The Convergence Time of a Blind adaptive Equalizer

Monika Pinchas

Abstract—In a typical communication system, the blind adaptive equalizer is followed by a decision device where we have to wait until the blind adaptive equalizer has converged to a residual inter-symbol interference (ISI) that makes the decision process of the decision device applicable and reliable. Up to now there is no algorithm that supplies without the knowledge of the initial ISI the convergence time of a blind adaptive equalizer where the blind adaptive equalizer leaves the system with a relative low residual ISI that makes the decision process of the decision device applicable. In this paper, we consider the two independent quadrature carrier input case and type of blind adaptive equalizers where the error that is fed into the adaptive mechanism which updates the equalizer's taps can be expressed as a polynomial function of the equalized output up to order three. We propose an algorithm that supplies without the knowledge of the initial ISI the convergence time of a blind adaptive equalizer that depends on the input signal statistics and properties of the chosen equalizer. It should be pointed out that the convergence time is supplied during the deconvolutional process. Simulation results confirm the efficiency of the proposed algorithm.

Keywords—Convergence time, Switching time, Blind Adaptive Equalizer, Deconvolution

I. Introduction

In this paper we consider the blind adaptive equalization issue where the error that is fed into the adaptive mechanism which updates the equalizers taps can be expressed as a polynomial function of the equalized output up to order three as is the case in the multimodulus blind equalization algorithm (MMA) [1], [2], [3]. Usually, the equalized output is sent to a decision device in order to get the desired result. But, in order to get a reliable output from the decision device, the equalized output driven to the decision device, should contain a relative low residual ISI that makes the decision process of the decision device applicable and reliable. Thus, first we have to wait for the blind adaptive equalizer to converge to the residual ISI that makes the decision process of the decision device applicable and reliable and only then starting with the decision process. This means, that having the convergence time of a blind adaptive equalizer could be very useful for making the right decision whether starting with the the decision process of the decision device or not. In the literature we may find the use of dual-mode (DM) switching methods where an acquisition algorithm (like MMA or constant modulus algorithm (CMA) [4]) is employed to ensure symbol error rate convergence during the initial acquisition state and then switched to a tracking algorithm (decision directed (DD) algorithm) when the output decision error rate is sufficiently low to achieve an accurate final solution [5]. According to [5], some examples are the classical Benveniste-Goursat (BG) algorithm [6], the dual-mode CMA (DM-CMA) [7], and its stop-and-go (SAG) extension SAG-DM-CMA [8].

Although the DM algorithm switches in to modes, it never stops adjusting the equalizer tap weights even when the adjustment is in the wrong direction [9]. If we can tell whether a particular adjustment is correct or not, we may improve the convergence behaviour by making only the right adjustment but bypassing those wrong ones [9]. Such a concept has been applied to blind equalization and is termed ”stop-and-go” [12], [11], [10], [9]. In the literature we have the work of [13] where a closed-form approximated expression was given for the convergence time (or number of iterations required for convergence). But, the proposed expression in [13] is based on the knowledge of the initial ISI which in general is unknown. Thus, this expression [13] is unable to help achieving an improved DM algorithm.

In this paper, we propose a novel algorithm (based on [14]) that supplies without the knowledge of the initial ISI the convergence time of a blind adaptive adaptive equalizer that depends on the input signal statistics and properties of the chosen equalizer. This new algorithm may lead in the future to a new family of DM algorithm where the switching to a tracking algorithm (DD algorithm) will be based on the convergence time of the blind adaptive equalizer.

The paper is organized as follows: After having described the system under consideration in Section II, we present in Section III the algorithm for the convergence time that depends on the input signal statistics and properties of the chosen equalizer. In Section IV simulation results are presented and the conclusion is given in Section V.

II. System Description

The system under consideration is described in Fig.1 with the assumptions following [14]:

1. The input sequence $x[n]$ belongs to a two independent quadrature carrier case constellation input with zero mean and variance σ_x^2 where $x_r[n]$ and $x_i[n]$ are the real and imaginary parts of $x[n]$ respectively.

