Performance Analysis of Doubly Fed Induction Generator - A Magnetic Saturation Perspective

Julia Tholath Jose and Chattopadhyay Adhir Baran

Abstract— Doubly fed induction generators (DFIGs) are commonly used in wind energy conversion systems due to their several advantages compared to fixed speed generator systems. Accurate prediction of steady-state and transient performance of doubly fed induction generator requires the proper representation of the magnetic saturation in the machine modeling. Two different magnetizing reactivities are considered in this paper to study the effect of main flux saturation on doubly fed induction generator dynamic response. The results indicate that saturation has noticeable effect in determining the rotor current under steady state operating conditions. The limits of reactive power capability of the DFIG is also found to be affected by saturation in the over excited mode. Additionally, analytical evaluations also show that accurate prediction of steady state stability limits are influenced by the saturation. This paper aims at providing an analytical overview of the impact of magnetic saturation in the performance of doubly fed induction generator.

Keywords— Doubly Fed Induction Generator, magnetic saturation, reactive Power Capability, short Circuit, voltage Sag, stability.

I. INTRODUCTION

IND power generation has grown significantly in the past decades due to the increasing demand of electrical power and the depletion of fossils fuels. The Global Wind Energy Council (GWEC) reports that 52GW of wind power was added in 2017, reaching a total installed capacity of 539 GW globally [1]. Doubly fed induction generator (DFIG) based wind turbines have gained importance in wind energy conversion system due to the benefits of variable speed operation, low power rating of converters and decoupled control of active and reactive power [2].

As stator and rotor windings of doubly fed induction generator are both connected to the grid, machine is more prone to grid disturbances. Accurate assessment of the performance of doubly fed induction generator under transient conditions is very important for design and coordination of control strategies and protection devices. Therefore, a theoretical analysis of the impact of saturation on the transient operation of doubly fed induction generator is needed [3].

Most of the models of DFIG are developed on the assumption of unsaturated magnetic core so that the magnetic flux is taken to be linearly proportional to the currents responsible for them. During rated operating conditions, the ferromagnetic core of DFIG is normally designed to work to some degree in the nonlinear region of saturation. This make sure of full utilization of the ferromagnetic material and reduces the cost of the system. But, the nonlinear nature of the magnetization characteristics is usually not considered in the dynamics and control of DFIG. When a large grid disturbance occurs, the stator and rotor currents will increase rapidly, causing the magnetic flux to vary non-linearly [3]. The parameters of the DFIG are subjected to variation due to the change of flux density caused by the grid voltage variations [4], [5]. The parameter variation of the DFIG poses challenge to the performance of control system of generator under grid voltage distortions and also affects the system stability. Hence, the magnetic nonlinearity due to saturation has to be considered in the control and dynamics of DFIG [6].

An integrated evaluation of the effect of saturation need a mapping of the magnetizing current to the flux over the entire region of operation of the machine. Mostly, the flux saturation affects the mutual inductance but to a less extent the leakage inductances. The saturation of leakage flux in doubly fed induction generators of MW range can be ignored [7] and hence the stator and rotor leakage inductances can be treated as constant during theoretical analysis. Depending on the problem, inductances based on total or incremental values of flux determined from magnetization characteristics may be appropriate.

The effects of magnetic saturation in electrical machines have been a significant topic of study and research from early 1980. Although considerable work has done in the area of magnetic saturation in induction machines, its effect is neglected in the case of doubly fed induction generator [8]-[15]. Only a few works in literature demonstrate the effect of magnetic saturation of doubly fed induction generator [16]-[18]. These studies mainly focused on the influence of magnetic saturation on the transient operation of DFIG. The effect of saturation on steady state conditions has not addressed. Moreover, a systematic theoretical analysis is still lacking in this field.

The objective of this paper is to analyze the impact of saturation on the performance of DFIGs during steady state as well as transient conditions. To achieve this goal, a saturated model of doubly fed induction generator using steady state and transient magnetizing inductances is employed. Nonlinear function interpolation of magnetizing characteristics is done
with the aim of determining the steady and transient magnetizing inductances.

