

# Distributed UFIR Filtering with Applications to Environmental Monitoring

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Received: August 30, 2020. Revised: March 27, 2021. Accepted: April 8, 2021. Published: April 12, 2021.

**Abstract**—Environmental monitoring requires an analysis of large and reliable amount of data collected through node stations distributed over a very wide area. Equipment used in such stations are often expensive that limits the amount of sensing stations to be deployed. The technology known as Wireless Sensor Networks (WSN) is a viable option to deliver low-cost sensor information. However, unpredictable issues such as interference from electromagnetic sources, damaged and unstable sensors and the landscape itself may cause the network to suffer from unstable links as well as missing and corrupted data. Therefore robust estimators are required to mitigate such effects. In this sense, the unbiased finite impulse response (UFIR) filter is used as a robust estimator for applications over WSN, especially when the process statistics are unknown. In this paper, we investigate the robustness of the distributed UFIR (dUFIR) filter with optimal consensus on estimates against missing and incorrect data. The dUFIR algorithm is tested in two different scenarios of very unstable WSN using real data. It is shown that the dUFIR filter is more suitable for real life applications requiring the robustness against missing and corrupted measurements under the unknown noise statistics.

**Index Terms**—WSN, robust estimation, distributed estimation, missing data, environmental monitoring.

## I. INTRODUCTION

The interest to environmental monitoring has grown in recent decades due to climate change and serious natural disasters. However, to provide a reliable monitoring a large amount of data is required for environmental scientist to be able to provide useful information about the behavior of physical variables in question, provide forecasting of such behaviors, and emit or validate recommendations that will lead to new legislation [1]–[3]. The collection of data is often performed manually at a local scale, which sometimes is a difficult task due to extreme environmental events. Also, harsh weather may affect the sensing stations causing a significant data loss.

The wireless sensor networks (WSN) are well suited for environmental data acquisition [4]–[7] and allow the implementation of distributed methods, which are known to be more robust than centralized approaches. The robustness can also be improved using the unbiased finite impulse response (UFIR) filtering approach, which is effective in harsh environments, where unpredictable issues such as interference from electromagnetic sources, damaged and unstable sensors and

the landscape itself may cause the network to suffer from unstable links as well as missing and corrupted data. In many cases, optimal estimation is required along with adequate sensor fusing [8]–[11] to be robust against missing data, model errors (mismodeling), and incomplete information about noise statistics.

When dealing with WSNs applications, it is important to consider the restrictions of WSNs due to limited battery life and processing power [10]. Such restrictions demand the development of appropriate algorithms to ensure the efficient use of these limited resources. In this sense, distributed filtering helps to improve battery life by minimizing the computational burden while performing real time estimation [12], [13]. Under the distributed filtering approach, each node is tasked to estimate  $\mathcal{Q}$  and a consensus protocol is implemented to average the estimates, measurements, or information [14], so that all nodes come to an agreement regarding a common value which is called the group decision value [15].

The Kalman filter (KF) is a very popular sensor fusion technique [16] mainly because of its optimal estimations and low computational burden [17]. These advantages have motivated many authors to address the consensus problem in WSNs under the KF approach. A KF-based structure proposed in [18] requires each node to locally aggregate data and the covariance matrices taken from the neighbors and, in a posterior step, compute estimates using a KF with a consensus term. In [19], the KF has been developed as a fusion technique for local estimation and a consensus matrix. In [20], a KF-based algorithm has been presented to address an issue with missing data.

Let us notice that the KF optimality is guaranteed only under the complete knowledge of the Gaussian noise statistics, adequate process modeling, and initial conditions [21]–[24]. Otherwise, the performance of the KF may drastically degrade and become unacceptable for real world WSN applications [25], [26]. It has been proven in [27]–[30] that, by using filters operating with finite data horizons, a better robustness can be achieved. Under such an assumption, in [31], a developed unbiased finite impulse response (UFIR) filter with consensus on measurements demonstrated better robustness than the KF for WSN. In [25], the UFIR approach was used to develop a distributed finite impulse response with consensus on estimates, but the consensus factor was obtained through a previous analysis and without a mathematical background. In

This investigation was partly supported by the Mexican CONACyT-SEP Project A1-S-10287, Funding CB2017-2018.

