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Bearings-only multitarget tracking based on Rao-Blackwellized particle CPHD filter

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Abstract—Following Mahler's framework for information fusion, this paper develops a implementation of cardinalized probability hypothesis density (CPHD) filter for bearings-only multitarget tracking. Rao-Blackwellized method is introduced in the CPHD filtering framework for mixed linear/nonlinear state space models. The sequential Monte Carlo (SMC) method is used to predict and estimate the nonlinear state of targets. Kalman filter (KF) is adopted to estimate the linear states with the information embedded in the estimated nonlinear states. The multitarget state estimates are extracted by utilizing the kernel density estimation (KDE) theory and mean-shift algorithm to enhance tracking performance. Moreover, the computational load of the filter is analyzed by introducing equivalent flop measure. Finally, the performance of the proposed Rao-Blackwellized particle CPHD filter is evaluated through a challenging bearings-only multitarget tracking simulation experiment.

Keywords—cardinalized probability hypothesis density, Rao-Blackwellized particle filter, bearings-only, multitarget tracking.

I. Introduction

IN bearings-only multi-target tracking, the number of targets is unknown and vary with time due to the uncertainty of target information. Moreover, the problem of model nonlinearity caused by coordinate transformation of target motion modeling and measurement modeling, and the physical characteristics of passive sensors themselves, as well as the incompleteness of measurement information, all bring great difficulties to target tracking. How to track multiple targets effectively based on bearings-only measurement information has always been a hot and difficult problem in both academic and engineering research [1,2].

Most traditional approaches for multiple targets tracking, assuming that each target moves independently of others, consider each target separately and track it with a separate filter. However this requires correct association of individual targets with their measurements, hence a heavy computational burden due to its combinatorial nature. Multiple hypotheses tracking (MHT) and its modified variations concern the propagation of association hypotheses in time [3]. The joint probabilistic data association filter (JPDAF) [4] uses observations weighted by their association probabilities. The alternative algorithms that avoid explicit associations between measurements and targets include symmetric measurement equations (SME) [5] and random finite sets (RFS) [6]. SME based approach obtains a new set of measurements by constructing a symmetric function of the original observations, then estimates the states of multiple targets simultaneously. In [7], a graph-based cooperative localization method is proposed which implements SME within factor graphs in order to overcome the data association challenge with a reduced bandwidth overhead.

Based on RFS, probability hypothesis density (PHD) filter propagates the posterior intensity function of the RFS of targets in time, which provides an efficient multitarget tracking algorithm for jointly estimating the number of targets and their states from a sequence of noisy measurement sets with data association uncertainty, missed detections and false alarms [8,9].The cardinalized probability hypothesis density (CPHD) filter, which can make full use of the information of multitarget density and does not need to limit the number of targets to obey Poisson distribution, has attracted more attention. Many scholars have carried out relevant research[10-14]. In [15], based on the famous Faà di Bruno determinant, a tractable recursion computation technique of the general cardinality prediction equation is presented. In [16], a robust CPHD based on interacting multiple model (RCPHD-IMM) is proposed for multiple maneuvering targets tracking under the Doppler blind zone of airborne pulse Doppler radar. In [17], to accommodate unknown target detection probability and nonnegative non-Gaussian parameters, a new implementation based on inverse gamma Gaussian mixtures is proposed, which introduces a location independent feature and gamma functions to determine detection probability incorporated into the recursions. In [18], a labeled box-particle CPHD filter is proposed for multiple extended targets tracking, which can improve the precision of estimating target number meanwhile achieve targets' tracks.

In this paper, a new multisensor particle CPHD filter is proposed, for the bearings-only multitarget tracking. For mixed linear/nonlinear state space models, the CPHD filter utilizes the idea of Rao-Blackwellized to enhance the estimating performance of the number of targets and their states, and uses the kernel density estimation (KDE) theory and mean-shift algorithm to extract target state estimates. In addition, the computational complexity of the proposed filter is analyzed.

The paper is organized as follows. Section II presents the problem formulation for the multisensor bearings-only multitarget tracking based on random finite set . Section III reviews the CPHD recursions briefly. Section IV describes the proposed particle CPHD filter in detail and provides the analysis on the computational complexity of the CPHD filter. The experimental results and performance analysis produced by the proposed filter are given in Section V. Our conclusions are summarized in Section VI.

II. Random Finite Set Model of Multitarget Dynamics

Consider the following passive multisensor bearings-only multitarget tracking system:

$$x_{n,k+1} = Fx_{n,k} + Gw_{n,k} \tag{1}$$

$$z^o_{m,k} = \begin{cases} h\left(x_{n,k}\right) + v^o_k, & \varsigma_{m,k} = n \\ clutter, & \varsigma_{m,k} = 0 \end{cases} \tag{2}$$

where $x_{n,k}$ is the system state vector of target n at time k, $x_{n,k} = \left[x_n(k), \dot{x}_n(k), y_n(k), \dot{y}_n(k)\right]^{\mathrm{T}}$, $w_{n,k}$ is a zero-mean white Gaussian noise with covariance matrix Q_n, $\left\{z^o_{m,k}, m = 1, \cdots, M_k\right\}$ is the set of target bearings generated by sensor o at time k which contains C_k clutters, $h\left(x_{n,k}\right) = \arctan\left(\frac{y(k) - y^o_n}{x(k) - x^o_n}\right)$, $\left(x^o_n, y^o_n\right)$ is the location of sensor o, $\varsigma_{m,k}$ represents the target indicator associated with measurement m. The measurement noise v^o_k is a zero-mean white Gaussian process with covariance matrix R^o, and it is uncorrelated with process noise $w_{n,k}$. Target bearing data measured at each sensor are combined by using the centralized fusion approach and are assumed to be associated.

