The use of relay shifting method of process identification for auto tuning of PID controller

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Abstract—The paper describes the use of a recently published method of relay feedback identification (called the shifting method) for estimation of the second order time delayed system model. An algorithm is designed for estimation of model parameters from two points of the frequency characteristics of the identified system. Both points are obtained from a single relay feedback test without any assumption about the model transfer function. The relay shifting method used here was modified by using an integrator in the feedback loop or an added time delay in closed loop. It allows to estimate points of frequency characteristics in positions more suitable for model fitting than the original shifting method approach. This modification enables a better estimate of the static gain even under constant load disturbance. The identified process model was used to calculate the PID controller parameters. The proposed solution is demonstrated on simulated and real examples.

Keywords—relay feedback identification, the shifting method, PID controller auto tuning.

I. INTRODUCTION

FOR optimal system control, we need to know the system properties. That is why the system identification is an important part of control engineering: correct identification is the first step in the tuning of controllers. The control of systems is in the industry commonly based on Proportional-Integral-Derivative (PID) controller. If we tune the controller manually, the tuning procedure is time consuming. The relay control method was published already in 1984 by Åström and Hägglund [1]. They used relay feedback for estimation of a system's critical gain and critical frequency in a closed loop. This method allows to calculate the same parameters as Ziegler-Nichols method [2], but without any knowledge of the system and in shorter time. Moreover, unlike most other methods, relay feedback identification allows a continuous control of the controlled plant. The method developed by Åström and Hägglund is simple and practical, therefore many methods based on relay feedback have been developed up to now [3], [4], [5].

The recently published shifting method [6] finds two points of a Nyquist frequency characteristics from a single relay feedback test. It allows estimating all parameters of second order plus time delay model (SOTD) [7], [8], [9]. The identified model can be used for calculation of PID controllers parameters according to many published methods, see for example [10].

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II. THE SHIFTING METHOD OF RELAY FEEDBACK IDENTIFICATION

We consider a stable process which can be described by a time invariant linear dynamic model around its operating point. This method allows to determine the process model from information obtained from a single relay feedback test. This model should be suitable for tuning PID controllers.

The shifting method [6] uses an asymmetrical relay with hysteresis (Fig. 1) for a process control close to the operating point. This technique generates a stable oscillation (Fig. 2) after the time t_L in the relay feedback experiment with period

$$T_p = T_1 + T_2 \tag{1}$$

where $T_1 \neq T_2$. The process can be described by a linear time invariant SISO (single input single output) model. The method

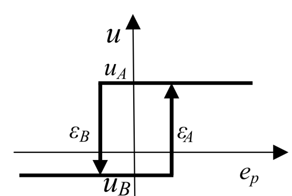

Fig. 1. The static characteristic of an asymmetrical relay with hysteresis.

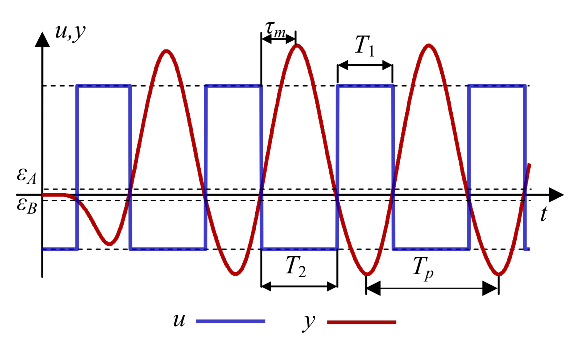

Fig. 2. The time courses u and y.

is based on estimation of two points $G(j\omega_1)$ a $G(j\omega_2)$ of the process frequency response $G(j\omega)$ related to the fundamental frequency ω_1 and the second harmonic ω_2 of the input/output signals where

$$\omega_1 = \frac{2\pi}{T_p} \tag{2}$$

$$\omega_2 = 2 \cdot \omega_1 \tag{3}$$

$$G(j\omega_1) = \frac{\int_t^{t+T_p} y(\tau)e^{-j\omega_1\tau}d\tau}{\int_t^{t+T_p} u(\tau)e^{-j\omega_1\tau}d\tau}, t \geq t_L, \tag{4}$$

$$G(j\omega_2) = \frac{\int_t^{t+T_p}\left(y(\tau) + y(\tau - \frac{T_p}{2})\right)e^{-j\omega_2\tau}d\tau}{\int_t^{t+T_p}\left(u(\tau) + u(\tau - \frac{T_p}{2})\right)e^{-j\omega_2\tau}d\tau}, \quad t \geq t_L, \tag{5}$$

where u is the system input and y is the system output. The static gain K of proportional system can be calculated from the following formula if the asymmetrical relay is used and we known the working point (u_0, y_0) [11].