2. The unknown channel $h[n]$ is a possibly nonminimum phase linear time-invariant filter in which the transfer function has no “deep zeros”, namely, the zeros lie sufficiently far from the unit circle.

3. The equalizer $c[n]$ is a tap-delay line.

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4. The noise $w[n]$ is an additive Gaussian white noise with zero mean and variance $\sigma_w^2 = E[w[n]w^*[n]]$ where $E[\cdot]$ is the expectation operator and $()^*$ is the conjugate operator on $()$.

The equalized output sequence is defined by:

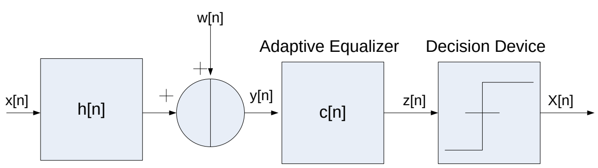

Fig. 1. Block diagram of a baseband communication system.

$$z[n] = (x[n] * h[n] + w[n]) * c[n] = x[n] + p[n] + \tilde{w}[n] \quad (1)$$

where "$*$" denotes the convolution operation, $p[n]$ is the convolutional noise (convolutional error) due to non-ideal equalizer's coefficients ($h[n] * c[n] \neq \delta[n]$) and $\tilde{w}[n] = w[n] * c[n]$. The equalized output sequence $z[n]$ is driven to the decision device. The output sequence of the decision device is denoted as $X[n]$. The adaptation mechanism of the equalizer is given by:

$$c_l[n+1] = c_l[n] - \mu \frac{\partial F(z[n])}{\partial z[n]} y^*[n-l] \quad (2)$$

where $l = 0, 1, 2..., (N-1)$, N is the equalizer's tap length, μ is the step-size parameter and $\frac{\partial F(z[n])}{\partial z[n]}$ is considered in this paper as was done in [14] as:

$$\frac{\partial F(z[n])}{\partial z[n]} = Re\left(\frac{\partial F(z[n])}{\partial z[n]}\right) + jIm\left(\frac{\partial F(z[n])}{\partial z[n]}\right)$$

$$Re\left(\frac{\partial F(z[n])}{\partial z[n]}\right) = a_1 z_r[n] + a_3 z_r^3[n] \quad (3)$$

$$Im\left(\frac{\partial F(z[n])}{\partial z[n]}\right) = a_1 z_i[n] + a_3 z_i^3[n]$$

where $z_r[n]$ and $z_i[n]$ are the real and imaginary parts of $z[n]$ respectively. The constants a_1 and a_3 depend on the chosen algorithm.

III. The Algorithm for the Convergence Time

It is well known ([14], [15], [16], [17]) that the convolutional noise probability density function (pdf) can be considered as Gaussian when the equalizer reaches a residual ISI level where the eye diagram can be considered as open. Namely, where relative reliable decisions can be carried out by the decision device. Recently, [14] considered the two independent quadrature carrier input case and type of blind adaptive equalizers where the error that is fed into the adaptive mechanism which updates the equalizers taps can be expressed as a polynomial function of the equalized output up to order three. A closed-form approximated expression was proposed in [14] for the step-size parameter that shows what are the constraints on the systems parameters (equalizers tap-length, input signal statistics, channel power and chosen equalization method) for which the assumption of a Gaussian model for the convolutional noise at the convergence state holds:

$$\mu \ll \frac{2\left|a_1 + 3a_3 n_2\right|}{3\left(\sigma_x^2 N \sum_{k=0}^{R-1} \left|h_k[n]\right|^2\right)\left|a_1^2 + 12a_3 a_1 n_2 + 15a_3^2 n_4\right|} \quad (4)$$

where $n_a = E[x_r^a[n]]$ $(a = 2, 4)$, $|(\cdot)|$ stands for the absolute value of $(\cdot)$ and R is the channel coefficient's length. Now, for the noiseless case we have that the equalized output ($z[n] = x[n] + p[n]$) reaches the sent sequence ($x[n]$) when the convolutional noise ($p[n]$) tends to zero. Thus, if we consider the following expression:

$$err = (\mu_p - \mu_x)^2 = (error)^2$$

with

$$\mu_p = \frac{2\left|a_1 + 3a_3 \tilde{n}_2\right|}{3\left(\sigma_x^2 N \sum_{k=0}^{R-1} \left|h_k[n]\right|^2\right)\left|a_1^2 + 12a_3 a_1 \tilde{n}_2 + 15a_3^2 \tilde{n}_4\right|}$$

$$\mu_x = \frac{2\left|a_1 + 3a_3 n_2\right|}{3\left(\sigma_x^2 N \sum_{k=0}^{R-1} \left|h_k[n]\right|^2\right)\left|a_1^2 + 12a_3 a_1 n_2 + 15a_3^2 n_4\right|} \quad (5)$$

where $\tilde{n}_a = \frac{1}{L}\sum_{b=0}^{b=L-1} z_r^a[n-b]$ and search during the equalization process the iteration number for which the normalized err (5) achieves in the average a small predefined threshold ε, we may get the convergence time (expressed in iteration number) of the equalizer.

IV. Simulation

In this section, we show via simulation results the efficiency of the proposed algorithm (5) for the 16QAM constellation input (a modulation using $\pm$ {1,3} levels for in-phase and quadrature components) and with the MMA algorithm [1], [2], [3]. The equalizer taps for the MMA algorithm [1], [2], [3] were updated according to:

$$c_l[n+1] = c_l[n] - \mu_{MMA}\left(a_{1_{MMA}} z_r[n] + a_{3_{MMA}} z_r^3[n] + j\left(a_{1_{MMA}} z_i[n] + a_{3_{MMA}} z_i^3[n]\right)\right) y^*[n-l] \quad (6)$$

where

$$a_{1_{MMA}} = -\frac{E[x_r^4[n]]}{E[x_r^2[n]]} = -\frac{E[x_i^4[n]]}{E[x_i^2[n]]}; \qquad a_{3_{MMA}} = 1 \quad (7)$$

Three channel cases were considered.

Channel1: (initial ISI $= 0.44$) where the channel parameters were determined according to [18]:

$h_n = (0 \quad \text{for} \quad n < 0; \quad -0.4 \quad \text{for} \quad n = 0;$

$0.84 \cdot 0.4^{n-1}$ for $n > 0$).

Channel2: (initial ISI $= 0.88$) where the channel parameters were determined according to [15]:
$h_n = [0.4851 - 0.72765 - 0.4851]$

Channel3: (initial ISI $= 0.5$) where the channel parameters were determined according to [19]:
$h_n = [-0.0144 \;\; 0.0006 \;\; 0.0427 \;\; 0.009 \;\; -0.4842 \;\; -0.0376 \;\; 0.8163 \;\; 0.0247 \;\; 0.2976 \;\; 0.0122 \;\; 0.0764 \;\; 0.011 \;\; 0.0162 \;\; 0.0063]$

For Channel1, Channel2 and Channel3, the equalizer's tap-length was set to 13, 13 and 21 respectively. The equalizer was initialized by setting the center tap equal to one and all others to zero. The simulation was carried out with SNR values of 30 and 20 dB. In order to calculate $\tilde{n}_a$, we used $L = 300$ unless otherwise stated. The threshold was set to $\varepsilon = 0.005$. At each simulation we used 100 Monte Carlo trials where at each trial we captured the iteration number for which the normalized err (5) reached $\varepsilon = 0.005$. In the following we denote the averaged iteration number (calculated out from 100 Monte Carlo trials) for which the normalized err (5) reached $\varepsilon = 0.005$ as "average-convergence-time".