In the MTT problem, with the appearance and disappearance of targets and clutter in the scene, the number of targets and the measurement generated by targets will change over time. The collections of target states and measurements at time k can be naturally expressed as finite sets, i.e.,

$$X_k = \left\{\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,N_k}\right\} \in \boldsymbol{F}\left(\chi\right) \tag{3}$$

$$Z_k = \left\{z_{k,1}, \ldots, z_{k,M_k}\right\} \in \boldsymbol{F}\left(Z\right) \tag{4}$$

where N_k and M_k are the number of targets and the number of measurements, respectively. $\boldsymbol{F}\left(\chi\right)$ and $\boldsymbol{F}\left(Z\right)$ are the collections of all finite subsets of χ and Z, respectively.

Given a multitarget state X_{k-1} at time $k-1$, the multitarget state X_k can be modeled by RFSs [10]

$$X_k = \left(\bigcup_{\mathbf{x} \in X_{k-1}} S_{k|k-1}\left(\mathbf{x}\right)\right) \bigcup \Gamma_k \tag{5}$$

where $S_{k|k-1}\left(\mathbf{x}\right)$ is the RFS of targets survived at time k from multitarget state X_{k-1}, and Γ_k is the RFS of targets that appear spontaneously at time k. In this paper, the target spawning is not considered for simplicity. The multitarget measurement Z_k is modeled by RFS

$$Z_k = K_k \bigcup \left(\bigcup_{\mathbf{x} \in X_k} \Theta_k\left(\mathbf{x}\right)\right) \tag{6}$$

where $\Theta_k\left(\mathbf{x}\right)$ is the RFS of measurements from multitarget state X_k and K_k is the RFS of measurements from clutter.

III. The CPHD filter

The PHD filter is proposed to alleviate the computational intractability in the multitarget Bayes filter. Instead of propagating the multitarget posterior density in time, the PHD filter propagates the posterior intensity, a first-order statistical moment of the posterior multitarget state. In essence, the CPHD filter is a high-order extension of the PHD filter, which jointly propagates the intensity function and cardinality distribution. The cardinality information can be expected to improve the accuracy and stability of the cardinality estimates, which in turn can improve the target state estimates [10].

Let $D_{k|k-1}$ and $p_{k|k-1}$ denote the intensity and cardinality distribution associated with the predicted multitarget state, respectively, and D_k and p_k denote the intensity and cardinality distribution associated with the posterior multitarget state, respectively. Then the CPHD recursions can be given as follows,

$$p_{k|k-1}\left(n\right) = \sum_{j=0}^{n} p_{\Gamma,k}\left(n-j\right) \Pi_{k|k-1}\left[D_{k-1}, p_{k-1}\right]\left(j\right) \tag{7a}$$

$$D_{k|k-1}\left(\mathbf{x}\right) = \int p_{S,k}\left(\zeta\right) \varphi_{k|k-1}\left(\mathbf{x} \mid \zeta\right) D_{k-1}\left(\zeta\right) d\zeta + \Gamma_k\left(\mathbf{x}\right) \tag{7b}$$

$$p_k(n)=\frac{\Upsilon_k^0\left[D_{k|k-1},Z_k\right](n)p_{k|k-1}(n)}{\left\langle\Upsilon_k^0\left[D_{k|k-1},Z_k\right],p_{k|k-1}\right\rangle} \tag{7c}$$

$$D_k(\mathbf{x})=\left[1-p_{D,k}(\mathbf{x})\right]\frac{\left\langle\Upsilon_k^1\left[D_{k|k-1},Z_k\right],p_{k|k-1}\right\rangle}{\left\langle\Upsilon_k^0\left[D_{k|k-1},Z_k\right],p_{k|k-1}\right\rangle}D_{k|k-1}(\mathbf{x})+\sum_{z\in Z_k}\Psi_{k,z}(\mathbf{x})\frac{\left\langle\Upsilon_k^1\left[D_{k|k-1},Z_k\setminus\{z\}\right],p_{k|k-1}\right\rangle D_{k|k-1}(\mathbf{x})}{\left\langle\Upsilon_k^0\left[D_{k|k-1},Z_k\right],p_{k|k-1}\right\rangle} \tag{7d}$$

where

$$\Pi_{k|k-1}\left[D,p\right](j)=\sum_{l=j}^{\infty}C_j^l\frac{\left\langle p_{S,k},D\right\rangle^j\left\langle 1-p_{S,k},D\right\rangle^{l-j}}{\left\langle 1,D\right\rangle^l}p(l) \tag{8}$$

$C_j^l=l!/j!(l-j)!$ denotes the binomial coefficient, $\langle\cdot,\cdot\rangle$ the inner product, $p_{\Gamma,k}(\cdot)$ the cardinality distribution of births, $p_{S,k}(\cdot)$ the probability of target existence, $\Gamma_k(\cdot)$ the intensity of spontaneous births,

$$\Upsilon_k^u\left[D,Z\right](n)=\sum_{j=0}^{\min(|Z|,n)}[(|Z|-j)!p_{Kk}(|Z|-j)P_{j+u}^n\times\frac{\left\langle 1-p_{D,k},D\right\rangle^{n-(j+u)}}{\left\langle 1,D\right\rangle^n}e_j\left(\Xi_k(D,Z)\right)] \tag{9}$$