$$K = G(0) = \frac{\int_t^{t+T_p}(y(\tau) - y_0)\,d\tau}{\int_t^{t+T_p}(u(\tau) - u_0)\,d\tau}, t \geq t_L \tag{6}$$

These three points can be used for fitting the model, see [7]. The values were determined without any assumptions about a model structure which is a great advantage of this approach.

A. The shifting method modifications

The position of the points $G(j\omega_1)$ and $G(j\omega_2)$ (Fig. 4) does not guarantee a precise estimation of the corresponding model in some cases. To estimate better positioned points $G(j\omega_1)$ and $G(j\omega_2)$ we can slightly modify the block diagram for the relay feedback test by the transport delay D (see [8]) or, alternatively, by the additional integrator (Fig. 3). The new position of the points is shown in Fig. 4 for both alternatives. We can then calculate the model of the identified system using these estimated points.

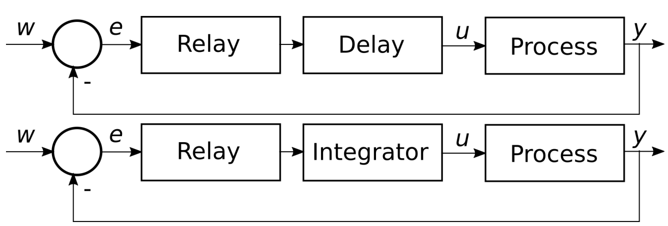

Fig. 3. Modified shifting method with a time delay or with an integrator.

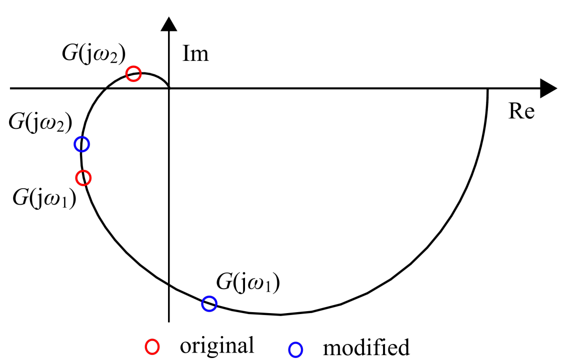

Fig. 4. Comparison of position of estimated frequency characteristics points from the original shifting method and the modified shifting method with a time delay or with an integrator.

III. Second order time delayed model fitting

Second order time delayed (SOTD) model with transfer function

$$M(s) = \frac{K \cdot e^{-s\cdot\tau_u}}{a_2 s^2 + a_1 s + 1}, \tag{7}$$

has four parameters - the static gain K, the parameters of a characteristic polynomial a_2, a_1 and the time delay τ_u. All parameters can be numerically calculated from estimated points $G(j\omega_1)$ and $G(j\omega_2)$ of frequency characteristics and values of their frequencies ω_1 and ω_2. We can used the criterion

$$Kr(K, a_2, a_1, \tau_u) = \sum_{i=1}^{2}(G(j\omega_i) - M(j\omega_i))^2 \tag{8}$$

for this purpose. The value of this criterion depends on values of model parameters K, a_2, a_1 and τ_u. Vector

$$\theta = \begin{bmatrix} K \\ a_2 \\ a_1 \\ \tau_u \end{bmatrix} \tag{9}$$

consists of the unknown values of these parameters. For a stable system, the value of the vector (9) that minimises the criterion (8) can be determined by

$$\begin{aligned} \hat{\theta} =& \underset{\theta \in D}{arg\,min} Kr(\theta) \\ D =& \{(K, a_2, a_1, \tau) : \\ & K > 0, a_2 > 0, a_1 > 0, \tau \in \langle 0, \tau_m \rangle\} \end{aligned} \tag{10}$$

