Hilbert-Huang Transform Based Time-Frequency Distribution and Comparisons with Other Three

Ming Li [1], Xue-Kang Gu [2], and Shen-Shen Yang [1]

***Abstract*—** Time-frequency distribution (TFD) of signals gains increasing applications in various areas of sciences and engineering for processing non-stationary signals and nonlinear signals. Traditional methods in the field are short-time Fourier transform (STFT), generalized TFDs in the Cohen class (GTFD), and wavelet transform (WT) based TFD. Recently, Huang et al. introduced a new method called Hilbert-Huang transform (HHT). This is an adaptively data-driven approach without the limitations caused by various window functions for STFT, different kernels for GTFD, and different mother functions for WT. This paper discusses four types of TFDs with demonstrations, providing a case to show that HHT based TFD may have high resolution.

***Keywords*—** Time-frequency distribution, Hilbert-Huang transform, wavelet based time-frequency distribution, generalized time-frequency distribution, short-time Fourier transform based time-frequency distribution.

I. Introduction

SUPPOSE $\{X(t)\}$ for $t \in \mathbb{R}$ is a stationary process. Then, its autocorrelation function (ACF) is given by

$$\mathrm{E}[X(t)X(t+\tau)] = r_{XX}(\tau),$$

where E stands for mean operation and $\tau \in \mathbb{R}$ is time lag. Power spectral density (PSD) function of $\{X(t)\}$, denoted by $S_{XX}(\omega) = \mathrm{F}[r(\tau)]$ is time invarying, where F means the Fourier transform and $\omega \in \mathbb{R}$. To be precise,

$$S_{XX}(\omega) = \mathrm{F}\left[r_{XX}(\tau)\right] = \int_{-\infty}^{\infty} r_{XX}(\tau) e^{-j\omega\tau} d\tau. \quad (1)$$

Hence, the PSD of a stationary process is analyzed in frequency domain without time information.

Computational methods of the PSDs for stationary processes have been well studied. However, spectral analysis of non-stationary processes and nonlinear signals is worth studying in practice, such as the response of structures in a given random loading environment, see e.g. [1-3].

The PSD of a non-stationary process is time varying. That is, it is also a function of time in addition to frequency. More precisely, the PSD of a non-stationary process $\{x(t)\}$ should be expressed by $S_{xx}(t, \omega)$, which is called evolutionary spectrum. In the field of signal processing, it is usually termed time-frequency (TF) distribution (TFD). By analogy with stationary processes, the PSD of a non-stationary process is defined as

$$S_{xx}(t,\omega) = \mathrm{F}\left[r_{xx}(t,\tau)\right] = \int_{-\infty}^{\infty} r_{xx}(t,\tau) e^{-j\omega\tau} d\tau. \quad (2)$$

Hence, the PSD of a non-stationary process is analyzed in a TF plane. It can be used to study statistical properties of $x(t)$. For instance, the instantaneous mean square value at t is given by

$$\mathrm{E}\left[x^2(t)\right] = r_{xx}(t,0) = \int_{-\infty}^{\infty} S_{xx}(t,\omega) d\omega. \quad (3)$$

Some early work regarding the mathematical properties of TFD was reported by Priestley [4], Loynes [5], Mark [6], Shinozuka [7,8]. There are several types of methods that are commonly used in engineering to describe $S_{xx}(t, \omega)$, such as Priestley's evolutionary spectrum [4], Mark's physical spectrum [6], short-time Fourier transform (STFT) [9,10], TFDs in the Cohen class [11], and wavelet-based TFD [12]. Recently, Huang and et al. introduced a new type of TFD based on Hilbert-Huang transform (HHT) [13,14]. Its applications in practice are noticeable, see e.g. Du and Yang [15], Yang and Gao [16,17].

Recall that resolution is a standard issue in spectrum analysis (Gonçalvès and Flandrin [18]). In this paper, we provide our experimental comparison of frequency resolution for four types of TFDs, namely, STFT based TFD, wavelet (Daubechies) based TFD, Choi-Williams TFD in the Cohen class, and HHT based TFD. The results in this paper demonstrate that HHT based TFD has higher resolution than other three.

The rest of paper is organized as follows. We give comparison demonstration in Section 2 together with the brief description of each computational method. Conclusions are given in Section 3.