II. DOUBLY FED INDUCTION GENERATOR

Doubly fed induction generator is basically a wound rotor induction machine in which stator is directly connected to the grid and the rotor is interfaced to the grid through a bidirectional converter [19]. The configuration of doubly fed induction generator-based wind energy conversion system is shown in Fig. 1.

![Fig. 1. Doubly fed induction generator-based wind turbine system](image)

A. Mathematical Model of Unsaturated DFIG

The voltage and flux linkage equations of doubly fed induction generator modeled in $dq$ synchronous rotating frame is as follows [20]:

\[
\begin{align*}
\psi_{sq} &= R_s i_{sq} + \frac{d}{dt} \psi_{sq} + \omega_s \psi_{sd} \\
\psi_{sd} &= R_s i_{sd} + \frac{d}{dt} \psi_{sd} - \omega_s \psi_{sq} \\
\psi_{rq} &= R_r i_{rq} + \frac{d}{dt} \psi_{rq} + (\omega_s - \omega_r) \psi_{rd} \\
\psi_{qr} &= R_r i_{qr} - \frac{d}{dt} \psi_{rq} - (\omega_s - \omega_r) \psi_{sd} \\
\end{align*}
\]

(1)

The expression of stator and rotor flux linkages are expressed as follows:

\[
\begin{align*}
\psi_{sq} &= L_s i_{sq} + L_m i_{rq} \\
\psi_{sd} &= L_s i_{sd} + L_m i_{rd} \\
\psi_{rq} &= L_r i_{rq} + L_m i_{sq} \\
\psi_{rd} &= L_r i_{rd} + L_m i_{st} \\
\end{align*}
\]

(2)

where $v_s = v_{sq} + jv_{sq}$ and $v_r = v_{rq} + jv_{rq}$ are the stator and rotor voltages respectively; $i_s = i_{sq} + ji_{sq}$ and $i_r = i_{rq} + ji_{rq}$ are the stator and rotor currents respectively; $\psi_s = \psi_{sq} + j\psi_{sq}$ and $\psi_r = \psi_{rq} + j\psi_{rq}$ are the stator and rotor fluxes respectively; $R_s$ and $R_r$ are the stator and rotor resistances respectively; $L_s = L_{sq} + L_m$ and $L_r = L_{rq} + L_m$ are the stator and rotor inductances respectively; $L_{mq}$ and $L_{md}$ are the stator and rotor leakage inductances respectively; $L_m$ is the magnetizing inductance; $\omega_s$ and $\omega_r$ are the synchronous and rotor speed respectively.

B. Mathematical Model of Saturated DFIG

The mathematical model of doubly fed induction machine shown in (1)-(2) is based on constant value of magnetizing inductance assuming machine is unsaturated.

Due to saturation the linear equation relating the magnetization flux and current has to be modified. The leakage inductances can be considered constant while magnetizing inductance varies with saturation. It is figured that the magnetizing inductance is a non-linear function of the magnetizing current. When machine is operating in the saturated region, small variations in the amount of flux causes considerable variations in the mutual inductance. The variation of magnetizing inductance is shown in Fig. 2 [21].

![Fig. 2. Magnetization characteristics showing steady state and transient saturated reactance](image)

As far as steady state is considered, the saturated magnetizing inductance is proportional to the chord slope of the saturation characteristic.

The chord slope reactance of magnetization characteristics is represented by:

\[
L_{ms} = \frac{\Delta \psi_{m0}}{\Delta i_{m0}} \quad (3)
\]

In transient analysis of DFIG the incremental inductances are also required.

\[
\begin{align*}
\frac{d}{dt} \psi_m &= \frac{d\psi_m}{dt} + \frac{d\psi_m}{di_m} \frac{di_m}{dt} \\
\frac{d}{dt} \psi_m &= L_{mt} \frac{di_m}{dt} \\
\end{align*}
\]

(4)

The transient magnetizing inductance $L_{mt}$ is proportional to the tangential slope of the magnetization characteristic.

