Improved Spectral Characteristics of Bandpass FIR Filter using a Novel Adjustable Window Function

P.Kamala Kumari and Dr.J. Beatrice Seventline

Abstract— The window method is a very simpler and popular procedure to design a Finite impulse response (FIR)filter. Of the existing window techniques, the Kaiser window outperforms in main lobe width but fails in computational complexity and higher relative side lobe attenuation. This paper presents a novel adjustable window function formed by combining Gaussian window, Lanczos window and Dolph-Chebyshev window (GLC) with a controlling parameter to meet the specifications for various applications. This parameter adjusts the spectral characteristics and shape of the window function according to the desire of the designer. The spectral characteristics of the proposed window have been analyzed and its performance has been compared with Gaussian Hann and Lanzcos Blackman windows. From MATLAB simulation results, it is observed that the proposed GLC window results in greater performance with respect to relative side lobe attenuation. Furthermore, it has been used to design a FIR bandpass filter to justify its performance and their comparative analysis is presented. The FIR bandpass filter designed with the proposed GLC window provided better results in terms of ripple ratio.

Keywords— Dolph-Chebyshev window, Gaussian window, Lanczos window, spectral characteristics. Window function.

I. Introduction

DIGITAL filters are extensively used in a variability of applications. Mostly, the final goal is to filter the input signal to obtain a kind of frequency selectivity on the spectrum. Digital filters are broadly classified as: the recursive filter and the non-recursive filter. These are frequently stated as infinite impulse response (IIR) filters and finite impulse response (FIR) filters, respectively[1].

For designing digital FIR filters, one of the simplest and convenient method is window method. A window is sequence of coefficients that satisfy certain requirements and is finite in nature[2]. In this procedure an ideal frequency -selective filter is chosen and its impulse response is obtained which is infinite. It is essential to make this infinite impulse response finite. In order to

obtain this, the ideal response must be truncated.

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The desired frequency response $H_d(e^{j\omega})$ and its corresponding impulse response $h_d(n)$ are determined using the relation (1)[3]

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n} d\omega \quad (1)$$

The infinite duration impulse response $h_d(n)$ is made finite by multiplying with a window function $w_f(n)$ of finite length N given by (2)

$$h(n) = h_d(n) \times w_f(n) \quad (2)$$

where

$$w_f(n) = \begin{cases} w(n), & 0 \le n \le N \\ 0, & Otherwise \end{cases} \quad (3)$$

The two desired specifications of a window function are lesser main lobe width and ripple ratio. But these specifications are contrary. A window with a thinner main lobe has a poor side lobe rejection and vice versa.[4]. There are many window functions like hamming window, Hanning window, Blackman window and adjustable window functions like Kaiser window, Gaussian window, Dolph-chebyshev filter and Lanczos window.

In literature, to achieve better performance, the combination of different windows was done. A new variable window function obtained by a combination of Lanczos and Blackman window was proposed by Tapash et al.(2017)[5] and its spectral performance was compared with Gaussian and Kaiser windows. It is observed that the main lobe width is almost same as that of the other windows but the ripple ratio is smaller than Gaussian and Lanczos window. A product of Gaussian and Hanning window was proposed by Vivek Kumar et al.[6] and its spectral performance was compared with Kaiser window. Mitun Shil et al.[7] proposed a adjustable window in combination with tan hyberbolic function and a weighted cosine series. The results are compared with hamming and Kaiser window. The proposed window[7] has a good linear phase and 0% leakage factor. Hrishi Rakshit et al.[8] given comaprision of various window functions for designing a low pass filter. [8] stated that Kaiser window is the most supercilious among other windows in terms of main lobe width and Dolph-Chebyshev window shows better response in stop band attenuation. From the literature, it is observed that out of all the windows, Kaiser window outperforms other windows in terms of main lobe width, but is computationally complex due to the calculation of Bessel functions.

