

Determination the Modes Characteristics in the Complex Cross-Section Waveguides

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Abstract— In this research the Expressions that determine the eigenfunctions and modes eigenvalues in waveguides with a composite sectorial cross-section are obtained. The possibility of characteristics changing for eigenvalues by changing the parameters that characterizing the cross-sectional shape was studied. The field modes of waveguide based on Ritz method was analyzed.it was determined the characteristics of the quasi – H_{mn} modes in a cruciform sector waveguide, and quasi – H_{mn} modes in a composite sector waveguide with an arbitrary number of sectors.it was also shown the advantage of using the cross-section waveguide in single mode optical fiber wavelength range. The eigenvalues (λ) and the normalized coefficients (a) for quasi – H_{mn} modes in terms of Bessel functions (Q_m, P_m) and their combinations was obtained.

Keywords—Signal Processing, Signals, Communications, Signal Processing Systems, Communication Systems, Waveguides, cross section, eigenfunctions, eigenvalues, mode.

I. INTRODUCTION

In communication and signal processing technology, as well as in several other electrical devices, the using of metal and dielectric waveguides is enough widespread. In this case, most often waveguides have rectangular or circular cross-sectional shape.

The task of determining the eigenfunctions and eigenvalues of modes in such waveguides is relatively simple to solve since the lines that bounding the cross-sectional contour coincide with the coordinate surfaces of the rectangular and cylindrical coordinate systems, which does not cause difficulties in imposing boundary conditions for determining the integration constants when solving the wave equation [1, 2]. In this case, as a rule, the variable separation method is used [1].

A special case of guiding systems in particular, waveguides having a complex cross-sectional shape, where the boundaries partially coincide with the coordinate surfaces of the selected coordinate system, for example as a cruciform L-shaped, H-shaped and others. In this case, to solve the wave equation, it is possible to apply approximate solution methods, such as the method of partial domains [2], the method of associated equations [3], and others. In addition to the above waveguide cross-sectional shapes, waveguides that having a

composite sectorial cross-sectional shape are practically interesting to research, a special case of such shape is a waveguide with a cross sectorial sectional shape (fig. 1). The advantage of such a cross-sectional shape is that their electromagnetic waves as studies have shown [4], preserve the structure of the waveguides wave field with a circular cross-sectional shape and, at the same time, such a shape allows changing the characteristics of the (eigenvalues) within specified limits by changing the parameters characterizing the cross-sectional shape [5,6,7].

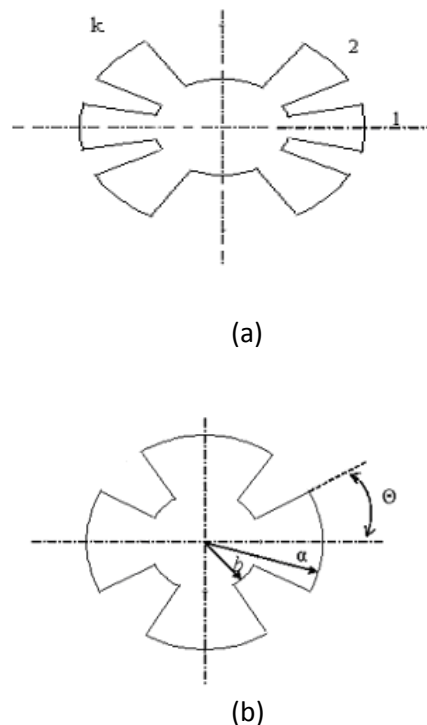


Fig. 1: waveguide with a cross sectorial sectional shape

The dielectric waveguide which having the shape shown in (Fig. 1b) can be used in the optical wavelength range [8,9]. In other words, if the core of the optical fiber is given a shape like this figure, then in a single-mode operation of optical fiber the coupling between ordinary and extraordinary waves will significantly decrease in comparison with a fiber with a circular cross-section, and the crosstalk between the waves will increase [4,10,11]. As a result, the polarization mode

dispersion of the signal in it can be significantly reduced. Besides, the mode selection for directional couplers with good mode selectivity can be created based on cross sector of sectional shape [12,13].