[32], a distributed UFIR (dUFIR) filter has been developed and tested over WSN for a rapid maneuvering object to show a better performance than the KF and  $H_\infty$  filter.

The issue of missing data can be addressed using the KF and UFIR approaches with an appropriate prediction stage [33]. A moving average estimator has been designed in [34] for weak observability. A consensus finite-horizon  $H_\infty$  approach was developed in [35] under missing data. In [36], an extended KF was modified with this aim and a KF was developed to address intermittent observations in [37].

For a single sensor, in [26] a UFIR filter proved better robustness against missing data and in [38], a UFIR filter was developed under delayed and missing data. Regarding to WSNs, a version of the dUFIR filter with a prediction option was developed and tested in [39] to provide a better robustness.

Let us notice that real life applications, such as whether monitoring, often suffers from missing or false data and uncertain noise. The issue is illustrated in Fig 1, where the real temperature data are taken from a whether station. In

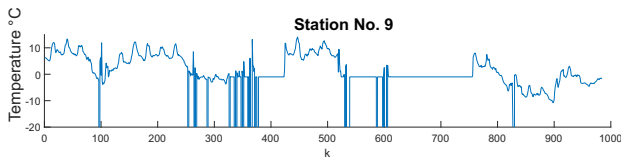


Fig. 1. Missing data and uncertain (colored) noise in temperature measurement data taken from [40].

the first 300 samples, one watches for noisy measurements with missing data and around  $k = 400$ , the sensor generates incorrect measurement of  $-1C$ .

In this work we present an application of the dUFIR under unknown statistics and demonstrate with simulations its minimum link requirements, robustness and data reconstruction capabilities under real missing data and false measurements.

## II. DUFIR FILTER FOR WSN UNDER MISSING DATA

Consider dynamics of a quantity  $Q$  measured over a distributed WSN and represent it with the following discrete  $K$ -states space equations,

$$x_k = F_k x_{k-1} + B_k w_k, \quad (1)$$

$$\bar{y}_k^{(i)} = H_k^{(i)} (F_k x_{k-1}), \quad (2)$$

$$y_k^{(i)} = \gamma_k (H_k^{(i)} x_k + v_k^{(i)}) + (1 - \gamma_k) \bar{y}_k^{(i)}, \quad (3)$$

$$y_k = H_k x_k + v_k, \quad (4)$$

where  $x_k \in \mathbb{R}^K$ ,  $u_k \in \mathbb{R}^M$ ,  $F_k \in \mathbb{R}^{K \times K}$ ,  $E_k \in \mathbb{R}^{K \times M}$ , and  $B_k \in \mathbb{R}^{K \times L}$ . The  $i$ th,  $i \in [1, n]$ , is a part of the WSN regarded as an undirected graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where each vertex  $v^{(i)} \in \mathcal{V}$  is a node and each link is an edge of a set  $\mathcal{E}$ , for  $i \in \mathcal{I} = \{1, \dots, n\}$  and  $n = |\mathcal{V}|$  with  $J$  inclusive neighbors.

Each node measures  $x_k$  by  $y_k^{(i)} \in \mathbb{R}^p$ ,  $p \leq K$ , with  $H_k^{(i)} \in \mathbb{R}^{p \times K}$ . Local data  $y_k^{(i)}$  are united in the observation vector  $y_k = [y_k^{(i)T} \dots y_k^{(j)T}]^T \in \mathbb{R}^{Jp}$  with  $H_k = [H_k^{(i)T} \dots H_k^{(j)T}]^T \in \mathbb{R}^{Jp \times K}$ . Noise vectors  $w_k \in \mathbb{R}^L$  and

$v_k = [v_k^{(1)T} \dots v_k^{(n)T}]^T \in \mathbb{R}^{Jp}$  are zero mean, not obligatorily white Gaussian, uncorrelated, and with the covariances  $Q_k = E\{w_k w_k^T\} \in \mathbb{R}^{L \times L}$ ,  $R_k = \text{diag}[R_k^{(1)T} \dots R_k^{(n)T}]^T \in \mathbb{R}^{Jp \times Jp}$ , and  $R_k^{(i)} = E\{v_k^{(i)} v_k^{(i)T}\}$ . A binary variable  $\gamma_k$  serves as an indicator of whether a measurement exist ( $\gamma_k = 1$ ) or not ( $\gamma_k = 0$ ), in which case the measurement prediction  $\bar{y}_k^{(i)}$  (2) is used by substituting  $x_{k-1}$  with the estimate.