In this paper, a Novel adjustable window function has been proposed as combination of three windows i.e., Gaussian, Lanczos and Dolph-Cbeyshev which yields better performance with less complexity. This paper compares the performance of the proposed GLC window with the window functions introduced in [5] and [6] and further for designing a FIR bandpass filter. The proposed GLC window results superior performance with respect to relative side lobe attenuation. The FIR bandpass filter designed with the proposed GLC window gives better results in both ripple ratio and side lobe roll off ratio. This paper is arranged as follows.

Section II defines the spectral properties of a typical window function. Section III gives the explanation of the proposed GLC window and its spectral performance for diverse values of controlling parameter, r. Section IV demonstrates the comparison of the proposed GLC window with other variable window functions. Section V illustrates the performance of FIR bandpass filter designed using the proposed GLC window function and its spectral characteristics are compared with other adjustable window functions. Section VI gives the conclusion of the paper.

II. Spectral Properties of Window function for FIR filter design

The normalized amplitude spectrum of a typical window in dB range is shown in the Fig.1.[9]

The spectral characteristics that are frequently used to distinguish the performance of the window function are Ripple ratio (R), Main lobe width (Mw) and side lobe roll-off ratio, S. These parameters can be defined from the Fig.1 as

Ripple ratio, RR= $S_1 - 0 = S_1$.

Main lobe width, Mw = $2 \times W_R$.

Side lobe roll-off ratio = = S_1 -S_L.

Where,

S_1 = Maximum side lobe amplitude in dB

S_L = Main lobe amplitude in dB

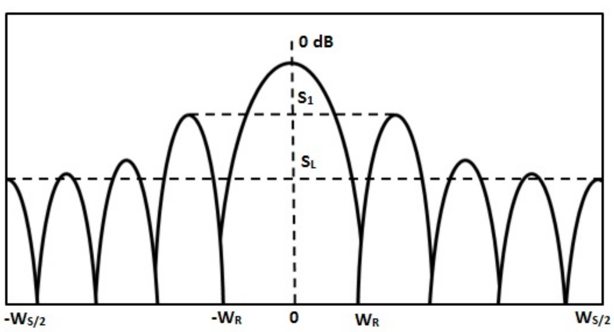

Fig.1: A typical window's normalized amplitude spectrum.

The main lobe width determines the transition width between the passband and stopband, the ripple ratio affects the ripples in the passband and stopband and the side-lobe roll-off ratio determines the distribution of the stopband energy. The desired properties at window amplitude spectrum are narrow width, small ripple ratio and large side-lobe roll-off ratio.For a given length, N of window, narrower main lobe, the ripple ratio increases. Therefore, there is a trade-off between these parameters [8].

III. Proposed GLC Window

The proposed GLC window is a combination of Gaussian ,Lanczos and Dolph Chebyshev window functions, given in (4),(5) and (6) respectively.

The Gaussian window function takes the form of (4)

$$w_1(n) = e^{\frac{1}{2}(\alpha\frac{n}{N/2})^2} \quad (4)$$

Here, α is inversely proportional to the standard deviation of Gaussian random variable and the value is fixed at α=2.3[10]

The Lanczos window function is given by (5)

$$w_2(n) = sinc^M(\frac{2n}{N} - 1) \quad (5)$$

The value of M is fixed at 2.[5]

The optimal Dolph Chebyshev window[8],[7] transform can be written in closed form (6)

$$w_0(n) = \frac{\cos(Ncos^{-1}[\beta \cos(\frac{\pi k}{N})])}{\cosh[Ncosh^{-1}(\beta)]} \quad (6)$$

where

$$\beta = \cosh[\frac{1}{N}cosh^{-1}(10^{\alpha})] \quad (7)$$

The inverse Discrete Fourier Transform of $w_0(n)$ yields zero phase Dolph-Chebyshev window $w_3(n)$. The parameter α controls the side lobe attenuation.

Mathematically, the proposed GLC window function is expressed as follows

$$w(n) = [w_1(n) * w_2(n) * w_3(n)]^r \quad (8)$$

Here 'r' is the adjustable parameter of the window. It controls the stop-band attenuation and also the shape of the window.

For a window length of N=61, as the value of r increases the window becomes narrower as shown in the Fig. 2.

From Table I it is perceived that as the value of r increases, the main lobe width rises and the relative side lobe attenuation decreases. The same can be observed from Fig. 3 also.

