

# Real Time PI Speed Control of a Series DC Motor

Jesús U. Liceaga-Castro and Irma I. Siller-Alcalá

**Abstract**—The design and implementation of a speed control system for a series DC motor is presented. The controller is the classical PI controller designed based on a linear approximation or model of the series DC motor. The control system also includes an "anti-windup" scheme in order to limit the effects of saturation due to limitations in voltage supply by power driver. Real time experimental results are included in order to support the proposal here presented.

**Keywords**— Series DC motor, PI control, Speed control systems, real time implementation.

## I. INTRODUCTION

THE design of speed control systems for electric motors either DC or AC, triphasic, brushless, variable reluctance, etc. had been tackled using several control strategies such as classical control, optimal control, non-linear control, fuzzy logic, etc. However, there is a kind of electric motors which despite its excellent features -high torque, low current consumption and small physical dimensions- has not been analyzed for control purposes with clear objectives of real time implementation. That is, early publications use the series DC motor as an academic example to assess control strategies for non-linear systems with not well-defined relative degree [1,2,3], while other reports tackled the problem of the design of speed control system for series DC motors but limited to digital simulations [4-11]. Meanwhile, real time implementation reports for series DC motor deals mainly with the design and assessment of the performance effects over the motor by the power driver [12-17].

In a previous report the non-linear modelling without magnetic saturation of series DC motor, together with an identification strategy based on the Strecj method was presented [18]. Also, a linearization around an equilibrium point was deduced and proved that linear models are appropriate for control objectives despite its non-linear characteristics. Therefore, the objective of this paper is to present a first approach and to prove, contrary to previous reports, that classical PI controllers are sufficient to control the rotor speed of series DC motors despite the non-linearities of the motor. The control system does not include revers motion

as this requires a more complex power driver; that is, a power driver capable of inverting motor polarization allowing inverse motion or magnetic induced torque inversion. The paper is presented as follows: In Section 2, the non-linear model without magnetic saturation is presented. In Section 3, the linearization and transfer function, based on the parameters reported in [18] is shown. In Section 4, the power driver is briefly described. In section 5, controller PI is designed together with its real time implementation. Section 6, shows real time experimental results. Finally, conclusions are presented in Section 7.

## II. SERIES DC MOTOR NON-LINEAR MODEL

Series DC motors get their name because field winding is connected in series to armature winding as shown in Figure 1. From this connection and based on the electric diagram of figure 2 the differential equations comprising the mechanical and electrical subsystems of the series DC motor are given by equations (1) and (2):

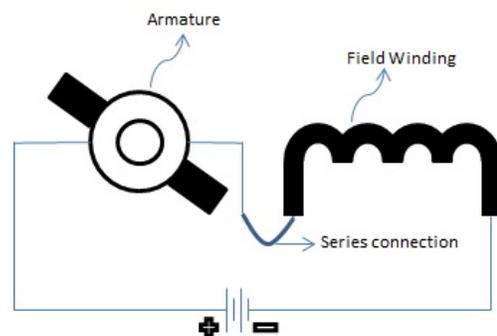


Fig. 1. Series connection of a DC motor

$$V(t) = R_a i_a(t) + R_f i_f(t) + L_a \frac{d}{dt} i_a(t) + L_f \frac{d}{dt} i_f(t) + E_a \quad (1)$$

$$T_e(t) = T_L(t) + b\omega(t) + J \frac{d}{dt} \omega(t) \quad (2)$$

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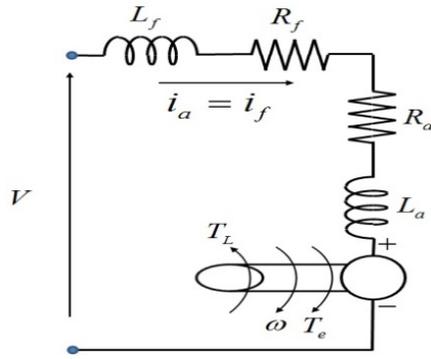