### A. Predictive Distributed UFIR Filter

To obtain optimum estimates and achieve a consensus on estimates, we formulate the distributed estimate as

$$\hat{x}_k^c = \hat{x}_k + \lambda_k^{\text{opt}} \Sigma_k, \quad (5)$$

where the centralized and individual estimates,  $\hat{x}_k$  and  $\hat{x}_k^{(i)}$  respectively, are obtained through

$$\hat{x}_k = K_{m,k} Y_{m,k}, \quad (6)$$

$$\hat{x}_k^{(i)} = K_{m,k}^{(i)} Y_{m,k}^{(i)} \quad (7)$$

and  $\Sigma_k = \sum_j (\hat{x}_k^{(j)} - \hat{x}_k^{(i)})$  is a consensus protocol that minimizes the disagreement between the first-order neighbors [18]. A consensus factor  $\lambda_k^{\text{opt}}$  is chosen such that the root mean squared error (RMSE) is minimized by

$$\lambda_k^{\text{opt}} = \arg \min_{\lambda_k} \{\text{tr } P(\lambda_k)\} \quad (8)$$

with  $P(\lambda_k) = E\{(x - \hat{x}^{ic})(x - \hat{x}^{ic})^T\}$  as the relevant error covariance.

### B. Batch Distributed UFIR Filter Design

To determine gains  $K_{m,k}$  and  $K_{m,k}^{(i)}$ , we express the model equations (1)–(4) in the extended state space form over horizon  $N$  as described in [25], [41],

$$X_{m,k} = A_{m,k} x_m + D_{m,k} W_{m,k}, \quad (9)$$

$$Y_{m,k} = C_{m,k} x_m + M_{m,k} W_{m,k} + V_{m,k}, \quad (10)$$

$$Y_{m,k}^{(i)} = C_{m,k}^{(i)} x_m + M_{m,k}^{(i)} W_{m,k} + V_{m,k}^{(i)}, \quad (11)$$

where  $X_{m,k} = [x_m^T \ x_{m+1}^T \ \dots \ x_k^T]^T$ ,  $Y_{m,k} = [y_m^T \ y_{m+1}^T \ \dots \ y_k^T]^T$ ,  $W_{m,k} = [w_m^T \ w_{m+1}^T \ \dots \ w_k^T]^T$ ,  $V_{m,k} = [v_m^T \ v_{m+1}^T \ \dots \ v_k^T]^T$ ,  $Y_{m,k}^{(i)} = [y_m^{(i)T} \ y_{m+1}^{(i)T} \ \dots \ y_k^{(i)T}]^T$ ,  $V_{m,k}^{(i)} = [v_m^{(i)T} \ v_{m+1}^{(i)T} \ \dots \ v_k^{(i)T}]^T$ , and the extended matrices are

$$A_{m,k} = [I \ F_{m+1}^T \ \dots \ (F_k^{m+1})^T]^T, \quad (12)$$

$$D_{m,k} = \begin{bmatrix} B_m & 0 & \dots & 0 & 0 \\ F_{m+1} B_m & B_{m+1} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F_{k-1}^{m+1} B_m & F_{k-1}^{m+2} B_{m+1} & \dots & B_{k-1} & 0 \\ F_k^{m+1} B_m & F_k^{m+2} B_{m+1} & \dots & F_k B_{k-1} & B_k \end{bmatrix}, \quad (13)$$

$$C_{m,k} = \bar{C}_{m,k} A_{m,k}, M_{m,k} = \bar{C}_{m,k} D_{m,k}, C_{m,k}^{(i)} = \bar{C}_{m,k}^{(i)} A_{m,k},$$

$$M_{m,k}^{(i)} = \bar{C}_{m,k}^{(i)} D_{m,k}, \text{ where}$$

$$\bar{C}_{m,k} = \text{diag}(H_m H_{m+1} \dots H_k), \quad (14)$$

$$\bar{C}_{m,k}^{(i)} = \text{diag}(H_m^{(i)} H_{m+1}^{(i)} \dots H_k^{(i)}), \quad (15)$$

$$\mathcal{F}_r^g = \begin{cases} F_r F_{r-1} \dots F_g, & g < r + 1 \\ I, & g = r + 1 \\ 0, & g > r + 1 \end{cases}. \quad (16)$$