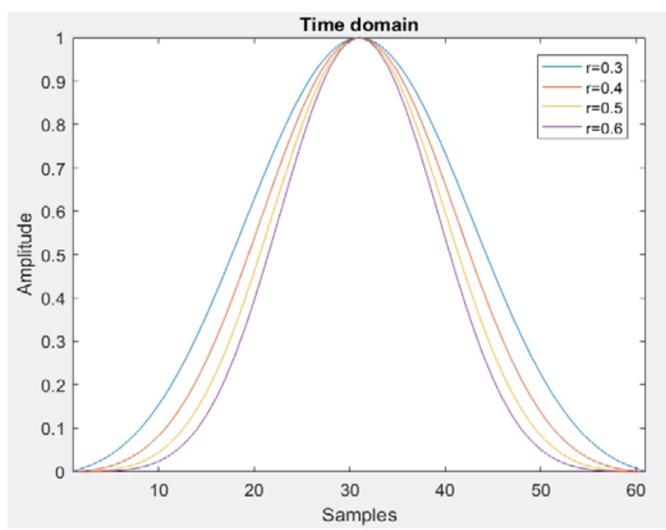

Fig.2: Shapes for proposed GLC window in time domain for different values of r

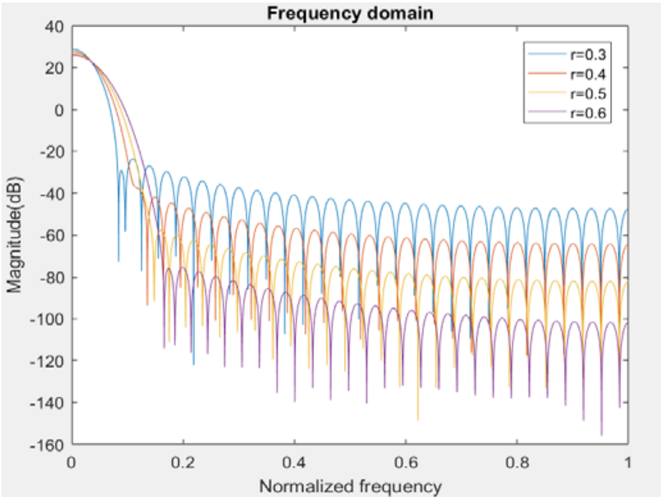

Fig.3: Proposed GLC window spectrum for different values of r

TABLE I: SPECTRAL PARAMETERS OF PROPOSED GLC WINDOW FOR DIFFERENT VALUES OF ɤ

Adjustable parameter, r	Main lobe width	Relative side lobe attenuation(dB)
r=0.3	0.046	-52.3
r=0.4	0.054	-69.6
r=0.5	0.058	-84.4
r=0.6	0.066	-101.3

IV. PERFORMANCE ANALYSIS OF GLC WINDOW

In this section, the spectral characteristics of the proposed GLC window is compared with window function formed as a combination of standard window such as Gaussian, Hanning and Lanczos window functions.

A. Gaussian and Hanning window.

The window function introduced in [2] is the product of Gaussian and Hanning window given as

$$w_4(n) = w_1(n) \times w_5(n) \qquad (9)$$

$w_1(n)$ is Gaussian window for α =2.3 is given by eq(4) and Hanning window [4]is given by(10)

$$w_5(n) = 0.5[\,1 - \cos(\tfrac{2n\pi}{N})] \qquad (10)$$

where the window length is N+1.

Fig. 4 shows that the proposed GLC window with r = 0.5 is slightly narrower than the product of Gaussian and Hanning window with N = 61.

From the Fig 5. it is observed that the proposed GLC window and the product of Gaussian and Hanning window have almost same main lobe width. But the proposed GLC window has better relative side lobe attenuation. Fig. 5 is summarized in Table II.