$P_j^n=n!/(n-j)!$ denotes the permutation coefficient,

$$\Psi_{k,z}(\mathbf{x})=\frac{\left\langle 1,K_k\right\rangle}{K_k(z)}g_k(z|\mathbf{x})p_{D,k}(\mathbf{x}) \tag{10}$$

$$\Xi(D,Z)=\left\{\left\langle D,\Psi_{k,z}\right\rangle:z\in Z\right\} \tag{11}$$

$e_j(\cdot)$ is the elementary symmetric function of order j and can be computed by

$$e_j\left(\{\rho_1,\rho_2,\cdots,\rho_m\}\right)=(-1)^j\alpha_{m-j}/\alpha_m \tag{12}$$

$\{\rho_1,\rho_2,\cdots,\rho_m\}$ are distinct roots of the polynomial $\alpha_m x^m+\alpha_{m-1}x^{m-1}+\cdots+\alpha_1 x+\alpha_0$. Z_k is the measurement set, $p_{K,k}(\cdot)$ denotes the cardinality distribution of clutter, $p_{D,k}(\cdot)$ is the probability of target detection and $K_k(\cdot)$ is the intensity of clutter measurements, and $\varphi_{k|k-1}(\mathbf{x}|\zeta)$, $g_k(z|\mathbf{x})$ denote the transition density and the measurement likelihood of each target, respectively. Assume that each target system model is Gaussian, then the target state transition function and measurement function are expressed by $f_{k-1}(\cdot)$ and $h_k(\cdot)$, respectively. $\varphi_{k|k-1}(\mathbf{x}|\zeta)=N(\mathbf{x};f_{k-1}(\zeta),Q_{k-1})$, $g_k(z|\mathbf{x})=N(z;h_k(\mathbf{x}),R_k)$, where $N(\mathbf{x};m,P)$ is the Gaussian density with mean m and covariance P, Q_{k-1} is the process noise covariance, and R_k is the measurement noise covariance.

IV. PROPOSED RAO-BLACKWELLIZED PARTICLE CPHD FILTER

In many applications, the target state space contains both linear and non-linear parts. Aiming at this kind of mixed linear/non-linear state model, the linear state and non-linear state of the target are estimated separately by combining Kalman filter and particle filter. In this paper, by using RBPF method, multitarget PHD and cardinality distribution are estimated. PF is used to estimate the nonlinear state of targets, and KF is adopted to estimate the linear states with the information embedded in the estimated nonlinear states. More details on RBPF can be found in [19]. Due to the complexity of multitarget tracking, the shape of PHD obtained by iterative estimation is very irregular. Using the mean-shift method, accurate peak position of density function can be extracted [20].

A. Description of the proposed filter

The target filtering model can be described as linear and non-linear forms, i.e.,

$$\mathbf{x}_k^n=f_{k-1}\left(\mathbf{x}_{k-1}^n\right)+A_{k-1}^n\mathbf{x}_{k-1}^l+B_{k-1}^n w_{k-1}^n \tag{13a}$$

$$\mathbf{x}_k^l=A_{k-1}^l\mathbf{x}_{k-1}^l+B_{k-1}^l w_{k-1}^l \tag{13b}$$

$$z_k=h_k\left(\mathbf{x}_k^n\right)+v_k \tag{13c}$$

where $\mathbf{x}_k^n$ and $\mathbf{x}_k^l$ denote the nonlinear and linear states, respectively, and $\left[\mathbf{x}_k^n,\mathbf{x}_k^l\right]^{\mathrm{T}}=\mathbf{x}_k$, w_k is the process noise given by

$$w_k=\begin{bmatrix}w_k^n\\ w_k^l\end{bmatrix}\sim N\left(w;0,\begin{pmatrix}Q_k^n & S_k\\ S_k^{\mathrm{T}} & Q_k^l\end{pmatrix}\right) \tag{14}$$

and the measurement noise $v_k\sim N(v;0,R_k)$. Also, assuming $\mathbf{x}_0^l\sim N\left(\mathbf{x};\mathbf{x}_{0|-1}^l,P_{0|-1}^l\right)$ and that $\mathbf{x}_0^n$ has arbitrary fixed probability density function (PDF). Then, the process model of Eq.(13) can be rewritten as

$$\mathbf{x}_k^l=A_{k-1}^l\mathbf{x}_{k-1}^l+B_{k-1}^l w_{k-1}^l \tag{15a}$$

$$\overline{z}_k=A_{k-1}^n x_{k-1}^l+B_{k-1}^n w_{k-1}^n \tag{15b}$$

where $\overline{z}_k=x_k^n-f_{k-1}\left(x_{k-1}^n\right)$. It can be observed that Eq.(15) describes a linear Gaussian model and the states can be estimated by the Kalman filter (KF). The nonlinear states can be estimated by utilizing the sequential Monte Carlo (SMC) method. The particles predicted from time $k-1$ to time k meets the Gaussian distribution, i.e.,

$$p\left(\mathbf{x}_k^n\mid\mathbf{x}_{k-1}^n\right)=N\left(\mathbf{x};f_{k-1}\left(\mathbf{x}_{k-1}^n\right)+A_{k-1}^n\hat{\mathbf{x}}_{k-1|k-2}^l,R^n\right) \tag{16}$$

where $R^n = A_{k-1}^n P_{k-1|k-2}^l \left(A_{k-1}^n\right)^{\mathrm{T}} + B_{k-1}^n Q_{k-1}^n \left(B_{k-1}^n\right)^{\mathrm{T}}$, $\hat{\mathbf{x}}_{k-1|k-2}^l$ and $P_{k-1|k-2}^l$ denote the one-step-ahead predictions of the linear states and its covariance, respectively.