The purpose of this research is the determination the proper functions and proper values of modes in a waveguide with a cross sectorial of cross-sectional shape which is extremely important from both theoretical and practical points of view.

II. METHOD AND PROCEDURE

At the first stage of the study, let us determine the characteristics of the modes in a metal waveguide with a cross sectorial shaped of cross section. In this case, the method of variables separation cannot be used in solving the wave equation, since the lines that bounding the contour of the cross section of such a waveguide coincide with the coordinate surfaces of the cylindrical coordinate system only in some places. The method of partial regions in this case is extremely difficult to use because a rather complex shape of the waveguide cross-section composed from sectors. to analyze the mode fields in such a waveguide it is desirable to use a method that is versatile, relatively simple, and sufficiently accurate, these properties are largely possessed by the Ritz variational method [5, 6].

As it is known [2,14,15], determination the fields and finding the eigenfunctions and eigenvalues in a waveguide with ideal conducting walls lead to solving the scalar equation:

$$\Delta_{\perp} \psi + \chi^2 \psi = 0 \quad (1)$$

Where ∇_{\perp} - two-dimensional (transverse) Laplace operator.

ψ - the eigenfunction of the magnetic mode, representing the longitudinal component of the Hertzian magnetic vector, and related to the longitudinal magnetic field by the relation [16,17]:

$$H_z = \chi^2 \psi \quad (2)$$

χ - Value of own modes

$$\chi^2 = k^2 - \beta^2 \quad (3)$$

κ - wave number in free space.

β - the mode phase constant.

First, we define the eigenfunctions and eigenvalues of magnetic modes with the boundary condition:

$$\left. \frac{\partial \psi}{\partial n} \right|_L = 0 \quad (4)$$

L- waveguide cross-section contour.

n - outward normal to the contour.

The solution for equation (1) will be sought based on the Ritz variational method [5,16]. According to the variational method, the task of determining the eigenvalues of modes is lead to study the extremum of the functional:

$$\chi^2 = \iint (\nabla_{\perp} \psi)^2 dS \quad (5)$$

provided that:

$$\iint \psi^2 dS = 1 \quad (6)$$

Where ∇_{\perp} - two-dimensional Hamilton operator.

ψ - eigenfunction magnetic mode.

S- cross-sectional area of the waveguide.

Indeed, the Laplace operator is a self-adjoint lower bounded operator. Then the smallest eigenvalue of the two-dimensional Laplace operator [5]:

$$\alpha = \inf \frac{(\Delta_{\perp} \psi, \psi)}{(\psi, \psi)} \quad (7)$$

in parentheses is a scalar product and \inf is the infimum. Then (α) is the smallest eigenvalue of the operator Δ_{\perp} with condition of existing the element ψ_0 , then:

$$\alpha = \frac{(\Delta_{\perp} \psi_0, \psi_0)}{(\psi_0, \psi_0)} \quad (8)$$

If such an element ψ_0 exists, then the definition of the smallest eigenvalue of the operator (Δ_{\perp}) is leads to the determination of the lower bound of the quantity (7) or the same as the lower bound of the quantity:

$$(\Delta_{\perp} \psi, \psi) \quad (9)$$

with the additional condition

$$(\psi, \psi) = 1 \quad (10)$$

Multiplying scalar equation (1) by (ψ), we get:

$$\chi^2 = \frac{(\Delta_{\perp} \psi, \psi)}{(\psi, \psi)} \quad (11)$$

Using the well-known Green's formula [7], we get:

$$(\Delta_{\perp} \psi, \psi) = \iint_S (\text{grad}_{\perp} \psi)^2 dS - \int_L \psi \frac{\partial \psi}{\partial n} \partial l \quad (12)$$

The contour integral in expression (12) is equal to zero due to boundary condition (4). Then from formula (11) considering (10) and (12) we get:

$$\chi^2 = \iint_S (\text{grad}_{\perp} \psi)^2 dS = \iint_S (\nabla_{\perp} \psi)^2 dS \quad (13)$$

According to the Ritz method [5], the approximate solution (ψ_n) of equation (1) under condition (4) is in the form:

$$\psi_n = \sum_{i=1}^n a_i u_i \quad (14)$$

The sequence of sufficiently smooth coordinate functions $\{u_i\}$ must be a complete linearly independent system. according to the Ritz method, these functions are not required to satisfy the natural boundary condition.