Fig. 2. Electric diagram of a series DC motor

As  $i_a(t) = i_f(t)$  equations (1) and (2) reduce to:

$$V(t) = (R_a + R_f)i(t) + (L_a + L_f)\frac{d}{dt}i(t) + E_a \quad (3)$$

$$T_e(t) = T_L(t) + b\omega(t) + J\frac{d}{dt}\omega(t) \quad (4)$$

Where,  $\omega(t)$  is the rotors speed,  $E_a$  represents the counter electromotive force EMF,  $T_L(t)$  is the load torque,  $i(t) = i_a(t) = i_f(t)$  is the current,  $b$  is the friction coefficient,  $J$  is the rotor's inertia and  $T_e(t)$  is the electromagnetic torque produced by the motor.

The EMF  $E_a$  and  $T_e(t)$  depend both on the air-gap flux, that is:

$$E_a(t) = \omega(t) \Phi(i) \quad (5)$$

$$T_e(t) = i(t) \Phi(i) \quad (6)$$

The flux is a function of the current  $i(t)$  so equations (1)-(4) are non-linear. Also, it is common practice to approximate the flux by a linear relation when the magnetic saturation is neglected, that is:

$$\Phi(i) = k_0 i(t) \quad (7)$$

Where,  $k_0$  is the mutual inductance between the armature and field coils.

Finally, the nonlinear model of a series DC motor without saturation is given by:

$$V(t) = Ri(t) + L\frac{d}{dt}i(t) + \omega(t)i(t)k_0 \quad (8)$$

$$i^2(t)k_0 = T_L(t) + b\omega(t) + J\frac{d}{dt}\omega(t) \quad (9)$$

where,  $R = R_a + R_f$  and  $L = L_a + L_f$

### III. NON-LINEAR MODEL LINEARIZATION

A linear approximation of equations (8) and (9) around any equilibrium point, as pointed out in [18], represent a better model for certain control objectives. Rearranging equations (8)

and (9) as follows:

$$\frac{d}{dt}i(t) = -\frac{R}{L}i(t) - \frac{k_0}{L}\omega(t)i(t) + \frac{1}{L}V(t) \quad (10)$$

$$\frac{d}{dt}\omega(t) = -\frac{b}{J}\omega(t) - \frac{1}{J}T_L(t) + \frac{k_0}{J}i^2(t) \quad (11)$$

and defining

$$\begin{aligned} a_1 &= \frac{k_0}{J}; & b_1 &= \frac{R}{L} \\ a_2 &= \frac{b}{J}; & b_2 &= \frac{k_0}{L} \quad \text{and} \quad \begin{aligned} x_1 &= \omega \\ x_2 &= i \end{aligned} \\ a_3 &= \frac{1}{J}; & b_3 &= \frac{1}{L} \end{aligned} \quad (12)$$

The nonlinear state space representation of the series DC motor is given by:

$$\begin{aligned} \dot{x}_1 &= a_1 x_2^2 - a_2 x_1 - a_3 T_L \\ \dot{x}_2 &= -b_1 x_2 - b_2 x_1 x_2 + b_3 V \end{aligned} \quad (13)$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_1 x_2^2 - a_2 x_1 - a_3 T_L \\ -b_1 x_2 - b_2 x_1 x_2 + b_3 V \end{bmatrix} = f(x, u) \quad (14)$$

The equilibrium points  $(x_1^0, x_2^0)$  of equation (13) are given by:

$$x_2^0 = \sqrt{\frac{a_2 x_1^0 + a_3 T_L}{a_1}}; \quad V = \frac{x_2^0 (b_1 + b_2 x_1^0)}{b_3} \quad (15)$$

The linear approximation of (13) around the equilibrium point (15) is given by:

$$\dot{x} = Ax + Bu; \quad y = Cx \quad (16)$$

Where

$$\begin{aligned} A &= \left. \frac{\delta f(x, u)}{\delta x} \right|_{x_1^0, x_2^0} = \begin{bmatrix} -a_2 & 2a_1 x_2^0 \\ -b_2 x_2^0 & -(b_1 + b_2 x_1^0) \end{bmatrix}; \\ B &= \left. \frac{\delta f(x, u)}{\delta u} \right|_{x_1^0, x_2^0} = \begin{bmatrix} -a_3 & 0 \\ 0 & b_3 \end{bmatrix}; \\ C &= [1, 0] \end{aligned} \quad (17)$$