The proposed filtering algorithm consists of prediction and updation parts.

Prediction:

Suppose that the survival and detection probabilities are state independent, i.e.

$$p_{S,k}\left(\mathbf{x}\right) = p_{S,k}, \quad p_{D,k}\left(\mathbf{x}\right) = p_{D,k} \tag{17}$$

Suppose at time $k-1$ that the posterior intensity $D_{k-1}\left(\mathbf{x}\right)$ and the posterior cardinality $p_{k-1}\left(n\right)$ are given, and that $D_{k-1}\left(\mathbf{x}\right)$ can be characterized by the set of particles and weights $\left\{\omega_{k-1}^{(i)}, \mathbf{x}_{k-1}^{(i)}\right\}_{i=1}^{L_{k-1}}$, i.e. $D_{k-1}\left(\mathbf{x}\right) = \sum_{i=1}^{L_{k-1}} \omega_{k-1}^{(i)} \delta\left(\mathbf{x} - \mathbf{x}_{k-1}^{(i)}\right)$. The target states contain nonlinear states and linear states, i.e., $\mathbf{x}_{k-1}^{(i)} = \left[\mathbf{x}_{k-1}^{n,(i)}, \hat{\mathbf{x}}_{k-1|k-2}^{l,(i)}\right]^{\mathrm{T}}$. Then, the predicted intensity can be expressed by

$$D_{k|k-1}\left(\mathbf{x}\right) = \sum_{i=1}^{L_{k-1}} p_{S,k} \omega_{k-1}^{(i)} f_{k|k-1}\left(\mathbf{x} \mid \mathbf{x}_{k-1}^{n,(i)}, \hat{\mathbf{x}}_{k-1|k-2}^{l,(i)}\right) + \Gamma_k\left(\mathbf{x}\right) \tag{18}$$

To obtain a particle approximation of $D_{k|k-1}\left(\mathbf{x}\right)$, we apply Rao-Blackwellized technique to each of its terms. Firstly, for the existing target at time k, we can obtain the nonlinear states of predicted particles from Eq.(18), i.e.,

$$\left\{\mathbf{x}_k^{n,(i)}\right\}_{i=1}^{L_{k-1}} \sim N\left(\mathbf{x}; f_{k-1}\left(\mathbf{x}_{k-1}^{n,(i)}\right) + A_{k-1}^n \hat{\mathbf{x}}_{k-1|k-2}^{l,(i)}, R^{n,(i)}\right) \tag{19}$$

The linear states of predicted particles can be calculated by KF,

$$G_{k-1}^{(i)} = P_{k-1|k-2}^{l,(i)} \left(A_{k-1}^n\right)^{\mathrm{T}} \times \left[A_{k-1}^n P_{k-1|k-2}^{l,(i)} \left(A_{k-1}^n\right)^{\mathrm{T}} + B_{k-1}^n Q_{k-1}^n \left(B_{k-1}^n\right)^{\mathrm{T}}\right]^{-1} \tag{20a}$$

$$\hat{\mathbf{x}}_{k|k-1}^{l,(i)} = A_{k-1}^l \times \left[\hat{\mathbf{x}}_{k-1|k-2}^{l,(i)} + G_{k-1}^{(i)}\left(\mathbf{x}_k^{n,(i)} - f_{k-1}\left(\mathbf{x}_{k-1}^{n,(i)}\right) - A_{k-1}^n \hat{\mathbf{x}}_{k-1|k-2}^{l,(i)}\right)\right] \tag{20b}$$

$$P_{k|k-1}^{l,(i)} = A_{k-1}^l \left(P_{k-1|k-2}^{l,(i)} - G_{k-1}^{(i)} A_{k-1}^n P_{k-1|k-2}^{l,(i)}\right)\left(A_{k-1}^l\right)^{\mathrm{T}} + B_{k-1}^l Q_{k-1}^l \left(B_{k-1}^l\right)^{\mathrm{T}} \tag{20c}$$

Secondly, for the target that appears spontaneously at time k, suppose $\Gamma_k\left(\mathbf{x}\right) = \Gamma_k^n\left(\mathbf{x}_k^n\right) \Gamma_k^l\left(\mathbf{x}_k^l\right)$, choose $\Gamma_k^n\left(\mathbf{x}_k^n\right)$ as importance density function and draw a set of particles $\left\{\mathbf{x}_k^{n,(i)}\right\}_{i=L_{k-1}+1}^{L_{k-1}+J_k}$. The linear states of the particles are initialized as $\left\{\hat{\mathbf{x}}_{k|k-1}^{l,(i)}, P_{k|k-1}^{l,(i)}\right\}_{i=L_{k-1}+1}^{L_{k-1}+J_k} = \left\{\overline{\mathbf{x}}_{0|-1}^l, \overline{P}_{0|-1}^l\right\}$.