Referring to [25], equation (5) can now be rewritten as

$$\hat{x}_k^c = K_{m,k} Y_{m,k} + J \lambda_k^{\text{opt}} K_{m,k} Y_{m,k} - J \lambda_k^{\text{opt}} K_{m,k}^{(i)} Y_{m,k}^{(i)}. \quad (17)$$

Since we are interested in a robust UFIR filter that ignores the initial values, the unbiasedness condition must hold for the distributed, centralized and individual estimates,

$$E\{\hat{x}_k^c\} = E\{\hat{x}_k\} = E\{\hat{x}_k^{(i)}\} = E\{x_k\} \quad (18)$$

where

$$x_k = \mathcal{F}_k^{m+1} x_m + \bar{D}_{m,k} W_{m,k} \quad (19)$$

with  $\bar{D}_{m,k} = [\mathcal{F}_k^{m+1} B_m \mathcal{F}_k^{m+2} B_{m+1} \dots F_k B_{k-1} Bk]$ . The corresponding gains are defined by

$$K_{m,k} = G_k C_{m,k}^T, \quad (20)$$

$$K_{m,k}^{(i)} = G_k^{(i)} C_{m,k}^{(i)T}, \quad (21)$$

where  $G_k = (C_{m,k}^T C_{m,k})^{-1}$  and  $G_k^{(i)} = (C_{m,k}^{(i)T} C_{m,k}^{(i)})^{-1}$ .

In real world applications, the nodes of the WSN may be unable to implement equation (17) due to large-dimension matrices and operations involved into the limited memory resources of the smart sensors. Therefore, below we develop an iterative form of (17) which fits better with the WSNs resources.

### C. Optimum $\lambda_k$

The optimal value of  $\lambda_k^{\text{opt}}$  is obtained by solving

$$\frac{\partial}{\partial \lambda_k} \text{tr} P_k = 0 \quad (22)$$

where  $P_k = E\{\varepsilon_k \varepsilon_k^T\}$  is the error covariance matrix and  $\varepsilon_k = x_k - \hat{x}_k^c$  is the estimation error. The resulting batch form of  $\lambda_k^{\text{opt}}$  is

$$\lambda_k^{\text{opt}} = -\frac{1}{J} (K_{m,k} \bar{R}_{m,k} K_{m,k}^T - G_k G_k^{(i)-1} K_{m,k}^{(i)} \times \bar{R}_{m,k}^{(i)} K_{m,k}^{(i)T}) (K_{m,k} \bar{R}_{m,k} K_{m,k}^T - 2G_k G_k^{(i)-1} \times K_{m,k}^{(i)} \bar{R}_{m,k}^{(i)} K_{m,k}^{(i)T} + K_{m,k}^{(i)} \bar{R}_{m,k}^{(i)} K_{m,k}^{(i)T})^{-1}. \quad (23)$$

where

$$\bar{R}_{m,k} = E\{v_{m,k} v_{m,k}^T\} = \text{diag}(R_m \dots R_k),$$

$$\bar{R}_{m,k}^{(i)} = E\{v_{m,k}^{(i)} v_{m,k}^{(i)T}\} = \text{diag}(R_m^{(i)} \dots R_k^{(i)}),$$

$$\tilde{R}_{m,k}^{(i)} = E\{v_{m,k} v_{m,k}^{(i)T}\} = \text{diag}(\tilde{R}_m^{(i)} \dots \tilde{R}_k^{(i)}).$$

If, for some particular application, the network and the process dynamics are both time invariant,  $\lambda_k^{\text{opt}}$  is also time invariant, to mean that equation (23) can be computed beforehand and embedded into the nodes.