Figure 2 describes the averaged ISI as a function of iteration number for a 16QAM source input going through channel1 for the $SNR = 30$ dB case. It should be pointed out that at approximately -16 dB the eye diagram starts to be open thus relative reliable decisions can be carried out by the decision device. According to figure 2, the residual ISI of -16 dB is reached at approximately 400 iteration number. Please note that the calculated average-convergence-time was 457 which is very close to that obtained from figure 2. Figure 3 describes the averaged normalized err (5) as a function of iteration number for a 16QAM source input going through channel1 for the $SNR = 30$ dB case. The zoomed in version of figure 3 is given in figure 4 where we can see that around the iteration number of 475, the threshold value of $\varepsilon = 0.005$ is reached. Thus the calculated average-convergence-time is very close to the obtained result from figure 4.

Next we turn to the $SNR = 20$ dB case. Figure 5 describes the averaged ISI as a function of iteration number for a 16QAM source input going through channel1 for the $SNR = 20$ dB case. According to figure 5, the residual ISI of -16 dB is reached at approximately 480 iteration number. Please note that the calculated average-convergence-time was 520 which is very close to the obtained result from figure 5. Figure 6 describes the averaged normalized err (5) as a function of iteration number for a 16QAM source input going through channel1 for the $SNR = 20$ dB case. The zoomed in version of figure 6 is given in figure 7 where we can see that around the iteration number of 520, the threshold value of $\varepsilon = 0.005$ is reached. Thus the calculated average-convergence-time is very close to the obtained result from figure 7.

Next we turn to test the algorithm with channel2. Figure 8 describes the averaged ISI as a function of iteration number for a 16QAM source input going through channel2 for the $SNR = 30$ dB case. According to figure 8, the residual ISI of -16 dB is reached at approximately 580 iteration number. Please note that the calculated average-convergence-time was 579 which is very close to the obtained result from figure 8. Figure 9 describes the averaged normalized err (5) as a function of iteration number for a 16QAM source input going through channel2 for the $SNR = 30$ dB case. The zoomed in version of figure 9 is given in figure 10 where we can see that around the iteration number of 600, the threshold value of $\varepsilon = 0.005$ is reached. Thus the calculated average-convergence-time is very close to the obtained result from figure 10.

Next we turn to test the algorithm with channel3. Figure 11 describes the averaged ISI as a function of iteration number for a 16QAM source input going through channel3 for the $SNR = 30$ dB case. According to figure 11, the residual ISI of -16 dB is reached at approximately 1020 iteration number. Please note that the calculated average-convergence-time was 847 which is not far away from the obtained result from figure 11. Figure 12 describes the averaged normalized err (5) as a function of iteration number for a 16QAM source input going through channel3 for the $SNR = 30$ dB case. The zoomed in version of figure 12 is given in figure 13 where we can see that around the iteration number of 925, the threshold value of $\varepsilon = 0.005$ is reached. Thus the calculated average-convergence-time is not far away from the obtained result from figure 13.

In the following, we increase for the previous case, the value for L and calculate again the average-convergence-time for each new value for L. Please note that L means how many samples we take for calculating $\tilde{n}_a$. On one hand we wish to have a low value for L since this means that we need less samples to store (we need less memory). But on the other side, if accuracy is more important then we wish to have a higher value for L. For $L = 400$ the calculated average-convergence-time was 893. For $L = 500$ the calculated average-convergence-time was 944. For $L = 600$ the calculated average-convergence-time was 978. For $L = 700$ the calculated average-convergence-time was 1055. We see indeed that if we use $L = 700$ instead of $L = 300$ then we are much closer to the obtained result from figure 11 for reaching the residual ISI of -16 dB (1020 iteration number).

V. Conclusion

In this paper, we proposed a novel algorithm that supplies without the knowledge of the initial ISI the convergence time of a blind adaptive equalizer that depends on the input signal statistics and properties of the chosen equalizer. This algorithm is valid for the real or two independent quadrature carrier input case and type of blind adaptive equalizers where the error that is fed into the adaptive mechanism which updates the equalizers taps can be expressed as a polynomial function of the equalized output up to order three. With this new algorithm we are now able to know the approximated time where the blind adaptive equalizer leaves the system with a relative low residual ISI that makes the decision process of the decision device applicable and reliable.

