,which can be restated as $|N(w)\cap F|\le 3$. Since $|N(w)|\ge s+1$ and $t\ge s>2$, we can derive $|N(w)|>3$. Therefore, w has at least one neighbor which is not in F.

Hence,each vertex of $V(ECQ(s,t))-F$ has at least one neighbor which is not in F. F is a conditional vertex cuts of $ECQ(s,t)$ and $|F|=2s$.

Thus, $k_C(ECQ(s,t))\le|F|=2s$ for $t\ge s>2$. □

Theorem 9. $k_C(ECQ(s,t))\ge 2s$, $t\ge s>2$.

Proof. By contradiction,we assume an arbitrary vertex subset F with $|F|\le 2s-1$ is a conditional vertex cut of $ECQ(s,t)$. By theorem 2, we partition $ECQ(s,t)$ into two $ECQ(s-1,t)$ subgraphs, denoted by L and R, where $V(L)=\{0a_{s-2}...a_0b_{t-1}...b_0c\}$ and $V(R)=\{1a_{s-2}...a_0b_{t-1}...b_0c\}$. Let $F_0=F\cap L$ and $F_1=F\cap R$. Because $F_0\cap F_1=\varnothing$ and $|F|\le 2s-1$, either $|F_0|<s$ or $|F_1|<s$. Without loss of generality, we assume that $|F_1|<s$. By theorem 4, we have $k(R)=s$. Thus, $R-F_1$ is connected.

Next, we will prove each vertex of $L-F_0$ is connected to $R-F_1$. Let u be an arbitrary vertex of $L-F_0$. There are two cases to be considered.

Case1. $u\in A$.

By the definition of conditional connectivity, each vertex of $V(L)-F_0$ has at least one neighbor not in F. As shown in Fig.2, there are three subcases to be considered.

Subcase1.1. $N(u)\cap B\notin F$.

Let $v=N(u)\cap B$, we have $v\notin F$. As shown in Fig.5, by the definition of $ECQ(s,t)$, u has $s-1$ neighbors in A and v has t neighbors in B. By theorem 5, we have $(N(u)\cap A)\cap(N(N(v)\cap B)\cap A)=\varnothing$. If u cannot connect to $R-F_1$, then each horizontal straight-line in subgraphs P and Q has at least one vertex in F (see Fig.5). Thus, we have $|F|\ge s-1+1+t\ge 2s$ which contradicts the assumption that $|F|\le 2s-1$.

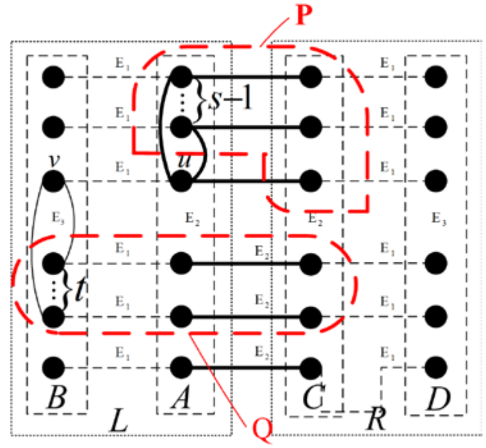

Fig. 5. Illustration for subcase1.1

Subcase1.2. $N(u)\cap B\in F$ and $N(u)\cap A\not\subset F$.

Let v be an arbitrary vertex of $N(u)\cap A$ with $v\notin F$. As shown in Fig.6, u has $s-2$ neighbors in A besides v, and v has $s-2$ neighbors in A besides u. We have $|N(u)\cap N(v)|=0$ because $ECQ(s,t)$ is triangle-free. If u cannot connect to $R-F_1$, then each horizontal straight-line in subgraph P has at least one vertex in F (see Fig.6). If $N(v)\cap B\in F$, we have $|F|\ge s-2+1+s-2+1+|N(u)\cap B|+|N(v)\cap B|=2s$ which contradicts the assumption that $|F|\le 2s-1$. Therefore, u is connected to $R-F_1$. Otherwise $N(v)\cap B\notin F$, the proof procedure is similar to that of Subcase1.1.

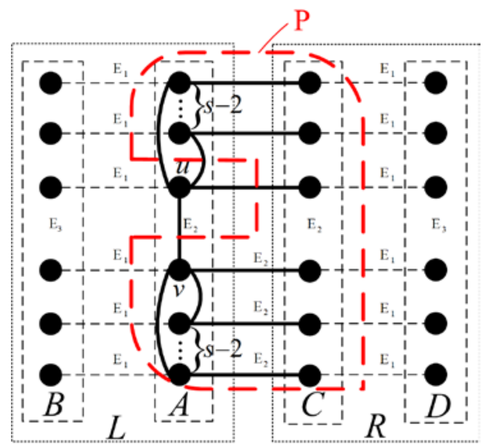

Fig. 6. Illustration for subcase1.2

Subcase1.3. $N(u)\cap A\in F$ and $N(u)\cap B\subset F$

By the definition of conditional connectivity,we have $N(u)\cap C\notin F$. Let $v=N(u)\cap C$, we have $v\notin F$. Therefore, u is connected to $R-F_1$ by (u,v).

Case2. $u\in B$.

Subcase2.1. $N(u)\cap A\notin F$.

The proof procedure is similar to that of Subcase1.1.

Subcase2.2. $N(u)\cap A\in F$.