### D. Iterative Distributed UFIR filter

An iterative algorithm for the centralized estimates  $\hat{x}_k$  can be derived following the procedure described in [41], including a variable  $l$  that starts at  $l = k - N + K + 1$  and ending in  $l = k$ . The recursions are given by

$$G_l = [H_l^T H_l + (A_l G_{l-1} A_l^T)^{-1}]^{-1}, \quad (24)$$

$$\hat{x}_l = A_l \hat{x}_{l-1}, \quad (25)$$

$$\hat{x}_l = \hat{x}_l^- + G_l H_l^T (y_l - H_l \hat{x}_l^-). \quad (26)$$

The initial values  $G_{l-1}$  and  $\hat{x}_{l-1}$  are computed at  $s = k - N + K$  in batch forms as

$$G_s = (C_{m,s}^T C_{m,s})^{-1}, \quad (27)$$

$$\hat{x}_s^c = G_s C_{m,s}^T Y_{m,s}. \quad (28)$$

The individual estimates  $\hat{x}_k^{(i)}$  are provided by

$$G_l^{(i)} = [H_l^{(i)T} H_l^{(i)} + (A_l G_{l-1}^{(i)} A_l^T)^{-1}]^{-1}, \quad (29)$$

$$\hat{x}_l^{(i)-} = A_l \hat{x}_{l-1}^{(i)}, \quad (30)$$

$$\hat{x}_l^{(i)} = \hat{x}_l^{(i)-} + G_l^{(i)} H_l^{(i)T} (y_l^{(i)} - H_l^{(i)} \hat{x}_l^{(i)-}) \quad (31)$$

with the initial values

$$G_s^{(i)} = (C_{m,s}^{(i)T} C_{m,s}^{(i)})^{-1}, \quad (32)$$

$$\hat{x}_s^{(i)} = G_s^{(i)} C_{m,s}^{(i)T} Y_{m,s}^{(i)}. \quad (33)$$

A pseudo code of the designed iterative dUFIR algorithm with consensus on estimates is listed as Algorithm 1.

## III. APPLICATIONS TO ENVIRONMENT MONITORING

We consider temperature measurements provided in 2007 at the Grand-St-Bernard pass at 2400 m between Switzerland and Italy as part of the Sensorscope project, which aims to develop a large-scale distributed environmental measurement system centered on a wireless sensor network. Measurements were recorded individually by low-cost sensing stations and are available from [40]. In this work, we examine only the stations shown in Fig. 4 and Fig. 7, depicted as red dots.

Measurements were performed each two seconds during two months. For each sensor, the average of the measurements was computed each hour along with the error variance. In Fig 2, we show the resulting standard deviations for each sensor, where stations 2 and 9 demonstrate large temperature deviations for unknown reasons.

The eleven stations present a similar behavior on their measurements. However, some of them present large gaps of information and a very unstable performance. The individual one-hour average temperature measurements of selected stations are sketched in Fig. 3 where it is important to notice that stations 2 and 9 also conduct incorrect measurements of  $-1^\circ\text{C}$  that cannot be regarded as missing data.