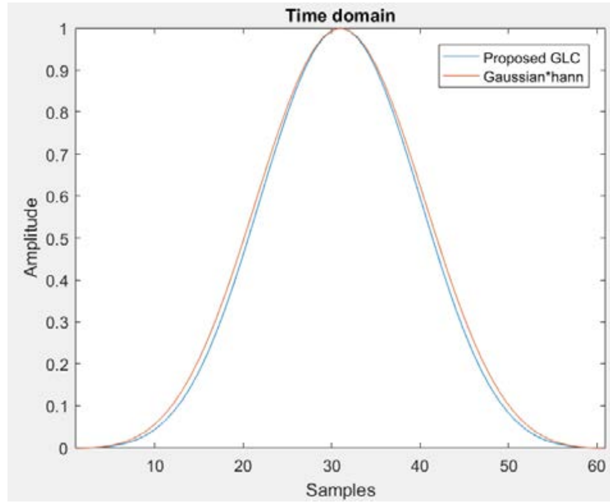

Fig.4: Comparison of proposed GLC window with window gaussian*hann in time domain

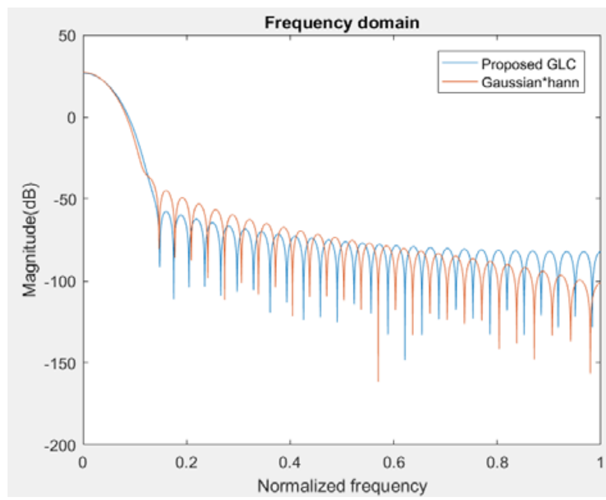

Fig.5: Comparison of proposed GLC window with window gaussian*hann in frequency domain.

TABLE II: COMPARISON OF SPECTRAL PARAMETERS OF PROPOSED GLC WINDOW WITH GAUSSIAN*HANN WINDOW

Window	Main lobe width	Relative side lobe attenuation(dB)
Proposed GLC window	0.058	-84.4
Gaussian*Hann	0.058	-72

B. Lanzcos and Blackman window

The adjustable window function introduced in [5] is a combination of Lanczos and Blackman window given as(11)

$$w_6(n) = [w_2(n)w_7(n)(\tfrac{1}{n}w_2(n) - 1)]^p \qquad (11)$$

Here p= 0.65 is the controlling parameter of the window. $w_2(n)$ is Lanczos window for M=2 is given by eq(5) and $w_7(n)$ is Blackman window is given by(12)

$$w_7(n) = 0.42 - \cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos(\frac{4\pi n}{N-1}) \qquad (12)$$

Fig. 6 shows that the proposed GLC window with r = 0.5 is slightly narrower than the combined Lanzcos and Blackman window with N = 61.

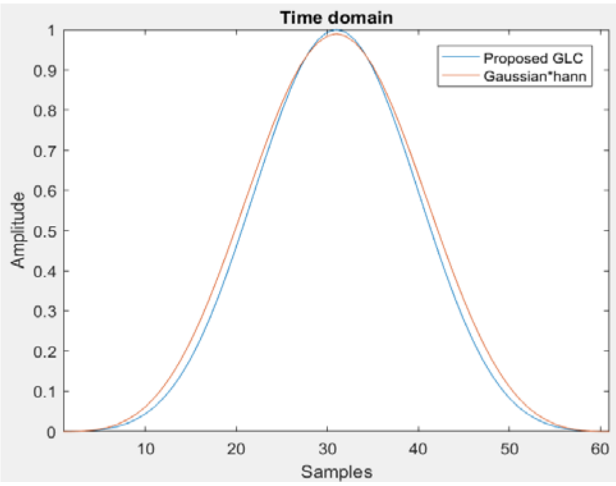

Fig.6: Comparison of proposed GLC window with window Lanczos*Blackman in time domain

From the Fig 7. it is observed that the proposed GLC window and the combination of Lanczos[11] and Blackman window[12]have almost same main lobe width. But the proposed GLC window has relative side lobe attenuation of -84.4dB which is much better. Fig. 7 is summarized in Table III.