The weights of particles are computed by

$$\omega_{k|k-1}^{(i)} = \begin{cases} p_{S,k} \omega_{k-1}^{(i)} & \left(i = 1, \cdots, L_{k-1}\right) \\ \dfrac{1}{J_k} & \left(i = L_{k-1}+1, \cdots, L_{k-1}+J_k\right) \end{cases} \tag{21}$$

The cardinality distribution $p_{k|k-1}\left(n\right)$ can be predicted by Eq.(7a).

Updation:

Suppose at time $k-1$ that the prediction intensity $D_{k|k-1}\left(\mathbf{x}\right)$ and the prediction cardinality $p_{k|k-1}\left(n\right)$ are given, and that $D_{k|k-1}\left(\mathbf{x}\right)$ is characterized by $\left\{\omega_{k|k-1}^{(i)}, \mathbf{x}_k^{(i)}\right\}_{i=1}^{L_{k-1}+J_k}$. Applying the KDE theory [21] and Eq.(7d) yields the updated intensity, i.e.,

$$D_k\left(\mathbf{x}\right) = \sum_{i=1}^{L_{k-1}+J_k} \omega_k^{(i)} \kappa_{\sigma_d}^{\mathbf{x}}\left(\mathbf{x} - \mathbf{x}_k^{(i)}\right) \tag{22}$$

where $\mathbf{x}_k^{(i)} = \left[\mathbf{x}_k^{n,(i)}, \hat{\mathbf{x}}_{k|k-1}^{l,(i)}\right]^{\mathrm{T}}$, $\kappa_{\sigma_d}^{\mathbf{x}}$ is Parzen-Rosenblatt kernel of appropriate dimensions (a Parzen-Rosenblatt kernel is a bounded positive and symmetric function for which $\int \kappa\left(\xi\right) d\xi = 1$ and, $\left\|\xi\right\|^{d_\xi} \kappa\left(\xi\right) \to 0$ as $\left\|\xi\right\| \to \infty$, where d_ξ denotes the dimension of variable ξ and $\left\|\cdot\right\|$ is the squared norm), σ_d is the kernel bandwidth depended on d, the dimensions of $\mathbf{x}$ and $\mathbf{x}^{1:d}$, and the weight associated with particle i is computed by

$$\omega_k^{(i)} = \omega_{k|k-1}^{(i)} \left[\frac{\left[1 - p_{D,k}\right] \left\langle \Upsilon_k^1\left[D_{k|k-1}, Z_k\right], p_{k|k-1} \right\rangle}{\left\langle \Upsilon_k^0\left[D_{k|k-1}, Z_k\right], p_{k|k-1} \right\rangle} + \sum_{z \in Z_k} \Psi_{k,z}\left(\mathbf{x}_k^{(i)}\right) \frac{\left\langle \Upsilon_k^1\left[D_{k|k-1}, Z_k \setminus \{z\}\right], p_{k|k-1} \right\rangle}{\left\langle \Upsilon_k^0\left[D_{k|k-1}, Z_k\right], p_{k|k-1} \right\rangle}\right] \tag{23}$$

It can be seen from Eq.(13c) that given $\mathbf{x}_k^n$, the measurement z_k is independent of $\mathbf{x}_k^l$. Then, $\Psi_{k,z}\left(\mathbf{x}_k^{(i)}\right)$ in Eq.(23) can be calculated by

$$\begin{aligned} \Psi_{k,z}\left(\mathbf{x}_k^{(i)}\right) &= \frac{p_{D,k} \left\langle 1, K_k \right\rangle}{K_k\left(z\right)} g_k\left(z \middle| \mathbf{x}_k^{n,(i)}, \mathbf{x}_k^{l,(i)}\right) \\ &= \frac{p_{D,k} \left\langle 1, K_k \right\rangle}{K_k\left(z\right)} g_k\left(z \middle| \mathbf{x}_k^{n,(i)}\right) \end{aligned} \tag{24}$$

The updated cardinality distribution $p_k\left(n\right)$ can be computed by Eq.(7c). Then, the target number can be calculated by

$$n_k = \sum_{n=1}^{\infty} n p_k\left(n\right) \tag{25}$$

The target state estimates are extracted by using mean-shift algorithm. Actually, mean-shift algorithm uses gradient iteration method to calculate extreme points of the PDF [22].

For the particle $\mathbf{x}_k^{(i)}$, the mean-shift vector $m\left(\mathbf{x}_k^{(i)}\right)$ can calculated by

$$m\left(\mathbf{x}_k^{(i)}\right)=\frac{\sum_{j=1}^{L_{k-1}+J_k}\omega_k^{(j)}\kappa_{\sigma_d}^{\mathbf{x}}\left(\mathbf{x}_k^{(i)}-\mathbf{x}_k^{(j)}\right)\mathbf{x}_k^{(j)}}{\sum_{j=1}^{L_{k-1}+J_k}\omega_k^{(j)}\kappa_{\sigma_d}^{\mathbf{x}}\left(\mathbf{x}_k^{(i)}-\mathbf{x}_k^{(j)}\right)}-\mathbf{x}_k^{(i)} \tag{26}$$

Equation (26) indicates that the mean-shift vector $m\left(\mathbf{x}_k^{(i)}\right)$ should be transferred to the spot of the maximum consistent change, which is also the direction of density gradient. The algorithm is to take $\mathbf{x}_k^{(i)}$ as a starting point, then move to the densest place, i.e., $\mathbf{x}_k^{(i)}\rightarrow\mathbf{x}_k^{(i)}+m\left(\mathbf{x}_k^{(i)}\right)$. After some iterations, the optimal locations of the intensity $D_k(\mathbf{x})$ can be obtained. Target states can be estimated from $D_k(\mathbf{x})$ by taking n_k local maxima with the highest weights.