II. Brief of Computational Methods for Four Types of TFDs with Comparison Demonstrations

While briefing the computational methods, we use the following signal

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$$x(t) = [\cos(0.1\pi t) + \cos(0.4\pi t)][u(t) - u(t - 100)] + [\cos(0.5\pi t) + \cos(0.9\pi t)][u(t - 200) - u(t - 300)], \qquad (4)$$

for the demonstration purpose, where $u(t)$ is the unit step function. Fig. 1 indicates its plot.

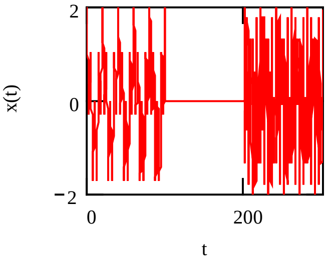


Fig. 1. Signal to be processed.

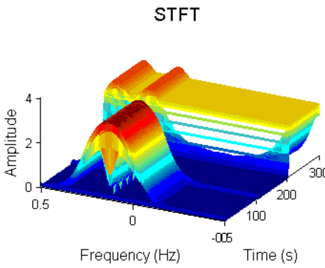


Fig. 2. STFT based TFD of $x(t)$.

A. STFT

Linear transforms were first introduced by Gabor [19]. The basic idea behind Gaor regarding TF description of a signal is to obtain a TFD of the signal by performing Fourier analysis on the signal as it is when observed through a set of identical windows that are translated with respect to each other in time. The functionality of windows is to localize the signal in TF plane. The window function Gabor suggested is Gaussian.

STFT is a kind of linear transforms. It is an extension of Gaor's transform [20]. Denote $S_x(t, \omega)$ the STFT of $x(t)$. Then,

$$S_x(t,\omega) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)e^{-j\omega\tau}d\tau, \qquad (5)$$

where $h(t)$ is a window function. Using the Hamming window of the window size 15, we obtain the STFT based TFD of $x(t)$ in (4), see Fig. 2.

Note that STFT stands for a set of methods. Different methods use different window functions. To select a window function so that it is optimally suitable for the signal to be processed may be an uneasy task in practice. In addition, the bandwidth of the analyzing functions is a constant that is independent of center frequency. Similarly, the time duration or window size of the analyzing functions is also a constant. To choose concrete values of the bandwidth and the time duration so that they are optimally suitable for the signal to be processed when a window function is given appears a hard problem.

B. Choi-Williams TFD in Cohen Class

Different from STFT, a class of TFDs is discussed by Cohen [11,21]. TFDs in the Cohen class are generalizations of the Wigner-Ville distribution that was first introduced by Wigner [22] in 1932 in quantum mechanics and Ville [23] in 1948 for TF analysis.

Denote $\mathrm{WD}_x(t,\omega)$ the Wigner distribution (WD) of a real-valued signal $x(t)$. Then,

$$\mathrm{WD}_x(t,\omega) = \int_{-\infty}^{\infty} x\left(t+\frac{\tau}{2}\right)x\left(t-\frac{\tau}{2}\right)e^{-j\omega\tau}d\tau. \qquad (6)$$

The main unsatisfactory point of WD in practice is that it produces cross-terms. In fact, using the Fourier series, one has $x(t) = \sum A_n \cos(n\omega_1 t + \varphi_n)$, where A_n and φ_n are the amplitude and the initial phase of the nth harmonic of $x(t)$, respectively. Thus,

$$x(t+\tau/2)x(t-\tau/2) = \sum A_n \cos[n\omega_1(t+\tau/2)+\varphi_n] \sum A_n \cos[n\omega_1(t-\tau/2)+\varphi_n]. \qquad (7)$$

In order to suppress cross-terms and to obtain auto-terms as many as possible, a kernel is utilized in the Cohen class. Denote $\Phi(u, \tau)$ the kernel function. In the general sense, we denote a TFD in the Cohen class by $\mathrm{GTFD}(t, \omega)$, which is written by

$$\mathrm{GTFD}(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t+\tau/2)x(t-\tau/2)\Phi(u,\tau)e^{j(ut-\tau\omega-ut)}dt\,d\tau\,du, \qquad (8)$$

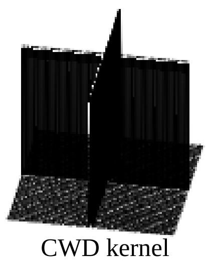
where $\Phi(u, \tau)$ satisfies the conditions mentioned in [11].