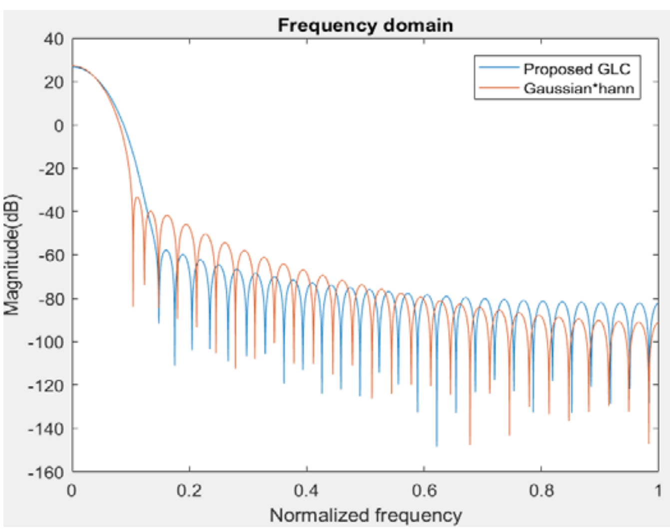

Fig.7: Comparison of proposed GLC window with window Lanczos*Blackman in frequency domain.

TABLE III: COMPARISON OF SPECTRAL PARAMETERS OF PROPOSED GLC WINDOW WITH LANCZOS*BLACKMAN WINDOW

Window	Main lobe width	Relative side lobe attenuation(dB)
Proposed GLC window	0.058	-84.4
Lanczos*Blackman	0.058	-60.5

This section summarizes that the proposed GLC window has better relative side lobe attenuation than the adjustable windows introduced in [5] and [6]. Since the three windows are adjustable, the adjustable parameters are fixed in such a way that for a constant main side lobe width i.e 0.058 and the comparison can be done based on relative side lobe attenuation.

V. APPLICATION

Bandpass filters are widely used in wireless transmitters and receivers, seismology, SONAR even in medical applications like Electrocardiogram, Electroencephalograph. Moreover, applications of digital bandpass filter banks include telecommunications (TDM-FDM translators), speech processing (phase vocoders) signal analysis and detection (spectrum analyzers).

In this section, the proposed GLC window is used to design a FIR bandpass filter and its performance is compared with the other variable windows.

The frequency response of an ideal bandpass filter is given as(13)[2]

$$H_{bp}(e^{j\omega}) = \begin{cases} 1, & \omega_{c1} \le |\omega| \le \omega_{c2} \\ 0, & |\omega| \le \omega_{c1} \text{ and } \omega_{c2} < |\omega| \le \pi \end{cases} \quad (13)$$

Its impulse response $h_{bp}(n)$ is obtained by taking DTFT to give the following:

$$h_{bp}(n) = \begin{cases} \frac{\omega_{c2}-\omega_{c1}}{\pi}, & n = 0 \\ \frac{\sin\omega_{c2}n}{\pi n} - \frac{\sin\omega_{c1}n}{\pi n}, & |n| > 0 \end{cases} \quad (14)$$

The FIR filter coefficients h(n) can be calculated from (14)