**Algorithm 1: Iterative dUFIR Filtering Algorithm**

**Data:**  $y_k, R_k^{(i)}, R_k, \lambda_k^{opt}$   
**Result:**  $\hat{x}_k$

```

1 begin
2   for  $k = N - 1 : \infty$  do
3      $m = k - N + 1, \quad s = m + K - 1;$ 
4      $G_s = (\mathcal{H}_{m,s}^T \mathcal{H}_{m,s})^{-1};$ 
5      $G_s^{(i)} = (\mathcal{H}_{m,s}^{(i)T} \mathcal{H}_{m,s}^{(i)})^{-1};$ 
6     if  $\gamma_k = 0$  then
7        $y_k^{(j)} = H_k^{(j)} F_k \hat{x}_{k-1}^{(j)};$ 
8     end if
9      $\tilde{x}_s = G_s \mathcal{H}_{m,s}^T Y_{m,s};$ 
10     $\tilde{x}_s^{(i)} = G_s^{(i)} \mathcal{H}_{m,s}^{(i)T} Y_{m,s}^{(i)};$ 
11    for  $l = s + 1 : k$  do
12       $\hat{x}_l^- = F_l \hat{x}_{l-1};$ 
13       $\hat{x}_l^{(i)-} = F_l \hat{x}_{l-1}^{(i)};$ 
14       $G_l = [H_l^T H_l + (F_l G_{l-1} F_l^T)^{-1}]^{-1};$ 
15       $G_l^{(i)} = [H_l^{(i)T} H_l^{(i)} + (F_l G_{l-1}^{(i)} F_l^T)^{-1}]^{-1};$ 
16       $\hat{x}_l = \hat{x}_l^- + G_l H_l^T (y_l - H_l \hat{x}_l^-);$ 
17       $\hat{x}_l^{(i)} = \hat{x}_l^{(i)-} + G_l^{(i)} H_l^{(i)T} (y_l^{(i)} - H_l^{(i)} \hat{x}_l^{(i)-});$ 
18    end for
19     $\hat{x}_k^c = (I + J \lambda_k^{opt}) \tilde{x}_k - J \lambda_k^{opt} \tilde{x}_k^{(i)};$ 
20  end for
21 end
22 † First data  $y_0, y_1, \dots, y_{N-1}$  must be available.
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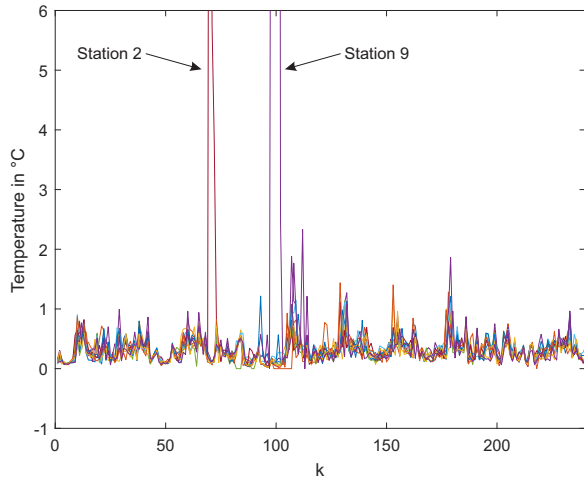


Fig. 2. Temperature standard deviations observed in the stations.

**A. Tuning dUFIR Algorithm**

To apply the dUFIR algorithm, we use model (1)–(4) with the following matrices [26],

$$A = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}, \quad H^{(i)} = [1 \quad 0],$$

where  $\tau = 1$  and  $B = I$ . As stated by (23), to compute  $\lambda_k^{opt}$  we need individual variances of the sensors, but this

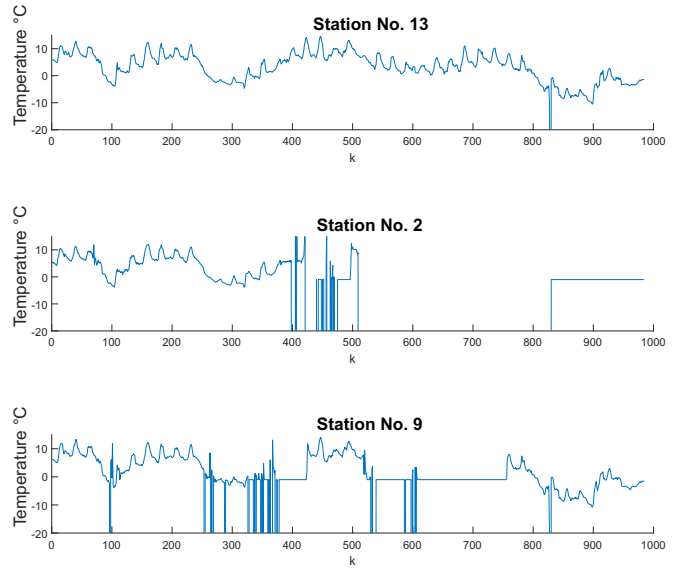


Fig. 3. Temperature measurements conducted by 11 stations.

information is not available in the data set. Furthermore, it is unclear if all sensors are of the same manufacturer. In Fig 2, we observe an abnormal behavior of the standard deviation at  $k = 70$  and  $k = 101$  for station 2 and 9 respectively, so we take the average from  $110 \leq k \leq 210$  and determine an estimated variance for each sensor. The results are shown in table I and the optimum horizon was measured to be  $N_{opt} = 37$  [28].