The literature regarding kernel design is rich, see e.g. Oh and Marks [24], Jeong and Williams [25]. Several commonly used kernels are Gaussian function discussed by Choi and Williams [26], Bessel function studied by Guo et al [27], corn-shaped kernel [24], and others, see [11] for details. The Gaussian function used in Choi-Williams TF distribution (CWD) is given by

$$\Phi_{\mathrm{CWD}}(u,\tau;\sigma) = e^{-\frac{u^2\tau^2}{\sigma}}, \qquad (9)$$

where σ is a scaling factor to control its attenuation rate. Fig. 3 indicates its plot for $\sigma = 1$. We use $\mathrm{CWD}(t, \omega)$ to represent Choi-Williams TFD. Fig. 4 shows CWD of $x(t)$ in (4).

Note that to select the optimal kernel for general signals may be difficult since kernel selection is signal dependent (Baraniuk and Jones [28,29]).



CWD

Fig. 3. Choi-Williams kernel for $\sigma = 1$.

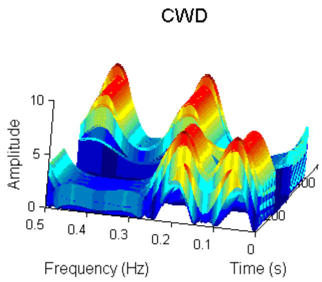


Fig. 4. CWD of $x(t)$ for $\sigma = 1$.

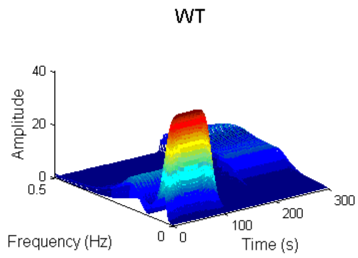


Fig. 6. WT based TFD of $x(t)$.

C. Wavelet Based TFD

WT of a signal $x(t)$ is defined by

$$\mathrm{WT}_x(t;a) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(\tau) g^*\left(\frac{\tau - t}{a}\right) d\tau, \tag{10}$$

where $g(t)$ is the basic wavelet or the mother function, $*$ represents the complex conjugate, $a > 0$ is a scalar. The constant $1/\sqrt{a}$ is used for energy normalization. Different numbers of a and t cause the property of multiresolution.

Note that $g(t)$ in (10) can be taken as an impulse function of a linear system. Denote f_0 the central frequency of the analysis system. Then, the scalar a can be expressed by

$$a = \frac{f_0}{f}. \tag{11}$$

Hence, WT is a tool for TF analysis of signals with the local frequency $f = af_0$, see e.g. Rioul and Vetterli [30], Strang and Nguyen [31].

WT receives considerable attention in the field of TF analysis, see e.g. Daubechies [32]. There are a number of mother functions in the field, such as Daubechies's 4-coefficient wavelet, Haar wavelet, Morlet wavelet, and so on [31].

Denote ϕ the mother function of Daubechies's 4-coefficient wavelet. Then, its plot is indicated in Fig. 5. We use it to obtain WT based TFD of $x(t)$ in (4) (Fig. 6).

Note that the selection of the optimal mother function for general signals may be a hard issue as different mother function corresponds to different kind of WT.

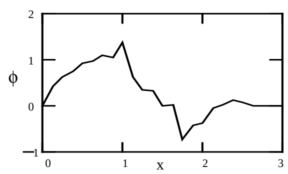


Fig. 5. Daubechies's 4-coefficient basic wavelet.

D. HHT Based TFD

By using HHT, a signal being analyzed can be represented in TF domain by combining the empirical mode decomposition (EMD) with the Hilbert transform (Huang et al. [13,14]). Different from three approaches mentioned previously, EMD is adaptively data-driven, see e.g. Flandrin [33,34].