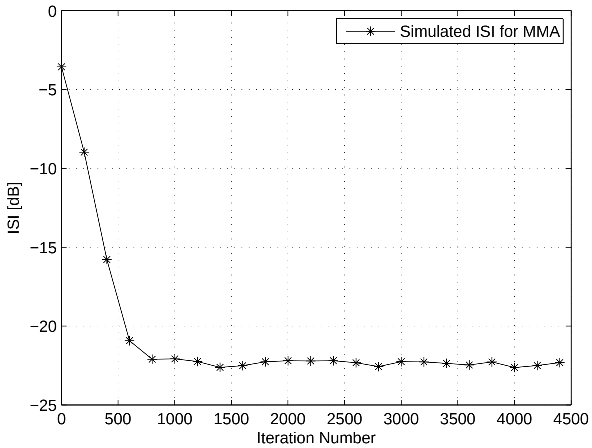

Fig. 2. ISI as a function of iteration number for a 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.0001$.

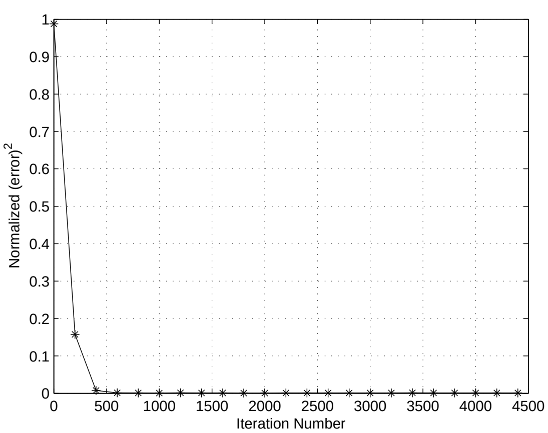

Fig. 3. Normalized err (5) as a function of iteration number for a 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.0001$.

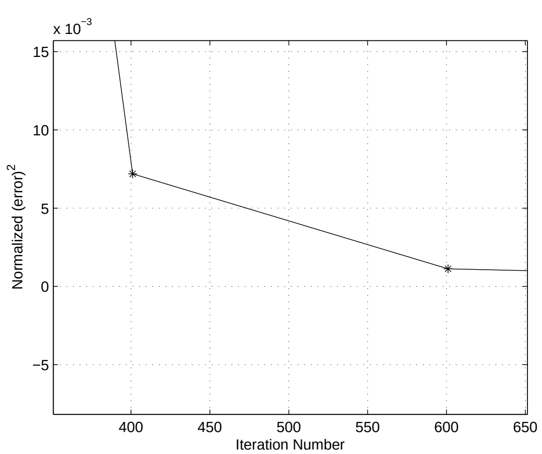

Fig. 4. Normalized zoomed in version for the err (5) as a function of iteration number for a 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.0001$.

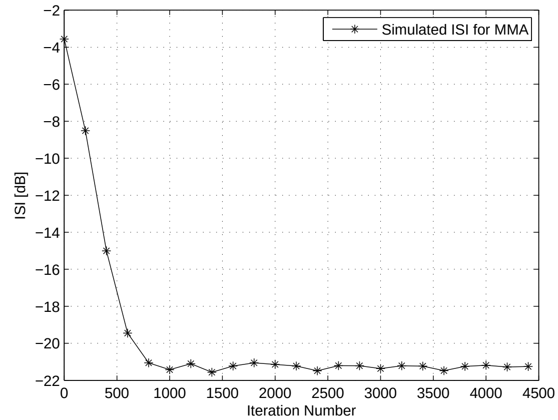

Fig. 5. ISI as a function of iteration number for a 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for a SNR=20 dB. $\mu_{MMA} = 0.0001$.