B. Complexity of the proposed particle CPHD filter

In this section the computational complexity of the proposed filter is analysed by introducing the equivalent flop (EF) measure [23]. The EF complexity for an operation is defined as the number of floating-point operations (flops) that results in the same computational time as the operation. A flop is here defined as one addition, subtraction, multiplication, or division of two floating-point numbers. As shown in Section IIB, the computational cost of the proposed CPHD filter mainly manifests in the update step. For each recursion, the EF complexity of the update step can be given by the following polynomial with the number of measurements M_k, i.e.,

$$\begin{aligned}\mathrm{EF}_{\mathrm{update}}&=M_k^3+\frac{11}{2}M_k^2+\left[\frac{1}{2}L_k^2+\left(c_1+\frac{5}{2}\right)L_k\right]M_k+\\&\left(N_{\max}+1\right)\left(\frac{4}{3}N_{\max}^2+\frac{25}{6}N_{\max}-10\right)+\left(c_2L_k+c_3\right)L_k\\&=O\left(M_k^3+L_k^2M_k+N_{\max}^3\right)\end{aligned} \tag{27}$$

where L_k denotes the number of particles at time step k, $N_{\max}$ is the maximum of target number, the coefficient c_1 is used for the calculation of the Gaussian likelihood, c_2 for the calculation of the mean-shift vector, c_3 for the resampling complexity. As shown above, the total EF complexity of the proposed CPHD filter is cubic in the number of measurements M_k, square in the number of particles L_k and cubic in the maximum of target number $N_{\max}$. In practice, the complexity can be reduced by using gating techniques as used in traditional tracking algorithms to eliminate those measurements not associated with targets and by using parallel implementations to reduce the number of operations in the resampling step[24].

V. Simulation Results

In this section, the performance of the proposed CPHD filter is shown via simulations. For multitarget tracking performance evaluation, the statistics of cardinality estimates and optimal subpattern assignment (OSPA) distance are used.

The OSPA distance is defined as

$$\bar{d}_p^{(c)}(X,Y)=\left(\frac{1}{n}\left(\min_{\pi\in\Pi_n}\sum_{i=1}^{m}d^{(c)}\left(\mathbf{x}_i,\mathbf{y}_{\pi(i)}\right)^p+c^p(n-m)\right)\right)^{1/p} \tag{28}$$

where $X=\{\mathbf{x}_1,\cdots,\mathbf{x}_m\}$ and $Y=\{\mathbf{y}_1,\cdots,\mathbf{y}_n\}$ are random finite subsets, $1\le p<\infty$, $c>0$, $m,n\in N_o=\{0,1,2,\cdots\}$, if $m\le n$, and $\bar{d}_p^{(c)}(X,Y)=\bar{d}_p^{(c)}(Y,X)$, if $m>n$; In this paper, the OSPA parameters are set as $p=2$ and $c=50$.

Consider a multitarget bearings-only tracking problem by using three passive sensors. The system model is described by Eq.(1) and (2), in which the number of targets varies with time. The specific parameters of the system are as follows:

$$F=\begin{bmatrix}1&1&0&0\\0&1&0&0\\0&0&1&1\\0&0&0&1\end{bmatrix},\qquad G=\begin{bmatrix}1/2&0\\1&0\\0&1/2\\0&1\end{bmatrix},$$

$$w_k\sim N\left(\cdot;0,\begin{bmatrix}\sigma_{w1}^2&0\\0&\sigma_{w2}^2\end{bmatrix}\right),\ \sigma_{w1}=\sigma_{w2}=0.01\mathrm{km/s^2}.$$

Targets are observed independently by the sensors. The locations of the sensors are set to $S_1(-8,-10)\mathrm{km}$, $S_2(8,-10)\mathrm{km}$, $S_3(0,13.8564)\mathrm{km}$, respectively. The measurement noise $v_k^o\sim N\left(\cdot;0,\sigma_v^2\right)$, $o=1,2,3$, $\sigma_v=0.01\mathrm{rad}$.

Assume no spawning, and that the spontaneous birth RFS obeys Poisson distribution with intensity

$$\Gamma_k(\mathbf{x})=0.2\times\sum_{i=1}^{4}N_i\left(\xi;m_\gamma^{(i)},P_\gamma\right) \tag{29}$$

where $m_\gamma^{(1)}=(-3.5,0,-2,0)^{\mathrm{T}}$, $m_\gamma^{(2)}=(3,0,0,0)^{\mathrm{T}}$, $m_\gamma^{(3)}=(-3.5,0,2,0)^{\mathrm{T}}$, $m_\gamma^{(4)}=(-5,0,5,0)^{\mathrm{T}}$, and $P_\gamma=diag([4,2,4,2])$. The probabilities of target survival and detection are $p_{S,k}=0.99$ and $p_{D,k}=0.98$, respectively. Clutter is modeled as a Poisson RFS with intensity $\lambda_k=3$ over the observation space. For the proposed CPHD filter, the number of particles associated with each birth is $J=500$.