According to the definition of conditional connectivity,we have $N(u)\cap B\not\subset F$. Let v be an arbitrary vertex of $N(u)\cap B$ with $v\notin F$. As shown in Fig.7, u has $t-1$ neighbors in B besides v and v has $t-1$ neighbors in B besides u . Because $ECQ(s,t)$ is triangle-free, we have $|N(u)\cap N(v)|=0$. If u cannot connect to $R-F_1$, then each horizontal straight-line in subgraph P has at least one vertex in F . We have $|F|\geq t-1+1+t-1+1=2t\geq 2s$ which contradicts the assumption that $|F|\leq 2s-1$.

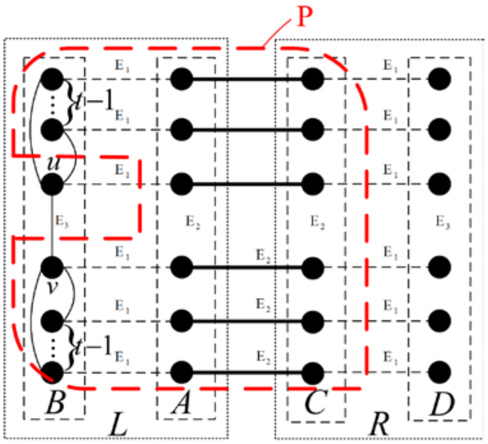

Fig.7. Illustration for subcase2.2

Therefore,each vertex in $L-F_0$ is connected to $R-F_1$. It follows that $ECQ(s,t)-F$ is connected and F is not a conditional vertex cut of $ECQ(s,t)$ which contradicts the assumption. Hence, $k_C(ECQ(s,t))>|F|$ and $k_C(ECQ(s,t))\geq 2s$, for $t\geq s>2$. □

Theorem 10. $k_C(ECQ(s,t))=2s\,,t\geq s>2$.

Proof. By theorem 8 and theorem 9,we can derive theorem 10.□

Thus, the conditional connectivity of $ECQ(s,t)$ is almost the same as that of $EH(s,t)$ [16], but much smaller than that of CQ_{s+t+1} [17].

We now use the above theorems to prove the upper and lower bound of $k_R(ECQ(s,t))$.

Theorem 11. $k_R(ECQ(s,t))\leq 2s\,,t\geq s>2$.

Proof. The proof procedure is similar to that of Theorem 8. But we needs to be consider whether any vertex $w\in F$ has at least one neighbor not in F . Due to $F=N(u)\cup N(v)-\{u,v\}$, as shown in Fig.4, either u or v is a neighbor of w , $u\in F$ and $v\in F$. Therefore, w has at least one neighbor not in F , F is a restricted vertex cut of $ECQ(s,t)$ with $|F|=2s$. As a result, $k_R(ECQ(s,t))\leq|F|=2s$, $t\geq s>2$.

Theorem 12. $k_R(ECQ(s,t))\geq 2s\,,t\geq s>2$.

Proof. The proof procedure is similar to that of Theorem 9. The difference is we should assume F is a restricted vertex cut of $ECQ(s,t)$.

Theorem 13. $k_R(ECQ(s,t))=2s\,,t\geq s>2$.

Proof. By Theorem 11 and Theorem 12, we can derive Theorem 13.

IV. Experiment Simulation

The conditional connectivity and restricted connectivity simulation algorithm of $ECQ(s,t)$ consists of the following 3 steps.

Step 1: Vertex-coding

Each vertex is represented by a s+t+1-bit binary string.

Step 2: Establish all the 2-tuple sets of the connected edge

Obtain the set of F , $|F|\leq t$ and each vertex of $V(ECQ(s,t))-F$ (or $V(ECQ(s,t))$)has at least one neighbor not in F . Then, establish all the 2-tuple sets of the connected edge after removing all vertices in F .

Step 3: Computing the connected components

Computing the connected components based on 2-tuple set of the connected edge. $k_C(ECQ(s,t))\geq t$ (or $k_R(ECQ(s,t))>t$) If there exists no 2-tuple set of the connected edge that has more than one connected component.

$k_C(ECQ(s,t))$ (or $k_R(ECQ(s,t))>t$) is the maximum integer. Table 1 shows the results of $k_C(ECQ(s,t))$ and $k_R(ECQ(s,t))>t$, with $2\leq s\leq t\leq 5$.

Table 1. The conditional connectivity and restricted connectivity simulation results of $ECQ(s,t)$ $(2\leq s\leq t\leq 5)$

ID	$ECQ(s,t)$		$k_C(ECQ(s,t))$	$k_R(ECQ(s,t))$
	s	t		
1	2	2	4	4
2	2	3	4	4
3	2	4	4	4
4	2	5	4	4
5	3	3	[illegible]	[illegible]
6	3	4	[illegible]	[illegible]
7	3	5	[illegible]	[illegible]
8	4	4	8	8
9	4	5	8	8
10	5	5	10	10

V. Conclusion

All kinds of connectivity of a interconnection network are the minimum number of vertex cut whose removal disconnects the network under different conditions, are directly related to its reliability and fault tolerability. Hence they are important indicators of interconnection networks' reliability evaluation.

In this paper, we determine the conditional connectivity and restricted connectivity of $ECQ(s,t)$, which is an important measure of the fault-tolerance of $ECQ(s,t)$. After analyzing the topology of $ECQ(s,t)$, we have proved the conditional connectivity and the restricted connectivity of $ECQ(s,t)$ are $2s$ by theoretical deduction and simulation experiments, for $t \geq s > 2$.

In the future, we may consider the diagnosability[18], conditional diagnosability[19], and g-good-neighbor conditional diagnosability[20] under the PMC and comparison model.

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