Considering the nonlinear variation of magnetizing flux, dynamic equations of doubly fed induction generator are modified as follows [22]:

\[
\begin{align*}
v_{sq} &= R_s i_{sq} + L_{ds} \frac{d}{dt} i_{sq} + L_{mt} \frac{d}{dt} i_{mq} + \omega_s (L_{s1} i_{sd} + L_{m1} i_{md}) \\
v_{sd} &= R_s i_{sd} + L_{ds} \frac{d}{dt} i_{sd} + L_{mt} \frac{d}{dt} i_{md} + \omega_s (L_{s1} i_{sq} + L_{m1} i_{mq}) \\
\end{align*}
\]
C. Determination of Magnetizing Inductance

The magnetizing inductance that varies with saturation can be identified from the magnetizing characteristics of the machine. The inductance $L_{ms}$ and $L_{mt}$ varies with the magnetizing current and appropriate method to identify them is to express the interpolating curve of the magnetization characteristic as non-linear function. The function adopted here for representing the magnetizing characteristics is as follows [15]:

$$\psi_m = a(1 - e^{-b i_m}) + c i_m$$  \hspace{1cm} (6)

The coefficients $a$, $b$, and $c$ can be obtained by minimization of nonlinear least squares using levenberg-marquardt algorithm.

$$L_m = \frac{\psi_m}{i_m} = a \left( \frac{1 - e^{-b i_m}}{i_m} \right) + c$$  \hspace{1cm} (7)

The expression for transient magnetizing inductance $L_{mt}$ can be obtained from interpolation function is as follows:

$$L_{mt} = \frac{\frac{d\psi_m}{di_m}}{a_i} = a_i e^{b_i i_m} + c_i$$  \hspace{1cm} (8)

The magnetization characteristics approximated by the mathematical function

$$\psi_m = a(1 - e^{-b i_m}) + c i_m$$

using experimental data from [6] is as shown in Fig. 3.

Fig. 3. Fitted magnetization characteristics

This method is appropriate in determining the steady state and transient inductance from the interpolated magnetizing curve. Fig. 4, shows the variation of steady state and transient magnetizing inductances with the no load current.

III. EFFECT OF MAIN FLUX SATURATION

Whenever magnetizing component of current is more, the core has a tendency to saturate and the flux is no longer linearly proportional to magnetizing current. Then the degree of non-linearity of the magnetization characteristics has to be tracked. In this section, the effect of main flux saturation on the steady state and the transient performance of DFIG has been investigated.

A. Reactive Power Capability

Wind energy conversion systems with DFIG is capable of controlling real and reactive power independently. The reactive power supplied or absorbed by a DFIG is expressed by [6]:

$$Q_s = 1.5 (v_d s i_q s - v_q s i_d s) \approx -1.5 v_q s i_d s$$  \hspace{1cm} (9)

where $v_{ds}$ = 0 in a stator flux oriented synchronous reference frame. If the reactive power supplied by the DFIG is zero (unity power factor), $i_{ds}$ must be equal to zero. The d axis component of stator flux is expressed as:

$$\psi_{ds} = L_{ds} i_{ds} + L_{m} i_{er}$$  \hspace{1cm} (10)

And the d axis component of rotor current is calculated by

$$i_{dr} = \frac{\psi_{ds}}{L_m} \approx \frac{v_{qs}}{\omega_L L_m}$$  \hspace{1cm} (11)

This shows that $i_{dr}$ is dependent on the mutual inductance of the generator. Hence, the d axis rotor current $i_{dr}$ required for a given reactive power output $Q_s$ depends on the value of mutual inductance. Therefore, control of the stator reactive power would result in considerable steady-state error if the effect of saturation on mutual inductance $L_m$ not considered. However, it is possible to control the stator reactive power without any steady state error using closed loop control method irrespective of the saturation.