With

$$u = [T_L \quad V]^T, \quad y = \omega(t) \quad (18)$$

If the load torque is assumed zero  $T_L = 0$

$$x_2^0 = \sqrt{\frac{a_2 x_1^0}{a_1}}; \quad V = \frac{x_2^0 (b_1 + b_2 x_1^0)}{b_3}; \quad B = \begin{bmatrix} 0 \\ b_3 \end{bmatrix} \quad (19)$$

These assumption leads to a model with one input, voltage  $V(t)$ , subjected to a torque perturbation.

The transfer function  $G(s)$  associated to the state space representation (17) based on the parameters of Table 1, [18], around the equilibrium point (19) with  $x_1^0 = 439.82$  RPM is:

$$G(s) = \frac{\omega(s)}{V(s)} = C(sI - A)^{-1} B = \frac{923.3}{s^2 + 626.6s + 67.58} \quad (20)$$

TABLE 1: Motor parameters

$k_0 = 0.17554 \text{ N-m/Wb-A}$
$R = 20.833\Omega$
$L = 156.24\text{mH}$
$b = 0.000026 \text{ N-m/(rad/sec)}$
$J = 0.0006206 \text{ Kg-m}^2$

The poles of transfer function (20) are  $\{-3256.2, -0.1\}$  so the series DC motor is stable and over damped. It is also possible to distinguish the two typical modes of a DC motor from the poles:  $(s + 0.1)$  representing the slow dynamic of the mechanical subsystem and  $(s + 3256.2)$  the fast dynamic of the electrical subsystem.

IV. POWER DRIVER

The experimental setup consists of a voltage source with a maximum voltage of 50 volts and a maximum current of 3 Amp, series DC motor, and a power driver with a USB communication. The power driver was designed considering the electric demand of the motor. Hence, the power driver is based on the MOSFET 12N65, which operates with a maximum current of 12Amp and a maximum input voltage of 650 Volts. An equally important element is the ACS711LC circuit used to monitor the power consumed by the motor.

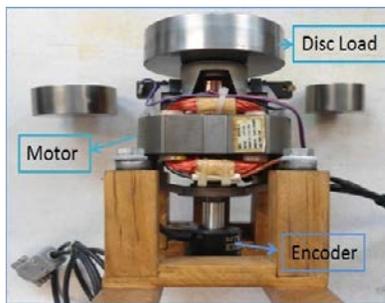


Fig. 3. Series DC motor

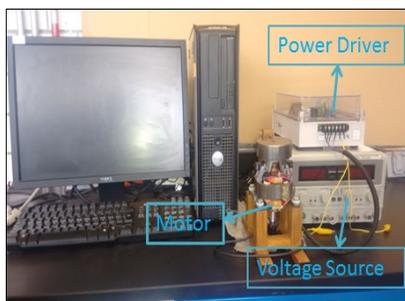


Fig. 4. Experimental Set Up

Inductive loads together with pulsed excitation signals generate reverse currents that may damage switching elements such as MOSFETs. Although the MOSFETs used in the implementation have an internal protection diode, two transistors FR307 of rapid recovery were added in order for additional protection, as shown in figure 5.

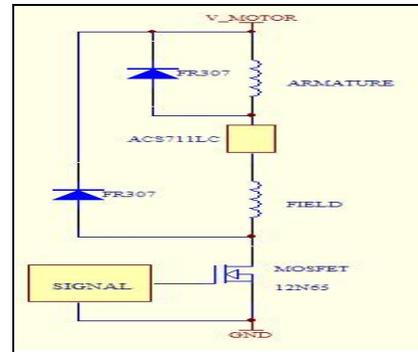


Fig. 5. Power driver

V. PI CONTROLLER DESIGN

The PI controller for the transfer function of the series DC motor of equation (20) was designed based on the frequency domain method of Bode-Shape. The objectives of design were set as: Bandwidth  $\omega_b = 1 \text{ rad/sec}$ , phase margin  $M_p > 80^\circ$  and gain margin  $M_G \rightarrow \infty \text{ dB's}$ . These conditions were set in order to assure good robustness conditions and an over-damped response with acceptable time response.