100 Monte Carlo (MC) runs are performed for each filter on the same target tracks and a comparison of the tracking performance is made between the proposed CPHD, PF-CPHD and RBPF-PHD filters. The true target tracks in x- and y-coordinates are shown in Fig.1.

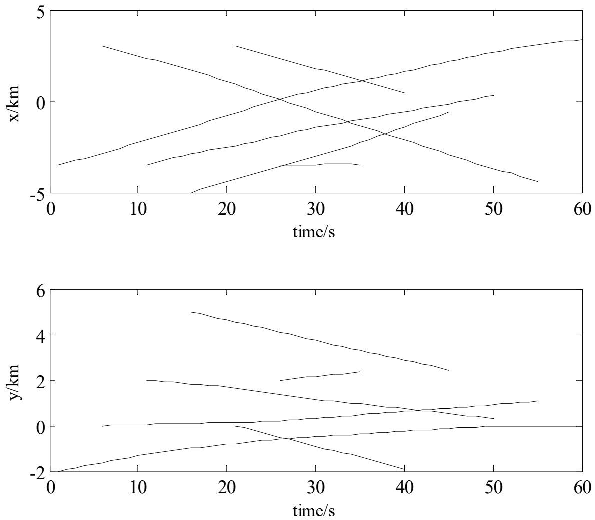

Fig. 1. The true target trajectory in x- and y-coordinates.

In Fig.2, the true number of targets at each time step is shown together with the mean of the estimated cardinality distribution for the filters. In addition, Fig.3 shows the comparison of standard deviation (STD) of cardinality distribution for the filters. These results demonstrate that the target number estimations of all these filters can converge to the correct number of targets. The RBPF-PHD filter has the fastest convergence speed for the estimation of target number. The proposed CPHD filter has the least variance to the target number estimation, which shows that the proposed algorithm is more reliable. Note that, these filters' correct convergence to the mean number of targets is only true as average behavior. In practical use, more attention may be paid to the robustness of the filtering algorithm since from the viewpoint of each single trial, the proposed CPHD filter's estimate is far more reliable and accurate.

In addition, from Fig.2, it can be seen that when the number of targets changes, the PF-CPHD and the proposed algorithms have a large delay in the estimation of target number , while the RBPF-PHD filter has a small delay. One possible reason for this is that the RBPF-PHD filter produces a cardinality estimate with a relatively high variance, thus it has low confidence in its estimate and is easily influenced by new incoming measurement information. So, the RBPF-PHD filter responds faster.

In Fig. 4, the MC average of the OSPA distances at each time step for these filters are shown. It can be seen that the proposed CPHD filter appears to perform better than the PF-CPHD and RBPF-PHD filters throughout the entire simulation due to that the proposed CPHD filter provides more accurate estimates of target states and target number. However, at several time instants when there are cardinality changes, the PF-CPHD and proposed CPHD filters are penalized much more heavily than the RBPF-PHD filter as a result of being slower to respond to the change.

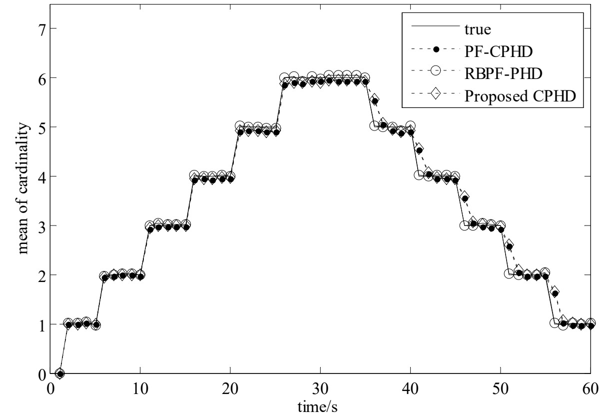

Fig. 2. The estimated mean of cardinality versus time.

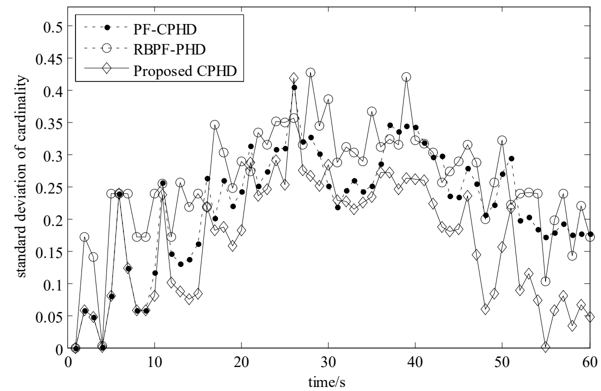

Fig. 3. The estimated STD of cardinality versus time.