The main limiting factors for the reactive power capability of doubly fed induction generator are stator voltage, stator and rotor currents which depends on the operating point. The saturation of the main flux is considered for calculating reactive current limits of DFIG [23].
The reactive current limitations set by the maximum rotor current can be determined by:

\[ P_r^2 + \left( Q_r + \frac{3u_s^2}{2\omega L_s} \right)^2 \leq \left( \frac{3u_s L_m I_{r\text{max}}}{2L_s} \right)^2 \]  

(12)

and due to the maximum stator current, boundaries are given by

\[ P_s^2 + Q_s^2 \leq \left( \frac{3u_L I_{s\text{max}}}{2} \right)^2 \]  

(13)

In the overexcited mode reactive power limits are set by rotor current and in the underexcited mode reactive power limits are set by stator current. The reactive power boundaries set by rotor current depends on mutual flux saturation. For accurate calculation of the reactive power limits, the variation of mutual inductance due to saturation should be considered. Overexcited mode denotes that as the main flux is increased, more reactive rotor current is required to provide a given reactive stator current compared to unsaturated condition. During under excited mode, reactive current capability is not affected by the mutual flux saturation as the stator current determines the reactive power limitations.

B. Rotor Current

The steady-state equivalent circuit of doubly fed induction generator shown in Fig. 5 is used to calculate the rotor current for a given power output and voltage [6].

\[ I_r = \frac{V_s}{R_s + j(X_{ds} + X_{ms}) - I_s} Z_{req} + jX_{ds} \]  

(14)

\[ Z_{req} = \frac{jX_{ms}(R_s + jX_{ds})}{R_s + j(X_{ms} - X_{ds})} \]  

(15)

This shows that rotor current \( I_r \) is dependent on the value of magnetizing reactance which in turn depends on the degree of saturation in doubly fed induction generator. Hence, for short circuit calculations prefault rotor current depends on the saturation of the machine.

C. Stability limits of DFIG

Small-signal stability of power systems can be analyzed by observing the eigenvalues of the state model of doubly fed induction generator. In this section the effect of machine inductances on the eigenvalues of the open-loop DFIG are observed. The machine inductances which depends on the saturation have impact on the stability of open loop DFIG.

D. Transient Performance of DFIG

The effect of saturation on the performance of DFIG is studied by analyzing a grid connected DFIG subjected to various transients such as short circuit and voltage sag.

The expression of stator and rotor currents as function of fluxes is given by [7]:

\[ \psi_r = -k_r \frac{1}{L_s} \psi_s + \frac{1}{L_s} \psi_s \]  

(16)

\[ \psi_s = \frac{L_{ds}}{L_{rs}} \psi_r - k_s \frac{1}{L_s} \psi_s \]  

(17)

where \( k_s = L_m/\omega_m \), \( k_r = L_r/\omega_m \), \( L_s = L_{sg} + L_{sr} \)

\[ \psi_s = L_{ds} \psi_r - \frac{L_{ds} L_m}{L_{rs}} \psi_s \]  

(18)

Short Circuit

Short circuit is one of the important reasons for interruption in the power system. As the stator voltage reduces to zero, the stator flux gradually reduces from the prefault value to zero, inducing a transient emf in the rotor windings. This transient stator flux can be expressed as:

\[ \psi_s = \psi_{s\text{q}0} e^{-t/T_s} \]  

(19)

This transient stator flux is a dc flux that decays with a time constant determined by stator parameters of DFIG as follows:

\[ T_s = \frac{L_{rs} L_{ds}}{R_s} \]  

(20)

The term \( L_{rs} = L_s - (L_{ms}^2/L_s) \) is similar to the transient inductance of a synchronous machine and is denoted as \( L_{\text{tr}} \):

\[ L_{tr} = \frac{L_{ds} L_m}{L_{rs} + L_m} \]  

(21)

This transient emf acts on the rotor windings to produce a transient rotor current of angular frequency corresponding to rotor speed.