A PI controller  $C(s)$  satisfying above conditions of design is given by:

$$C(s) = \frac{0.75s + 0.0675}{s} = 0.75 + \frac{0.0675}{s} \quad (20)$$

The Bode plots of the open loop control system are shown in figure 6. From these plots it is clear that the objectives of design are fulfilled.

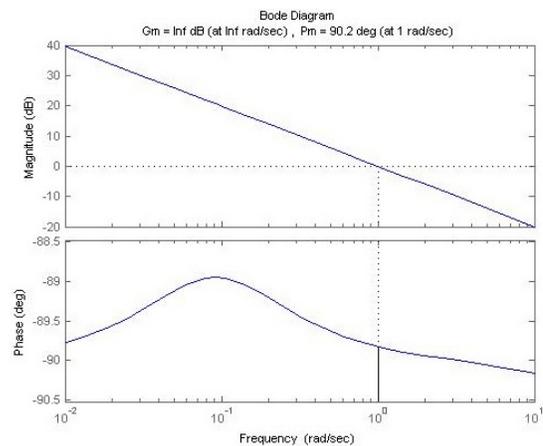


Fig. 6. Bode plots of C(s)G(s)

Since series DC motors cannot reverse their direction of rotation because it is not possible to reverse the induced magnetic torque unless motor polarization is inverted, the minimum output of the controller (20) is limited or saturated to  $0V$ . This is also necessary to avoid instability. On the other hand, it is well known that controllers with integral action in conjunction with saturation affect the transient response of any control systems. Therefore, controller (20) is implemented including an anti-windup scheme, as shown in figure 10.

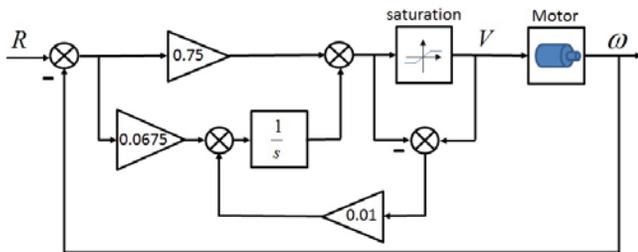


Fig. 7. Anti-windup PI controller

## VI. EXPERIMENTAL RESULTS

To verify the performance of the speed control system depicted in figure 7 several experiments are carried out. These experiments allow evaluating the control system over a wide operating range, regulation and tracking, and under unknown torque loads perturbations.

**EXPERIMENT 1:** In figure 8 the speed response to a square reference signal  $R(t)$  from  $0$  to  $630\text{rad/sec}$  is shown. From this plot it is possible to see that the speed control system complies with the specification of design when the speed reference is increased. However, this is not the case when the speed is reduced. This is due to the fact that the input signal  $V(t)$  is limited to a minimum of  $0V$ ; that is, in this condition the motor is actually operating open loop. Also, the range of variation of reference signal covers the full range of operation which, as mention before, is limited to a maximum voltage supply of  $50V$ . Input voltage  $V(t)$  and current consumption  $i(t)$  are shown in figure 8 and 9, respectively. Figure 8 shows that control variable -input voltage- is highly affected by sensor noise. Nonetheless, the current consumption is low, around  $0.25A$ , as shown in figure 9. This means that, electric subsystem remains out of magnetic saturation. Also, in figure 9, current peaks are due to non-recommended sudden changes in the reference signal. Nevertheless, despite the limitations in input voltage and noise effects the system behaves according to specifications along a wide range of operation.