**B. Network with 350 m Link Range**

To test the algorithm, we simulate a WSN for a maximum link distance of 350 m. The resulting topology is sketched in Fig. 4. The estimation results by Algorithm 1 are shown

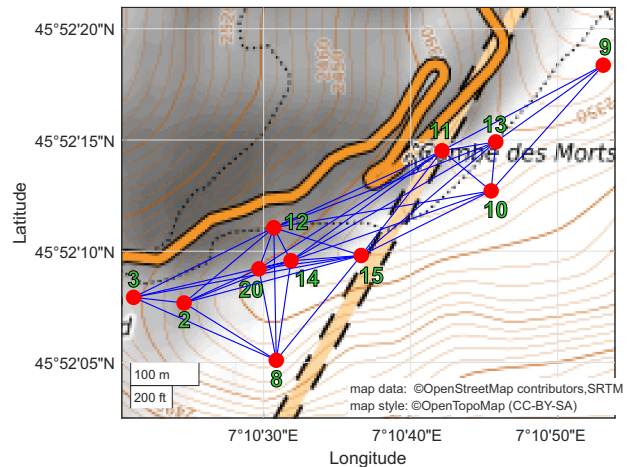


Fig. 4. Simulated WSN connections between sensing stations for a link distance of 350m.

in Fig. 5 for three sensing stations. Here, noise reduction is observed in all stations and yet large gaps are bridged over in the 2nd station (Fig. 5 b) and 9th station (Fig. 5 c).

TABLE I  
 INDIVIDUAL VARIANCES FOR EACH STATION.

Station	10	11	12	13	14	15	2	20	3	8	9
Variance	0.13	0.17	0.18	0.2	0.13	0.17	0.15	0.15	0.16	0.11	0.19

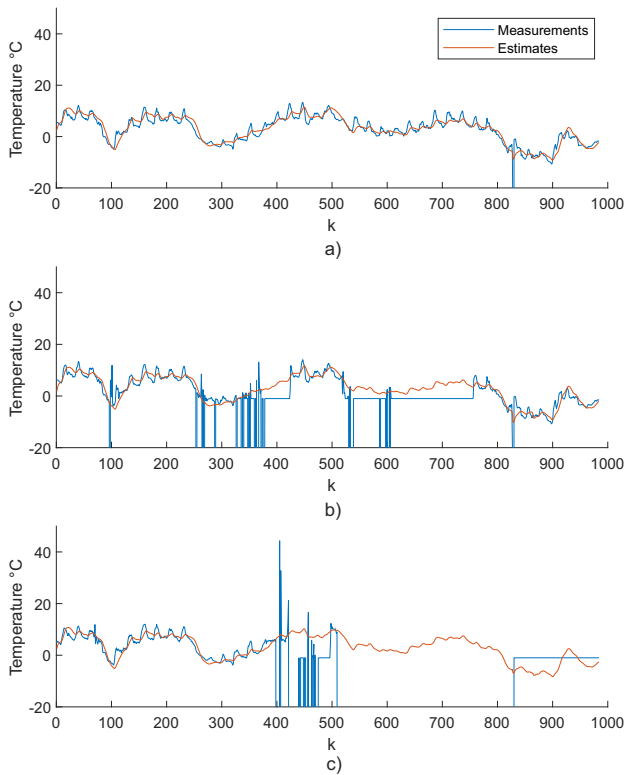


Fig. 5. Temperature measurements and estimates: a) 10th station, b) 9th station, and c) 2nd station.

A key difference between the 9th and 2nd stations is observed in the range of  $540 < k < 780$ . While measurements are completely lost in the 2nd station, a false measurement of  $-1^{\circ}\text{C}$  is recorded in the 9th station. The algorithm employs the prediction option only when missing data are detected and it considers a wrong measurements of  $-1^{\circ}\text{C}$  as true. However, due to the distributive nature of the dUFIR filter, the estimates of the 9th station do not get away from the remaining stations. This can be seen in Fig. 6a, where we show estimates of all of the stations. In Fig. 6b, the estimate variances are considered as an indicator of disagreements between the nodes. It can also be seen that much less disagreements are observed when the measurements are correct.

### C. Network with 200 m Link Range

Performance of the dUFIR Algorithm 1 depends on the amount of redundant available information. When the number of the links decreases, the disagreements and the estimation errors increase. In Fig. 7, we consider the same base stations but with a restriction link range of 200 m. In this case, a smaller number of the links are available and the 9th station has a single link rather than three links in the previous case