At steady state, the stator current induces mutual flux in the rotor. As the short circuit occurs, a transient rotor flux evolves in the rotor winding to maintain constant flux. This transient rotor flux \( \psi_r \) also rotates with angular velocity of the rotor. Thus, in a fixed reference frame, the rotor flux is expressed by

\[ \psi_r = \psi_{r\text{q}0} e^{-t/T_s} e^{-j\omega_m t} \]  

(22)

This flux is fixed with respect to rotor reference frame and decreases exponentially with time constant determined by the parameters of rotor windings is given by:

\[ T_r = \frac{L_{tr}}{R_r} \]  

(23)

where

\[ L_{tr} = L_{rs} + \frac{L_{ds} L_m}{L_{rs} + L_m} \]  

As with respect to stationary reference frame, this rotor flux rotates with the angular frequency corresponding to rotor speed \( \omega_m(t) \). Ignoring mechanical transients, the rotor flux is expressed by:

\[ \psi_r = \psi_{r\text{q}0} e^{-t/T_r} e^{-j\omega_m t} \]  

(24)
The stator short circuit current can be expressed by:

$$\tilde{I}_s = \frac{\tilde{V}_s}{j\omega_s L_s} e^{-t/T_s} - \frac{k_{r}}{L_s} \tilde{\psi}_m e^{-t/T_{cm}} e^{-j\omega_m t}$$  \hspace{1cm} (22)

The initial value of this transient stator short circuit current is determined by the magnitude of stator flux at the instant of short circuit and this initial flux in turn depends on the operational reactance of the stator circuit. The rate at which stator short circuit current decays depends on time constant $T_s$ which is determined by the stator parameters of the generator. The transient time constants determine the dynamic electromagnetic phenomena in electric machines. It is the rate at which magnetic flux linkage varies in stator and rotor windings of induction machine. Hence, these time constants are of great significance in determining the dynamic performance of doubly fed induction generator under varying electromagnetic conditions.

**Voltage Sag**

According to the IEEE definitions for power quality, the momentary voltage sag is defined as a decrease in the rms value of voltage or current at the power frequency between 0.1 and 0.9pu for durations of 0.5s to 3s. The steady state flux completely depends on the stator voltage. The stator flux before and during the voltage sag by neglecting stator resistance is expressed as [24]:

$$\tilde{\psi} = \begin{cases} 
\frac{V}{j\omega_s} e^{j\omega_s t} & \text{for } t < t_0 \\
\left(1-q\right)\frac{V}{j\omega_s} e^{j\omega_s t} & \text{for } t \geq t_0 
\end{cases}$$  \hspace{1cm} (23)

where $V$ is prefault value of rms grid voltage and $q$ is percentage voltage drop.

The steady state flux or forced flux at the moment of voltage sag depends on the grid voltage, thus $1-q$ times the prefault voltage. According to constant flux linkage theorem, the transient flux guarantees that no discontinuities appear in the magnetic state of the machine when there is a change in voltage. The natural flux $\tilde{\psi}_{no}$ is estimated from the initial conditions (at $t = t_0$) since the stator flux has to remain constant before and after the fault.

$$\tilde{\psi}_{sf}(t_0^-) = \tilde{\psi}_{sf}(t_0^+)$$  \hspace{1cm} (24)

$$\tilde{\psi}_{sf}(t_0^-) = \tilde{\psi}_{sf}(t_0^+) + \tilde{\psi}_{st}(t_0^-)$$  \hspace{1cm} (25)

The stator flux during the voltage sag is expressed by:

$$\tilde{\psi}_s(t) = \frac{1}{j\omega_s} \left(1-p\right) e^{j\omega_s t} + \frac{pV}{j\omega_s} e^{-t/T_s}$$  \hspace{1cm} (26)

This equation shows that the stator flux contains a rotational term that is decided by the stator voltage and a constant term due to the transient mode.

**IV. NUMERICAL INVESTIGATIONS**

The effect of saturation on doubly-fed induction generator has been evaluated by considering a grid connected doubly fed induction generator under steady state and transient conditions. The steady state stability is analyzed by observing the eigen values of the state space model of doubly fed induction generator. For, stable steady state operation, all eigenvalues $\lambda = \sigma \pm j\omega$ must be in the left half plane (LHP) i.e. $\sigma < 0$. The transient conditions considered here are voltage sag and short circuit. The effect of the transients is calculated analytically using saturated and unsaturated value of mutual inductances.

The parameters of DFIG considered here are shown in Appendix.