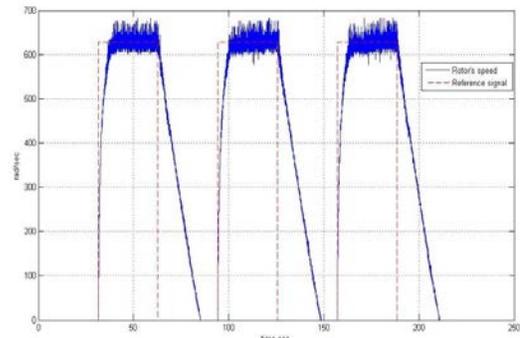


Fig. 8. Rotor speed response to a positive square reference signal

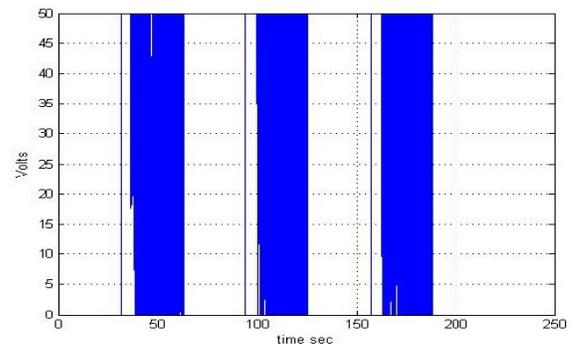


Fig. 9. Input voltage  $V(t)$

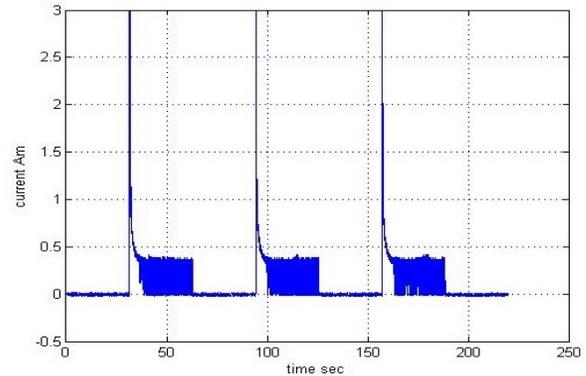


Fig. 10. Current consumption  $i(t)$

**EXPERIMENT 2:** In order to assess the performance of the control system under tracking conditions two strictly positive sinusoidal signals  $R(t)$  with frequencies  $0.33\text{rad/sec}$  and  $0.66\text{rad/sec}$  -frequencies consistent with bandwidth  $\omega_B = 1\text{rad/sec}$  - were applied.

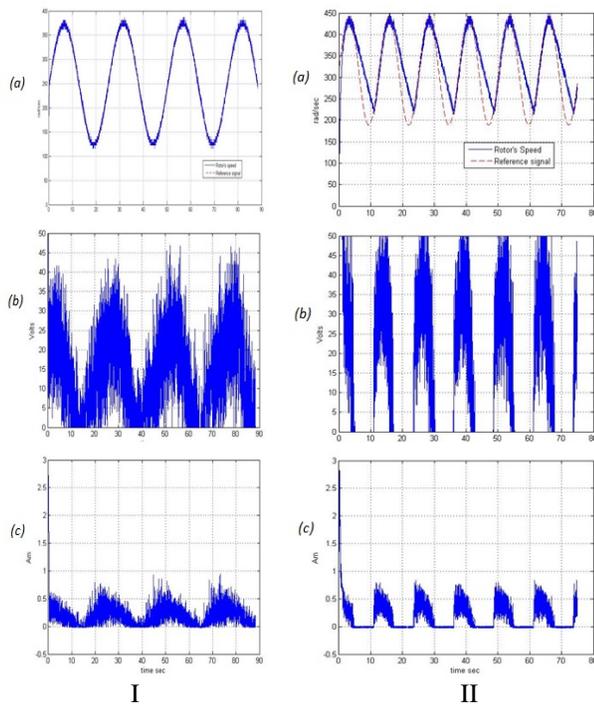


Fig. 11. System responses to sinusoidal signal references, (I)  $0.33\text{rad/sec}$ , (II)  $0.66\text{rad/sec}$