Denote H the operator of the Hilbert transform (HT). Then, $\mathrm{H}[x(t)]$ is given by

$$\mathrm{H}[x(t)] = y(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau. \tag{12}$$

As any analytic signal $z(t)$ can be expressed by the sum of its real part $x(t)$ and the imaginary part $y(t)$ that is the Hilbert transform of the real part (Papoulis [35, Chap. 7]), one has

$$z(t) = x(t) + jy(t). \tag{13}$$

In the polar coordinate system, the above can be rewritten by

$$z(t) = a(t)\exp[j\vartheta(t)], \tag{14}$$

where $a(t) = [x^2(t) + y^2(t)]^{0.5}$ and $\vartheta(t) = \tan^{-1}[y(t)/x(t)]$ are the instantaneous amplitude and the instantaneous phase of $z(t)$, respectively. Thus, $x(t)$ can be recovered from $z(t)$ by

$$x(t) = \mathrm{Re}[z(t)] = \mathrm{Re}\{a(t)\exp[j\vartheta(t)]\}. \tag{15}$$

Based on HT, therefore, the instantaneous frequency $\omega(t)$ is expressed by

$$\omega(t) = \frac{d\vartheta(t)}{dt} = \frac{\dot{y}(t)x(t) - y(t)\dot{x}(t)}{x^2(t) + y^2(t)}. \tag{16}$$

Note that to assure of the physical meaning of $\omega(t)$ requires that $\theta(t)$ must be a single-valued function over time, i.e., a mono-component function. However, $\theta(t)$ of a signal being processed in general may not be mono-component but multi-component. Thus, a method to decompose $x(t)$ into a series of mono-functions is desired. In this aspect, Huang et al. [13,14] developed a method called EMD that decomposes $x(t)$ into a series of mono-functions termed intrinsic mode functions (IMFs).

It is noted that, physically speaking, the necessary conditions to define a meaningful instantaneous frequency are that the signal being processed must be symmetric regarding the local zero mean, and have the same numbers of zero crossings and extrema. This implies that, in an IMF, the number of extrema and the number of zero crossings must be either equal or different at most by one in the whole data set, and the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero at every point. Those conditions are strict such that the resulting IMF may not satisfy them exactly in general. Hence, the result is generally nearly a mono-component function instead of a perfect one. However, this does not matter in practice when one considers that signals to be processed usually include a certain amount of noise or measurement errors in addition to computation errors in signal processing.

The procedure to decompose $x(t)$ into a series of IMFs is as follows.

- First, identify all local maxima from $x(t)$ and then connect them with the cubic spline line to form the upper envelope of $x(t)$. Denote the upper envelope of $x(t)$ by $x_{up}(t)$.
- Second, identify all local minima from $x(t)$ and then connect them with the cubic spline line to form the lower envelope of $x(t)$. Denote the lower envelope of $x(t)$ by $x_{low}(t)$.
- Third, compute the mean by
  $$m_{11}(t) = [x_{up}(t) + x_{low}(t)]/2 \tag{17}$$
  and construct a new signal $h_{11}(t)$ by
  $$h_{11}(t) = x(t) - m_{11}(t). \tag{18}$$

In the ideal case, $h_{11}(t)$ is an IMF since it satisfies all the conditions of IMF. In practice, however, there may exist overshoots and undershoots during processing. This is particular true for processing shock and impact signals in mechanics. Those overshoots or undershoots may distort the mean values, accordingly make the envelope mean differ from the true local mean, and, as a result, make $h_{11}(t)$ asymmetric. To deal with this practical issue, Huang et al. suggested the fourth step below.

- Fourth, repeat the shifting process (18) by taking $h_{11}(t)$ as a new signal. After kth iterations, we have
  $$h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t), \tag{19}$$

where $m_{1k}(t)$ is the mean envelope after the kth iteration, and $h_{1(k-1)}(t)$ is the difference between the signal and the mean envelope at the $(k-1)$th iteration. Define $h_{1k}(t)$ as the first IMF component. Express it by
$$c_1(t) = h_{1k}(t). \tag{20}$$

- Fifth, having separated $c_1(t)$ from $x(t)$, one has the residue given by
  $$r_1(t) = x(t) - c_1(t). \tag{21}$$

The criterion for stopping the iteration suggested by Huang et al. is like this. Given the standard deviation expressed by
$$\mathrm{SD} = \sum_{t=0}^{N} \frac{\left|h_{1(k-1)}(t) - h_{1k}(t)\right|^2}{h_{1(k-1)}^2(t)}. \tag{22}$$

Then, the iteration stops when SD is equal to or less than a predetermined value. Huang et al. suggested that, typically, SD $\approx$ 0.2 ~ 0.3, which is very rigorous limit for the difference between two consecutive iteration.