However, to improve the convergence of the series, we will choose them partially satisfying condition (4) on the part of the contour (L) of the waveguide cross section.

The coefficients (a_i) in solution (14) are chosen based on the minimum of function (13) taken in consideration (10). Thus, this is led to find the minimum function of (n)variables :

$$(\nabla_{\perp} \psi_n, \nabla_{\perp} \psi_n) = \sum_{i,j=1}^n (\nabla_{\perp} u_i \nabla u_j) a_i a_j \quad (15)$$

related by the equation:

$$(\psi_n, \psi_n) = \sum_{i,j=1}^n (u_i, u_j) a_i a_j = 1 \quad (16)$$

where in parentheses is a scalar product of functions.

$$F = (\nabla_{\perp} \psi_n, \nabla_{\perp} \psi_n) - \chi^2 (\psi_n, \psi_n) \quad (17)$$

and from condition $\frac{\partial F}{\partial a_i} = 0$, we obtain a symmetric system:

$$\sum_{i,j=1}^n a_j |\nabla_{\perp} u_i, \nabla_{\perp} u_j) - \chi^2 (u_i, u_j) = 0 \quad (18)$$

Where $i, j = 1, 2, 3, \dots, n$.

From the condition that the determinant of the linear system (18) homogeneous with respect to (a_j) equal to zero, we obtain the characteristic equation for (χ^2).

The smallest roots of system determinant (18) will be equal to the minimum of expression (15). And to find the approximate (k^{th}) eigenvalue, it is necessary to find the minimum of (15) with additional orthogonality conditions for eigen functions:

$$(\psi_l, \psi_l) = 1, (\psi_l, \psi_m) = 0, (m = 1, 2, 3, \dots, l - 1) \quad (19)$$

Where (ψ_m) - the approximate value of the (m^{th}) normalized function of the Laplace operator.

So, let us determine the eigenfunctions and eigenvalues of magnetic modes in a composite sector waveguide. Modes H_{mn} of a circular waveguide upon transition to a composite sector waveguide will excite its quasi - H_{mn} modes. We represent the eigenfunction of the quasi- H_{mn} mode in a waveguide with ($2k$) sectors in the form of a superposition of (k) degenerate quasi - H_{mn} modes which differing from each other in polarization:

$$\psi_{mn} = \sum_m a_m j_m(v_{mn} \frac{r}{a}) \sum_{j=1}^k \cos m \left[\phi + (j - 1) \frac{\pi}{k} \right] \quad (20)$$

Where $j_m(x)$ - first kind of Bessel function of order m .

v_{mn} - n^{th} root of equation $j'_m(x) = 0$.

a - the size of waveguide cross-section (fig. 1).

r, ϕ - coordinates of the cylindrical system.

a_m - coefficient normalized to the area (S) of the waveguide cross-section, determinable from condition (6).

For a composite sectorial waveguide with an arbitrary number of sectors ($2k$), it was possible to obtain relatively simple analytical expressions that determine the eigenfunctions and eigenvalues in terms of the Bessel

functions and their combinations in general form only for the quasi - H_{mn} modes. For quasi - H_{mn} modes with arbitrary (m), these expressions were obtained only for the cruciform sector waveguide (Fig. 1b).

III. RESULTS AND DISCUSSIONS

Thus, for $k = 2$, and from (20) we obtain an expression for the eigenfunction of the quasi - H_{mn} mode:

$$\psi_{mn} = \sum_m a_m j_m(v_{mn} \frac{r}{a}) (\cos m\phi - \sin m\phi) \quad (21)$$

This function is an approximate solution to equation (1) under conditions (2) and (19).