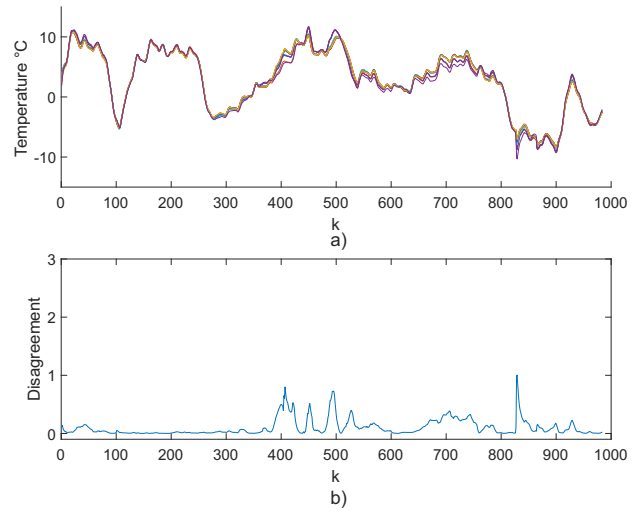


Fig. 6. Temperature measurements and estimates: a) all stations and b) disagreement between estimates.

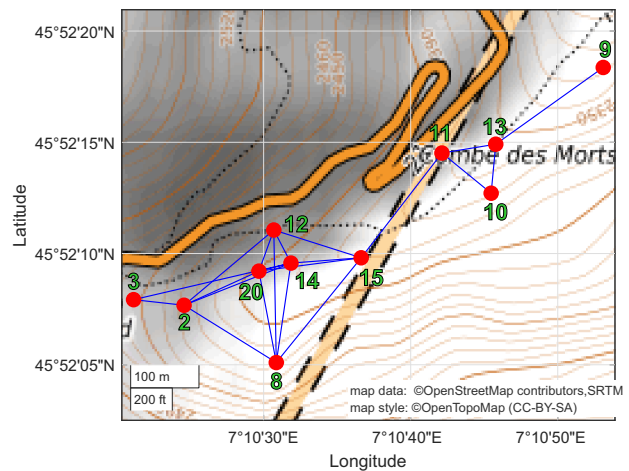


Fig. 7. Simulated network links between the sensing stations for the range of 200 m.

(Fig 4). Due to a lack of the redundant information and an inability to process wrong data as missing, estimates by the 9th station deviate from those by other stations (Fig 8a). Here, one can also see the effect of the 9th station on the performance of the 13th station. Under such circumstances, the consensus and prediction capabilities of the Algorithm 1 are not able to compensate incorrect data in the 9th station that results in growing disagreement between the estimates (Fig. 8b).

The simulations were performed using MATLAB®R2019a operating on AMD Ryzen 7 CPU(2.3 GHz) with 16.0 GB RAM. The total simulation time of the entire network was 34 seconds.

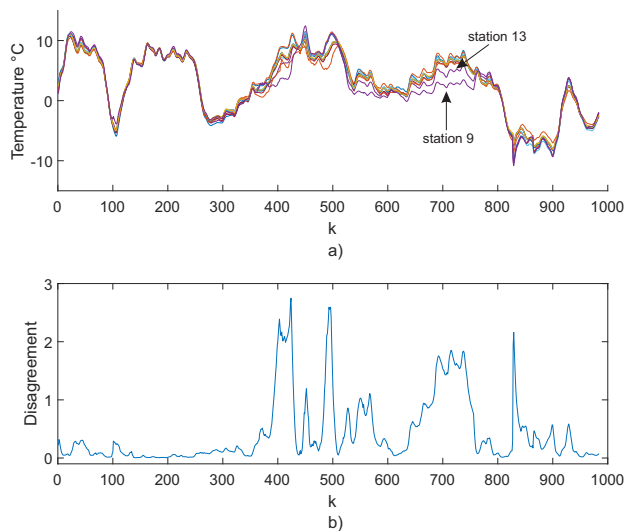


Fig. 8. Temperature measurements and estimates: a) all stations and b) disagreement between the estimates.

#### IV. CONCLUSIONS

In this paper, we have applied the developed dUFIR filtering algorithm to real temperature measurements with missing and incorrect data. We have discussed the dUFIR filter performance in two feasible scenarios of different numbers of links. As the main contributions of this work, we found that under the allowed minimum three links the dUFIR filter produces acceptable estimates and provides a good data reconstruction under a scenario where the process noise statistics are unavailable, proving that dUFIR presents a great advantage against the KF-based algorithms. We also demonstrate the robustness of the algorithm against incorrect measurements that cannot be regarded as missing data. We are already working in the development of the dUFIR filter with consensus on estimates for time-delayed signals and colored noise.

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