**A. Reactive Power Capability**

The active-reactive current diagram considering saturation is shown in Fig. 6. It is found that in the over excited mode maximum stator reactive current is influenced by magnetic saturation [23].

![Fig. 6 Reactive current limits](image)

**B. Steady State Rotor Current**

Steady state rotor current depends on the value of magnetizing reactance. Here, rotor current is calculated using unsaturated and saturated values of magnetizing reactance. The steady state rotor current calculated for both cases of magnetizing reactances is shown in Table I. This value of rotor current is used as the prefault rotor current for short circuit calculations.

<table>
<thead>
<tr>
<th>Case</th>
<th>$X_m = X_{mm}$</th>
<th>$X_m = X_{ms}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>$0.67 \angle -58.17$</td>
<td>$0.85 \angle -64.63$</td>
</tr>
</tbody>
</table>

**C. Stability of Open loop DFIG**

In small-signal studies the dynamic equations of DFIG (1) are linearized around an operation point $V_s = 1pu$ and slip $s=0.2$ and the eigenvalues of the state matrix obtained without and with saturation is shown in Table II. The effect of saturation is to reduce the magnetizing flux which in turn reduces the electromagnetic torque. Hence the system instability is due to the lack of electromagnetic torque.

<table>
<thead>
<tr>
<th>Without saturation</th>
<th>With saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_\Delta$</td>
<td>$-22.32 + j386.68$</td>
</tr>
</tbody>
</table>
D. Transient Performance of DFIG

Short Circuit

The transient per phase stator current plotted using unsaturated and saturated value of magnetizing inductances during a total voltage dip for open circuited rotor is shown in Fig.7. As the machine goes to saturation, magnetizing inductance decreases. The decrease in magnetizing inductance lead to smaller value of time constant. Hence, the transitory flux will decay much faster compared to unsaturated operating point.

![Fig. 7 Stator current after short circuit](image)

**Voltage Sag**

In this investigation the pre-fault value of the grid voltage is taken as 1 pu. A voltage sag of 70% is assumed to happen. The impact of saturation on the performance of doubly fed induction generator subjected to short circuit is shown in Fig. 8.

![Fig. 8 Stator current after voltage sag](image)

V. CONCLUSION

The impact of main flux saturation on steady state and transient performances of doubly fed induction generator has not obtained much attention so far. But, in real time scenario, generators are operated in saturation, so the nonlinearity should be considered for precise prediction of machine performance. This paper theoretically studied the impact of main flux saturation on the transient responses of a DFIG under voltage sag and short circuit at the generator terminals. Based on this theoretical study, following conclusions has been deducted.

1) It is also found that reactive power capability and rotor current needed for a given power output and voltage is influenced by the magnetic flux saturation.

2) In steady state analysis, the rotor current is influenced by magnetizing reactance.

3) The effect of magnetic flux saturation has noticeable effect on the stability of open loop DFIG.

4) The effect of magnetic flux saturation has noticeable effect on the stability of open loop DFIG.

5) The nonlinear model shows more damping than the linear model. For a fixed value resistance, the larger the inductance the slower will be the transient time. The stator and rotor transient currents obtained by considering saturation in the main flux paths decay faster than the ones calculated by ignoring the saturation.

### APPENDIX

Parameters of DFIG [6]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (3 phase)</th>
<th>3 pu Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>6,7723 VA</td>
<td>3 pu</td>
</tr>
<tr>
<td>Voltage(L-L)</td>
<td>V_s</td>
<td>230V</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>R_s</td>
<td>0.2178 ohm</td>
</tr>
<tr>
<td>Stator leakage reactance</td>
<td>X_{ls}</td>
<td>0.5319 ohm</td>
</tr>
<tr>
<td>Magnetizing reactance</td>
<td>X_{m}</td>
<td>15.34 ohm</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>R_r</td>
<td>0.2068 ohm</td>
</tr>
<tr>
<td>Rotor leakage reactance</td>
<td>X_{lr}</td>
<td>0.5319 ohm</td>
</tr>
</tbody>
</table>
Poles | 4
Stator/Rotor turns ratio | 1

REFERENCES