Denote R_i and I_i be the real and the imaginary part of the complex values $G(j\omega_1)$ and $G(j\omega_2)$

$$G(j\omega_i) = R_i + I_i \cdot j, i = 1, 2. \tag{11}$$

Then

$$\hat{\theta} = \underset{\tau \in \langle 0,\tau_m\rangle \& K,a_2,a_1>0}{arg\,min} Kr\left(\begin{bmatrix} (Z^T Z)^{-1} \cdot Z^T p \\ \tau \end{bmatrix}\right) \tag{12}$$

where

$$Z = \begin{bmatrix} cos(\omega_1\tau) & R_1\omega_1^2 & I_1\omega_1 \\ -sin(\omega_1\tau) & I_1\omega_1^2 & -R_1\omega_1 \\ cos(\omega_2\tau) & R_2\omega_2^2 & I_2\omega_2 \\ -sin(\omega_2\tau) & I_2\omega_2^2 & -R_2\omega_2 \end{bmatrix} \tag{13}$$

and

$$p = \begin{bmatrix} R_1 \\ I_1 \\ R_2 \\ I_2. \end{bmatrix} \tag{14}$$

IV. PID control of the identified systems

For PID controller tuning we used four methods based on SOTD models. The AMIGO method allows to tune PID control of non-oscillatory systems [12]. The second method is based on the phase margin criterion [13]. The dynamic inversion method can be used for control of non-oscillatory systems and oscillatory systems with damping bigger than 0.5 [14]. The simple control tuning rules allows to tune controllers for non-oscillatory systems [10].

A. The AMIGO tuning rules for PID controllers

The AMIGO PID controllers tuning rules are based on the model of a non-oscillatory system

$$M(s) = \frac{K \cdot e^{-s \cdot \tau_u}}{(T_1 s + 1)(T_2 s + 1)}, \tag{15}$$

where T_1 and T_2 are the time constants. The controller parameters can be calculated from equations [12]

$$r_0 = \frac{0.19}{K} + \frac{0.37T_1 + 0.18T_2}{K \cdot \tau_u} + \frac{0.02T_1T_2}{K \cdot \tau_u^2}, \tag{16}$$

$$r_i = \frac{0.48}{K \cdot \tau_u} + \frac{0.03T_1 - 0.0007T_2}{K \cdot \tau_u^2} + \frac{0.0012T_1T_2}{K \cdot \tau_u^3}, \tag{17}$$

$$T_i = \frac{r_0}{r_i}, \tag{18}$$

$$r_d = \frac{T_1 + T_2}{K(T_1 + T_2 + \tau_u)} \cdot \left(0.29\tau_u + 0.16T_1 + 0.2T_2 + \frac{0.28T_1T_2}{\tau_u}\right), \tag{19}$$

$$T_d = \frac{r_d}{r_0}, \tag{20}$$

for $T_1 > T_2$.

B. Phase margin criterion (PMC) based PID controllers tuning

This method sets PID parameters by requiring that the controlled closed loop phase margin value is $\pi/4$ [13]. Parameters calculation is based on the model (7) and model (15). The PID control parameters are estimated with the equations [13]

$$r_i = \frac{\pi}{4|K|\tau_u}, \tag{21}$$

$$T_i = \frac{r_0}{r_i}, \tag{22}$$

$$r_0 = (T_1 + T_2) r_i = a_1 \cdot r_i, \tag{23}$$

$$r_d = T_1T_2r_i = a_2 \cdot r_i, \tag{24}$$

$$T_d = \frac{r_d}{r_0}. \tag{25}$$

C. The dynamics inversion method (DIM) of tuning PID controllers

The method is based on the model of the oscillatory system

$$M(s) = \frac{K \cdot e^{-s \cdot \tau_u}}{T_0^2 s^2 + 2\xi_0 T_0 s + 1}, \tag{26}$$

where

$$0.5 < \xi_0 < 1. \tag{27}$$

is damping and

$$T_0 = \frac{1}{\omega_0}, \tag{28}$$

where ω_0 is the natural frequency [14]. The controller parameters can be calculated also from the model (7) using equations

$$T_i = 2\xi_0 T_0 + T = a_1 + T, \tag{29}$$

$$r_0 = \frac{aT_i}{K}, \tag{30}$$

$$a = \frac{1}{\alpha T + \beta \tau_u}, \tag{31}$$

$$T_d = \frac{T_0}{2\xi_0} + \frac{T}{4} = \frac{a_2}{a_1} + \frac{T}{4}, \tag{32}$$

where T is the sampling period of a discrete PID controller [14]. The function block of the PID controller works like a continuous controller, therefore

$$T = 0, \tag{33}$$

TABLE I
VALUES OF α AND β ACCORDING TO THE VALUE OF THE CONTROLLED VARIABLE OVERSHOOT κ [14]