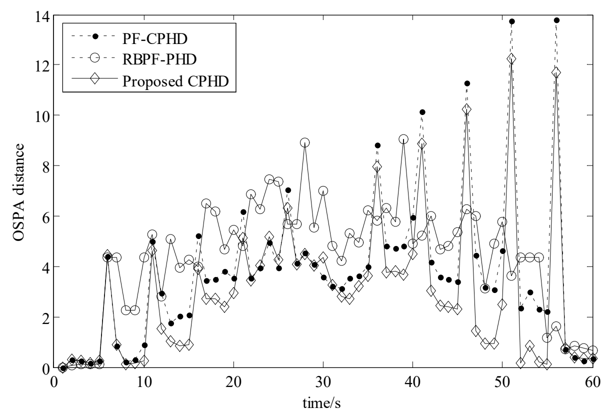

Fig. 4. MC average OSPA distance versus time.

In another experiment, all the above simulations are replicated for a varying number of particles J to verify the performance of the proposed filter. Figure 5 shows the mean of the MC average OSPA distance with different number of particles for the proposed CPHD filter. It can be observed that the average OSPA distance of the proposed CPHD filter is smaller than that of the PF-CPHD and RBPF-PHD filters. An increase in the number of particles reduces the OSPA distance, and increasing the number of particles further gives minor improvements. Note that as expected, with the increase of the number of particles, the operation time of each algorithm will also increase. Figure 6 shows the comparison of the average operation time of the three algorithms. In the proposed CPHD filter, due to the joint estimation of the target intensity and the probability distribution of the target number, and the use of the mean shift algorithm for state extraction, the operation time is the largest.

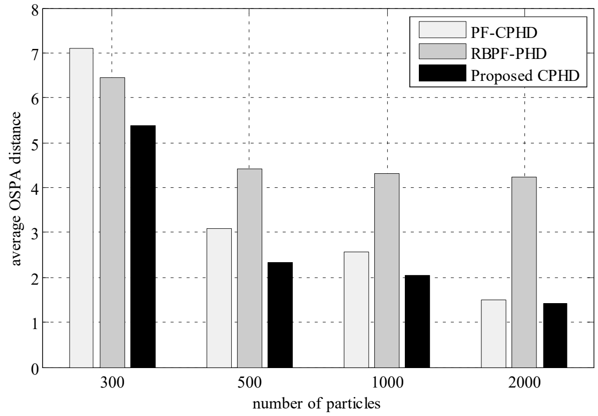

Fig. 5 Comparison of the average OSPA distance.

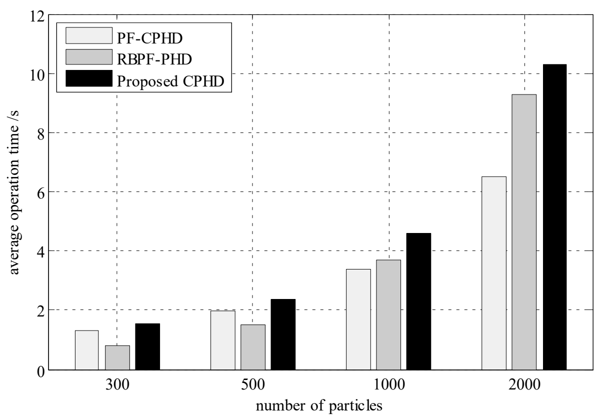

Fig.6 Comparison of average operation time.

In order to verify the relationship between the complexity of the proposed CPHD filter and the number of measurements, the simulation is redesigned. Assuming that the number of targets does not change with time, the number of measurements is increased by increasing the number of targets. The clutter intensity $\lambda_k = 3$, the particle number $J = 500$, and other simulation parameters are unchanged. The complexity of the algorithm is represented by the operation time. Figure 7 shows the relationship between the average operation time of the filter and the number of measurements. For the convenience of comparative analysis, a cubic curve equation about the number of measurements is drawn in the figure. It can be seen that the complexity of the proposed CPHD filter is related to multiple parameters, and approximately satisfies the cubic growth relationship with the increase of the number of measurements.

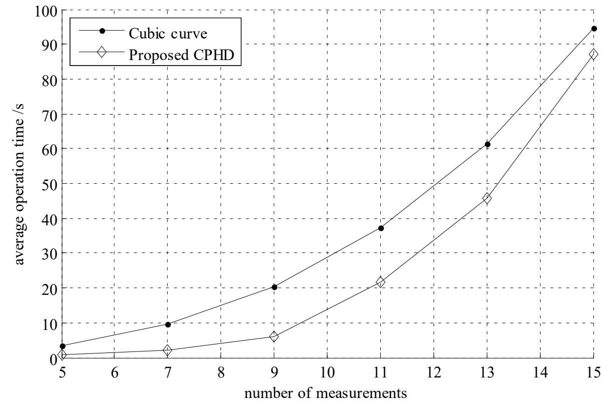

Fig.7 The relationship between the average operation time and the number of measurements.

VI. Conclusions

The CPHD filter has become one of the most acclaimed methods for multiple targets tracking in the presence of births, deaths, clutter and missed detections. This article proposed a new particle implementation framework for the CPHD filter as a solution to the multitarget tracking problem for the class of mixed linear/nonlinear state space models. Furthermore, KDE theory and mean-shift algorithm are adopted for the extraction of target states. The total computational complexity of the proposed CPHD filter is cubic in the number of measurements, square in the number of particles and cubic in the maximum of target number. Simulations verified that the proposed CPHD filter performs accurately and shows a dramatic reduction in the variance of the estimated number of targets when compared to the PF-CPHD and RBPF-PHD filters. The main drawback of the proposed CPHD filter is the calculation load, which is to be addressed in the future work. We will work on particle gate selection technology and better particle resampling technology.

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