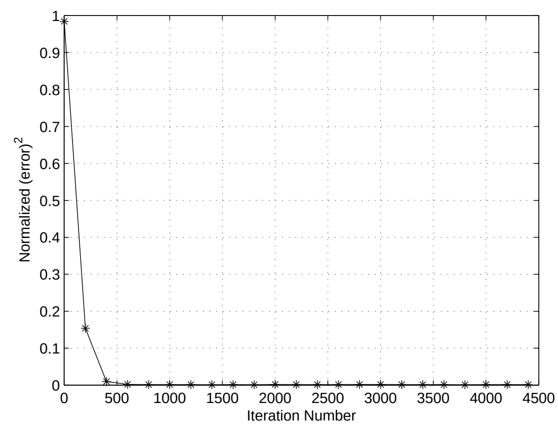

Fig. 6. Normalized err (5) as a function of iteration number for a 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for a SNR=20 dB. $\mu_{MMA} = 0.0001$.

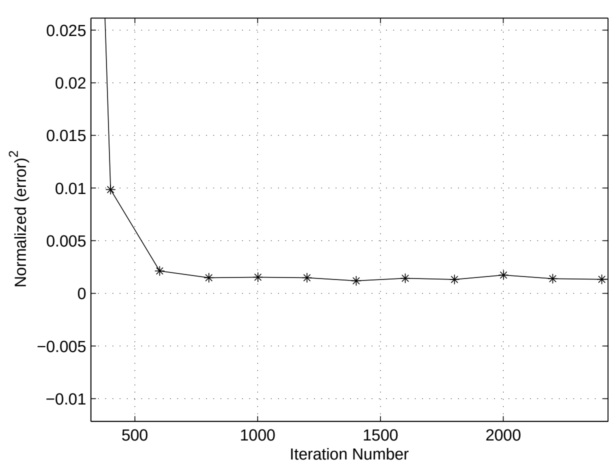

Fig. 7. Normalized zoomed in version for the err (5) as a function of iteration number for a 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for a SNR=20 dB. $\mu_{MMA} = 0.0001$.

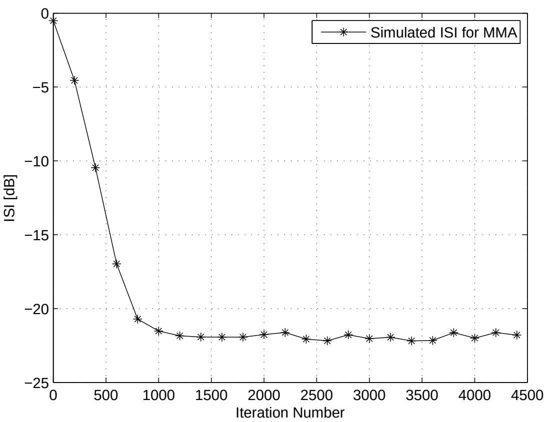

Fig. 8. ISI as a function of iteration number for a 16QAM source input going through channel2. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.0001$.

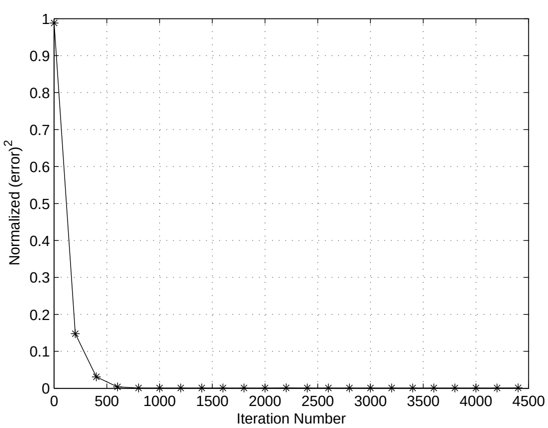

Fig. 9. Normalized err (5) as a function of iteration number for a 16QAM source input going through channel2. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.0001$.