κ	0	0.05	0.1	0.15	0.2	0.25
α	1.282	0.984	0.884	0.832	0.763	0.697
β	2.718	1.944	1.720	1.561	1.437	1.337

D. The simple control (SIMC) tuning rules for ideal PID controllers

The SIMC PID rules for PID controller are based on the non-oscillatory model (15) [10]. If $T_1 <= 8\tau_u$ and $T_1 > T_2$, then

$$r_0 = \frac{0.5}{K}\frac{T_1 + T_2}{\tau_u}, \tag{34}$$

$$T_i = T_1 + T_2 \tag{35}$$

$$T_d = \frac{T_2}{1 + \frac{T_2}{T_1}}[10]. \tag{36}$$

If $T_1. = 8\tau_u$ and $T_1 > T_2$, then

$$r_0 = \frac{0.5}{K}\frac{T_1}{\tau_u}\left(1 + \frac{T_2}{8\tau_u}\right), \tag{37}$$

$$T_i = 8\tau_u + T_2, \tag{38}$$

$$T_d = \frac{T_2}{1 + \frac{T_2}{8\tau_u}}[10]. \tag{39}$$

V. CONTROL OF SYSTEMS WITH PID CONTROLLERS TUNED ACCORDING TO IDENTIFIED MODELS

The modified shifting method was successfully used for identification of three simulated and one real system. Matlab/Simulink programming environment was used for identification of simulated processes. Simulated systems were identified by the shifting method with use of added integrator and the asymmetrical relay with hysteresis and parameters

$$\begin{aligned} u_A &= 2 \\ u_B &= -1 \\ \epsilon_A &= 0.1 \\ \epsilon_B &= -0.1. \end{aligned} \tag{40}$$

Real PLC Tecomat Foxtrot (Teco, Kolín, Czech republic) was used for identification of real process and control of all tested systems. The system control started from null conditions. The command variable was set to 5. Quality of the control was compared between the PID controller tuning methods based on the time of control T_c, the value of overshoot OS_c and the integral of time multiplied by squared error Er_c. We tuned the PID controller according to the identified models of the respective systems. We used the PLC Tecomat (Teco) controller with program based od the SimplePID function block from ModelLib library for the PID control. The SimplePID works with PID controllers with two degrees of freedom (PID 2DOF)

$$U(s) = r_0\{[bW(s) - Y(s)] + \frac{1}{T_I s}[W(s) - Y(s)] + T_D s\,[cW(s) - Y(s)]\} \tag{41}$$

with anti-windup. r_0 is the controller gain, b is the weight of the proportional part of the controller, $W(s)$ is the requested value of the system output, $Y(s)$ is the system output, T_I is the integration time constant, s is the operator of the Laplace transform, T_D is the derivative time constant and c is weight of the derivative part of controller. The controller uses the filtration of the derivative part

$$F(s) = \frac{1}{T_f s + 1} \tag{42}$$

and special values of the coefficients b and c

$$b = 1, \tag{43}$$

$$c = 0, \tag{44}$$

which are specific for a PI-D controller. The PI-D controller works according to the equation

$$U(s) = r_0\left\{E(s) + \frac{1}{T_I s}E(s) - \frac{T_D s}{T_f s + 1}Y(s)\right\}. \tag{45}$$

We calculated the PID parameters with the PLC Tecomat Foxtrot (TECO, Kolín, Czech Republic). The program for the PLC was written in the "Structured text" programming language according to IEC 61131-3 in the environment MOSAIC (TECO) [15].

The quality of control processes is compared according to the time of control T_c, the integral of time multiplied by squared error Er_c and value of the overshoot OS_c.