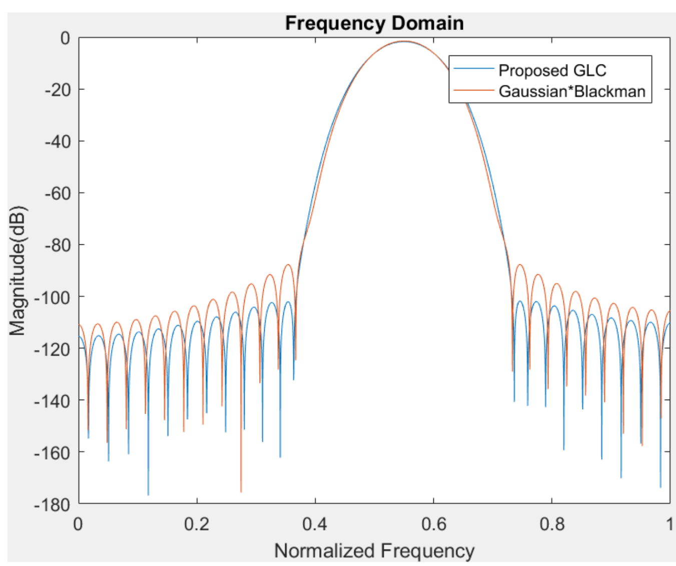

Fig.8: Comparison of spectral characteristics between the proposed GLC window and gaussian*Hann window in case of FIR bandpass filter design.

Fig. 8 and Fig.9 shows the comparison of spectral characteristics of a FIR bandpass filters designed by the proposed GLC window and the other variable windows. These figures are summarized in Table IV.

The simulation results are obtained using MATLAB are listed in Table IV. The FIR bandpass filter is designed with the cutoff frequencies ω_{c1} = 0.5 rad/sec and ω_{c2} = 0.6rad/sec. It is observed that the main lobe width of the proposed GLC window is nearly equal to the other adjustable windows. But the ripple ratio of the proposed GLC window is lesser than the other specified windows. And also the side lobe roll off ratio is better than the other windows.

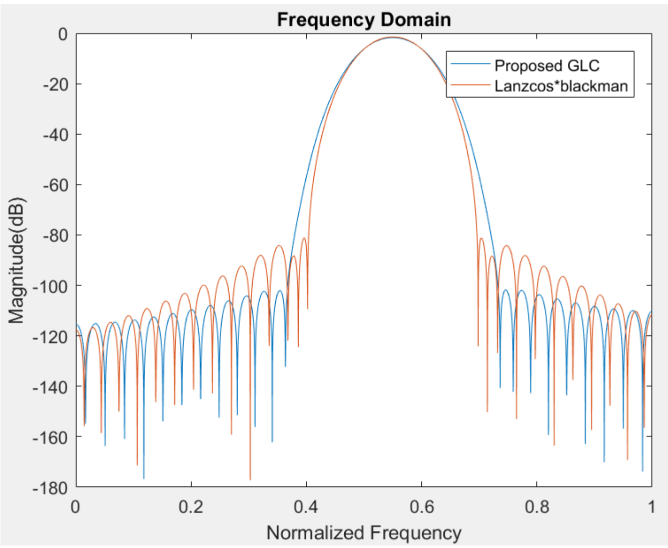

Fig.9: Comparison of spectral characteristics between the proposed GLC window and Lanczos*Blackman window in case of FIR bandpass filter design

TABLE IV: COMPARISON OF FIR BANDPASS FILTER CHARACTERISTICS BETWEEN THE PROPOSED GLC WINDOW BY OTHER ADJUSTABLE WINDOWS

window	Main lobe width	Ripple Ratio (dB)	Side lobe roll off ratio(dB)
Proposed r=0.5	1.993	-102.1	13
Lanczos*Hann	1.996	-81.23	35.6
Gaussian*Blackman	1.995	-87.72	23.08

VI. CONCLUSION

A novel adjustable window function with a controlling parameter that adjusts the spectral characteristics was presented in this paper. From the comparative study it can be concluded that the main lobe width of the proposed GLC window is nearly equal to the other adjustable windows. In order to increase the side lobe attenuation, a novel window which is the product of Gaussian, Lanczos and Dolph-Chebyshev windows has been designed. The relative side lobe attenuation is improved by 14.69% and ripple ratio by 28.3%. It has better relative side lobe attenuation (-84.4dB) than Gaussian Hanning window (-72dB) and Lanczos Blackman window (-60.5dB). The ripple ratio of the proposed GLC window(-102.1dB) is better than the other stated windows (-81.23dB and -87.72dB) but at the cost of lesser side lobe roll off ratio. In future, side lobe roll off ratio can be improved by making the adjustable window into a adaptable one. Further the proposed window can be extended for the design of a narrow band pass filter i.e., anti-notch filter.

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