- Sixth, treating $r_1(t)$ as a new signal and the above iteration procedure is repeated to extract the rest of IMFs to the signal $x(t)$ as
  $$\begin{cases} r_1(t) - c_2(t) = r_2(t) \\ \quad\vdots \\ r_{n-1}(t) - c_n(t) = r_n(t) \end{cases}. \tag{23}$$
- Seventh, the signal decomposition procedure ends when $r_n(t)$ becomes a monotonic function or a constant, which implies that no further IMFs can be extracted from $x(t)$.

Replacing (23) into (21), a series of IMFs of $x(t)$ are obtained. Therefore, $x(t)$ can be expressed as the combination of $c_i(t)$ plus the residue $r_n(t)$. That is,
$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t). \tag{24}$$

Now, performing HT on $c_i(t)$ yields
$$a_i(t) = \sqrt{[c_i(t)]^2 + \mathrm{H}[c_i(t)]^2},\ \vartheta_i(t) = \tan^{-1}\left\{\frac{\mathrm{H}[c_i(t)]}{c_i(t)}\right\}. \tag{25}$$

The instantaneous frequency is given by
$$\omega_i(t) = \frac{d\vartheta_i(t)}{dt}. \tag{26}$$

In the polar coordinate system, $x(t)$ is expressed by
$$x(t) = \mathrm{Re}\left(\sum_{i=1}^{n} a_i(t)\exp\left[j\int \omega_i(t)dt\right]\right) + r_n(t). \tag{27}$$

Ignoring the residue is practically allowed since it is either a monotonic function or a constant. Doing so yields
$$x(t) \approx \mathrm{Re}\left(\sum_{i=1}^{n} a_i(t)\exp\left[j\int \omega_i(t)dt\right]\right). \tag{28}$$

Let $a_i(t,\omega)$ be the combination of the amplitude $a_i(t)$ and the instantaneous frequency $\omega_i(t)$ of the ith IMF. Denote by $\mathrm{HHT}(t,\omega)$ the HHT of $x(t)$. Then,
$$\mathrm{HHT}(t,\omega) = \sum_{i=1}^{n} a_i(t,\omega). \tag{29}$$

Fig. 7 indicates IMFs and residue of $x(t)$ in (4). Fig. 8 is $\mathrm{HHT}(t,\omega)$ of $x(t)$ in (4).

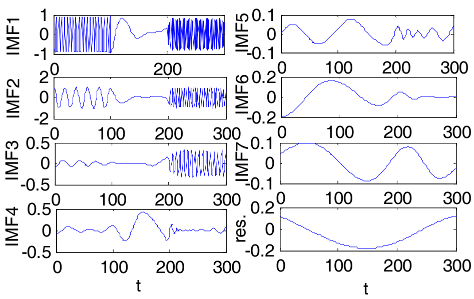


Fig. 7. IMFs of $x(t)$.

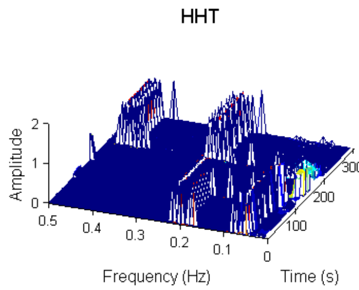


Fig. 8. HHT based TFD of $x(t)$.

E. Discussions

Judging from Figs. 2, 4, 6, and 7, one sees that $\mathrm{HHT}(t,\omega)$ apparently has higher frequency resolution than those with STFT, CWD and WT. In general, however, it is difficult to deal with the issue of resolution comparison among SFTF, WT and GTFD since there are different window functions for SFTF, various mother functions for WT and a number of kernels for GTFD, though, for the signal in (4), CWD appears better in frequency resolution than WT based one, which in turn is superior to STFT based one.

III. Conclusion

We have discussed 4 types of TFDs. The demonstrations with $x(t)$ given in (4) indicate that HHT based TFD has high resolution.

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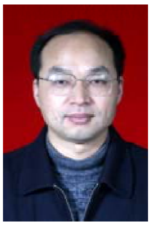
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