A. Example 1: simulated lag dominated process

The simulated lag dominated system

$$P_1(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)} \tag{46}$$

was identified by the modified shifting method with use of integrator, resulting in model

$$M_1(s) = \frac{1 \cdot e^{-0.011s}}{0.1s^2 + 1.1s + 1}.[9] \tag{47}$$

PID controller was tuned according to the identified model (47). Calculated parameters od the controllers and the parameters of the control process are presented in tab. II. The control processes of all four tuning methods are compared in Fig. 5.

TABLE II
EXAMPLE 1 - PARAMETERS OF PID CONTROLLERS AND CONTROL PROCESSES

	AMIGO	PMC	DIM	SIMC
r_0	51.99	78.54	36.79	97.11
T_I [s]	0.1364	1.1	1.1	0.188
T_D [s]	0.05196	0.09091	0.09091	0.04681
T_c [s]	1.38	7.38	7.42	1.48
Er_c [s]	445.5	2414	2490	447.4
OS_c [%]	0.98	0	0	0

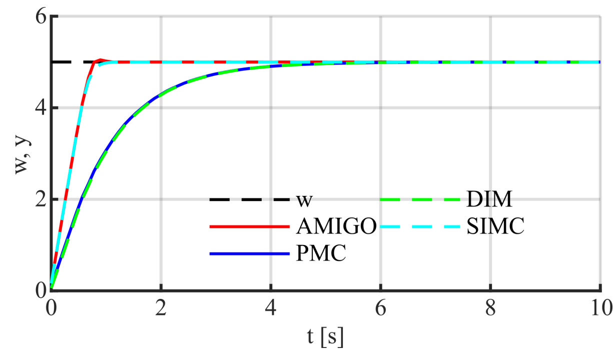

Fig. 5. Example 1 - results of system control with tuned PID controllers.

B. Example 2: simulated balanced process

Simulated balanced process with the transfer function

$$P_2(s) = \frac{1}{(s+1)^4} \tag{48}$$

was identified by the modified shifting method as a system with oscillatory model

$$M_2(s) = \frac{0.9535 \cdot e^{-0.956s}}{3.084s^2 + 2.942s + 1}.[9] \tag{49}$$

Because the model was identified as oscillatory, only PMC and DIM method of PID controllers tuning were used. Results of PID parameters calculation based on the identified model (49) and parameters of control process are presented in tab. III. The control processes are compared in Fig. 6 for both tuning methods.

TABLE III
EXAMPLE 2 - PARAMETERS OF PID CONTROLLERS AND CONTROL PROCESSES

	PMC	DIM
r_0	2.535	1.187
T_I [s]	2.942	2.942
T_D [s]	1.048	1.048
T_c [s]	15.17	19.00
Er_c [s]	46453	78854
OS_c [%]	1.8	1.2

C. Example 3: simulated delay dominated process

Simulated delay dominated system with the transfer function

$$P_3(s) = \frac{1 \cdot e^{-s}}{(0.05s+1)^2} \tag{50}$$

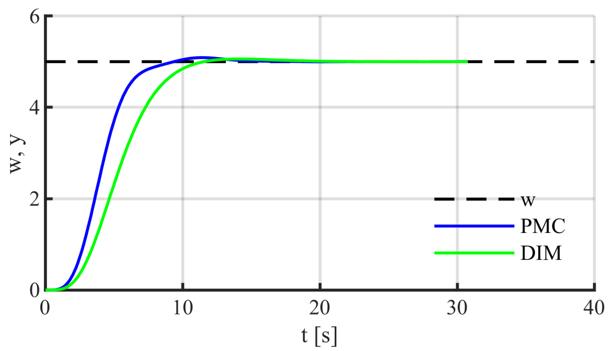

Fig. 6. Example 2 - results of system control with tuned PID controllers.

was identified by the modified shifting method as a system with the oscillatory model

$$M_3(s) = \frac{1 \cdot e^{-0.955s}}{0.00856s^2 + 0.1486s + 1}.[9] \quad (51)$$

Calculated parameters of PID controllers are based on the model (51). The controllers parameters and parameters of control process are in tab. IV. Because of identified oscillatory model only PMC and DIM method were used. Control processes are compared in Fig. 7 for both tuning methods.