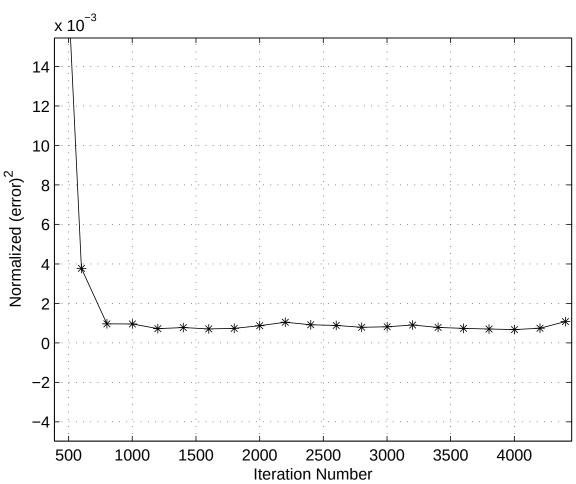

Fig. 10. Normalized zoomed in version for the err (5) as a function of iteration number for a 16QAM source input going through channel2. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.0001$.

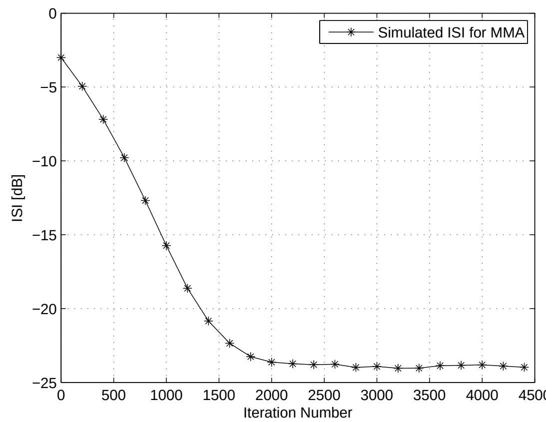

Fig. 11. ISI as a function of iteration number for a 16QAM source input going through channel3. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.00004$.

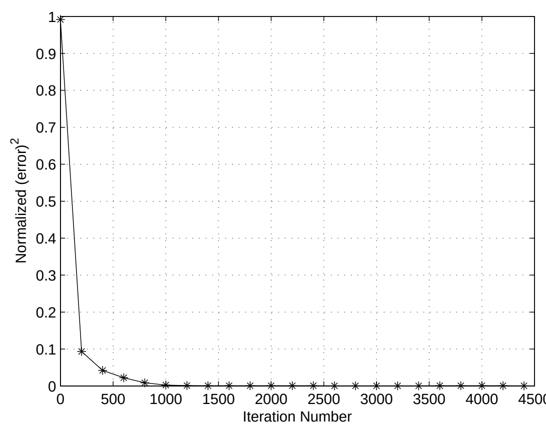

Fig. 12. Normalized err (5) as a function of iteration number for a 16QAM source input going through channel3. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.00004$.

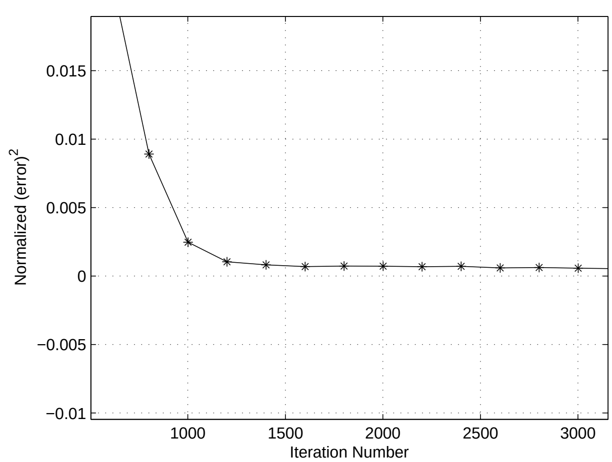

Fig. 13. Normalized zoomed in version for the err (5) as a function of iteration number for a 16QAM source input going through channel3. The averaged results were obtained in 100 Monte Carlo trials for a SNR=30 dB. $\mu_{MMA} = 0.00004$.

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