Figures 11 show the responses of the control system to the two strictly positive sinusoidal signals with frequencies of  $0.33\text{rad/sec}$  and  $0.66\text{rad/sec}$ , respectively. Figure 11.I shows the rotor speed response when the sinusoidal reference signal has a frequency of  $0.33\text{rad/sec}$ . It is clear from this plot that the control system has an excellent performance under tracking conditions. Also, and similar to the results of Experiment 1, the input voltage is highly affected by sensor noise, figure 11.I.b, and low current consumption, figure 11.I.c.

Figure 11.II shows the responses of rotor speed, input voltage and current consumption when the sinusoidal reference signal has a frequency of  $0.66\text{rad/sec}$ . Contrary to previous responses, in this case tracking performance is deteriorated in periods in which the speed is reduced but, alike to Experiment 1, this is due to limitation of the control variable -input voltage- which is limited to  $0V$ . However, in the periods when the speed is increased the control system presents excellent performance.

**EXPERIMENT 3:** In this experiment the control system is affected by an unknown constant torque load perturbation when reference signal is  $R(t)=314\text{ rad/sec}$ . The unknown perturbation load torque is applied at  $30\text{sec}$  and released at  $60\text{sec}$ . The responses of rotor speed  $\omega(t)$ , input voltage  $V(t)$  and current consumption  $i(t)$  are shown in figure 12. From figure 12.a is possible to see that control system is capable of compensate torque perturbation with low current consumption, figure 12.c, although the control signal -input voltage- in figure 12.b is still affected by sensor noise.

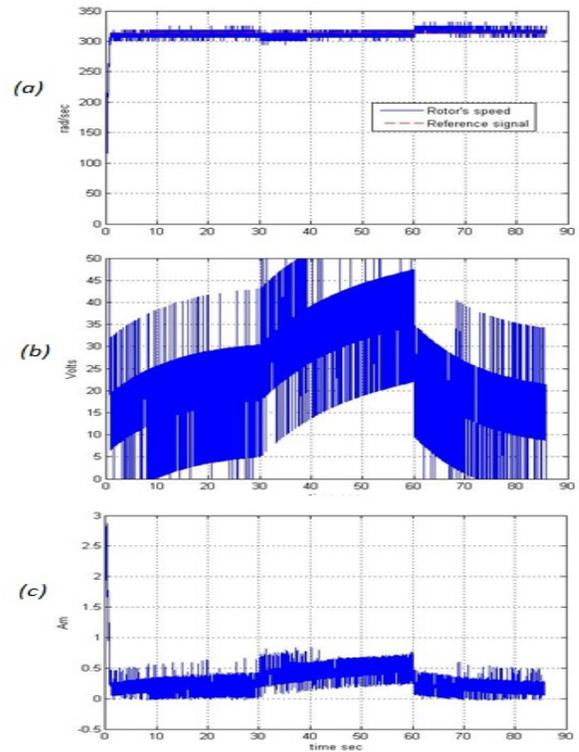


Fig. 12. System responses to unknown torque load perturbation

## VII. CONCLUSIONS

A speed control system for a series DC motor based on a classical PI controller was designed and implemented. It is proved that despite nonlinearities and non-well-defined relative degree of this kind of motors it is possible to design classical controllers, like the PID, based on linear models resulting in control systems with excellent performance along a wide range of operation. Moreover, by means of several real time experiments it is verified that the proposed control system has excellent performance under tracking and torque load perturbations conditions. It is also found that in order to fully exploit the capabilities of this kind of motors -high torque with low current consumption- it is necessary to improve the power driver so that it can reverse direction of induced magnetic torque and therefore direction of rotor rotation. This will also allow elude the use of the anti-windup scheme required to avoid instability and performance deterioration. Also, despite good control system performance it is necessary to implement a strategy such that sensor noise effects are reduced, this will improve steady state responses and will reduce current consumption. This improvement will be reported in a future report.

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