TABLE IV
EXAMPLE 3 - PARAMETERS OF PID CONTROLLERS AND CONTROL PROCESSES

	PMC	DIM
r_0	0.1222	0.05725
T_I $[s]$	0.1486	0.1486
T_D $[s]$	0.05760	0.05760
T_c $[s]$	9.34	21.31
Er_c $[s]$	3522	14035
OS_c $[\%]$	0	0

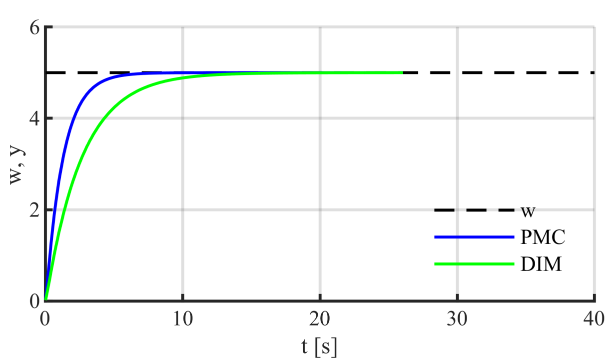

Fig. 7. Example 3 - results of system control with tuned PID controllers.

D. Example 4: real laboratory controlled plant

Real laboratory controlled plant called ”Air Aggregate” (Fig. 8) consists of a fan and a flow rate meter. Action variable is voltage on the fan (which regulates its power output) and controlled variable is voltage on the flow rate meter (which depends on the air flow rate created by the fan). The plant

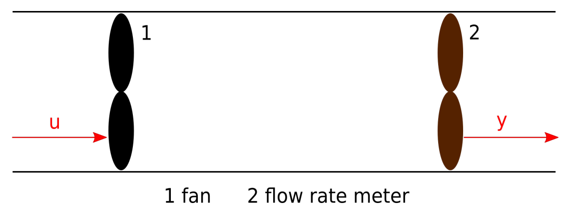

Fig. 8. Laboratory controlled plant ”Air Aggregate”.

was identified by the modified shifting method with added delay with the model

$$M_5(s) = \frac{1.969 \cdot e^{-3.89s}}{0.0044s^2 + 8.315s + 1}. \quad (52)$$

The resulting PID parameters and parameters of control process are presented in tab. V, the control processes are compared in Fig. 9.

TABLE V
EXAMPLE 4 - PARAMETERS OF PID CONTROLLERS AND CONTROL PROCESSES

	AMIGO	PMC	DIM	SIMC
r_0	0.4982	0.8526	0.3994	0.5428
T_I $[s]$	7.012	8.315	8.315	8.315
T_D $[s]$	1.708	0.00053	0.00053	0.00053
T_c $[s]$	50	50	67	48
Er_c $[V^2s]$	8.1e+03	1.0e+04	2.2e+04	1.6e+04
OS_c $[\%]$	38.7	2.2	0.6	1.5

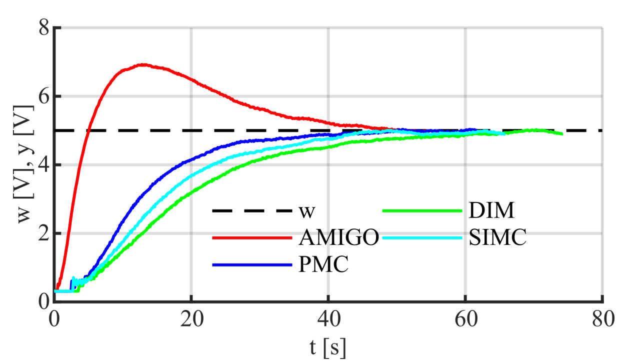

Fig. 9. Example 4 - results of system control with tuned PID controllers.

VI. CONCLUSION

Compared to the method used in [6], [7] and [8], we circumvented the need to know the working point (u0; y0), and the static gain is calculated from two measured points of the system’s frequency characteristics without using (6). We did so by applying the least-squares method to the parameter estimation. The static gain is notoriously hard to measure, but the proposed method allows to estimate the static gain even under the influence of the static disturbance. Control of identified processes with use of PID controllers tuned according to the found models ended successfully in all cases at the set point. The proposed auto tuning algorithm for PID controllers implemented in Tecomat Foxtrot PLC is now tested on both stable and unstable systems. The achieved results in comparison with alternative auto tuning algorithms support the assumption of a prospective extension of the proposed algorithm into common practice.

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