To determine ψ_{mn} and χ_{mn} , we can restrict ourselves to the approximation functions ($\cos m\phi j_m(v_{mn}r)$) and ($\sin m\phi j_m(v_{mn}r)$) considered for all pairs of indices of the orthogonal system [4]. For the quasi - H_{01} mode, the maximum correction to the eigenvalue does not exceed 6%, and to the eigenfunction - no more than 10%. Subsequent approximations will give even smaller corrections. Then, for the quasi - H_{mn} mode, considering the first approximation, expression (21) will be in the form:

$$\psi_{mn} = a_m j_m(v_{mn} \frac{r}{a}) (\cos m\phi + \sin m\phi) \quad (22)$$

The eigenvalue (χ_{mn}) of the quasi - H_{mn} modes of a cruciform sector waveguide is determined in the first approximation from relation (18):

$$\chi_{mn} = \sqrt{\frac{(\nabla_{\perp} u_i, \nabla_{\perp} u_i)}{(u_i, u_i)}} \quad (23)$$

In accordance with (23), we obtain an expression for the eigenvalue (χ_{mn}) of a cruciform sector waveguide:

$$\chi_{mn} = \frac{v_{mn}}{a} \left[\frac{(2\pi - 8\theta) 0.5 \mu^2 Q_m(v_{mn}\mu) + 4\theta Q_m(v_{mn})}{(2\pi - 8\theta) \mu^2 P_m(v_{mn}\mu) + 8\theta P_m(v_{mn})} \right] \quad (24)$$

Where θ - the angle characterizing the opening of the sectors (fig. 1).

$\mu = \frac{b}{a}$, a, b - dimensions of the cross-section of the waveguide (fig. 1).

In this case, the coefficient (a_{mn}) normalized in accordance with (4), is determined by the expression:

$$a_{mn} = \frac{\sqrt{2}}{a \sqrt{\mu^2 (2\pi - 8\theta) P_m(v_{mn}\mu) + 8\theta P_m(v_{mn})}} \quad (25)$$

Where:

$$Q_m = [J_{m-1}^2(x) - J_{m-2}(x)J_m(x) + J_{m+1}^2(x) - J_m(x)J_{m+2}(x)] \quad (26)$$

$$P_m(x) = [J_m^2(x) - J_{m-1}(x)J_{m+1}(x)] \quad (27)$$

Thus, from expressions (24) and (25), the values of the eigenvalues (χ) and the normalized coefficients (a) can be obtained in terms of the functions $Q_m(v_{mn}\mu)$ and $P_m(v_{mn}\mu)$ which contain some combinations of first kind of Bessel functions of order m .

Figure.2 shows, as an example, the dependence of (Q_m, P_m) on (μ) calculated by formulas (26) and (27), respectively.

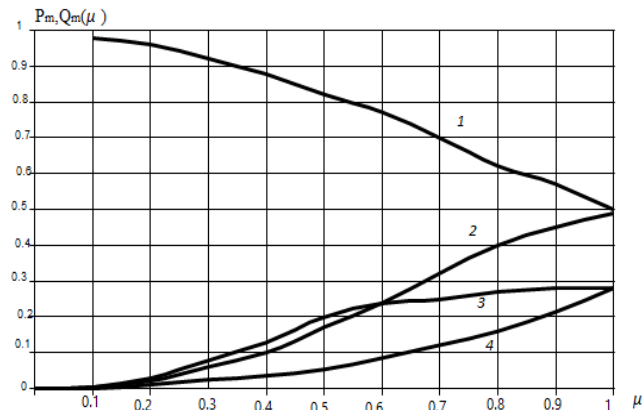


Fig. 2: The dependence of (Q_m, P_m) functions on (μ)

Where: (1)- $0.5Q_1(1.84\mu)$; (2)- $P_1(1.84\mu)$
 (3)- $0.5Q_2(3.054\mu)$; (4)- $P_2(3.054\mu)$

IV. CONCLUSION

- The obtained expressions in this research make it possible to determine the characteristics of the quasi - H_{mn} modes in a cruciform sector waveguide, as well as quasi - H_{mn} modes in a composite sector waveguide with an arbitrary number of sectors.

- To determine the eigenvalues (χ) and normalized coefficients of the eigenfunctions, it is necessary to know the parameters (μ) and (θ), which characterize the shape of the waveguide cross-section, as well as the functions $Q_m(v_{mn}\mu)$ proposed in this work.

- The characteristics of modes in a dielectric waveguide with a cross-shaped sectorial cross-section can be determined similarly to a metal waveguide, based on the Ritz method. This paper is very useful for signal processing and communications academic community.

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