A filtering procedure based on least squares and Kalman algorithm for parameter estimate in distance protection

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Abstract— A digital procedure aimed at improving the estimate of the parameters of a faulted line is suggested. The approach is particularly suitable to increase the performances of algorithms nowadays commonly adopted in distance protection especially when signals received by relays are very noisy and uncertainties are present in line parameters. The described procedure is based on a combined use of the weighted recursive least-square method and Kalman filter. The results of a simulation campaign carried out to investigate performances and capabilities of the estimator are also included in the paper. The extensive simulation studies indicated that the trip signal could be obtained in less than a quarter of the cycle, and therefore the method may prove useful in high speed digital relaying.

Keywords— Numerical distance protection, recursive least square method, parameter estimation.

I. Introduction

In the event of faults and transients occurring in both MV and HV transmission networks, accurate and quick responses of the distance relays are of crucial importance in order to maintain the system stable and reliable. Distance protection is usually based on the estimate of the line direct impedance between the relay and fault [1], [2], [3], [4], [5]. Since symmetrical electrical power lines exhibit constant kilometric impedance, the fault distance can be promptly evaluated once R_d and L_d line fault parameters are known. Although the working principle is relatively simple, when the relay is operating in the field a number of inconveniences may arise, such as the presence of non-linear loads, measurement noise, exponentially decaying current components, transients phenomena [8]. Other difficulties may involve the line parameter uncertainties due to the arc resistance, return path resistance, power swings, serial and derived compensation. Basic voltage and current components are usually extracted by means of the Discrete Fourier Transform [9], or other orthogonal series expansion (Walsh, Harr), and the Least Square Method [15]. These procedures exhibit good frequency but poor time resolution, which means that about one and a half cycle is required before the tripping command can be generated. Actually, time delay can be critical for protection selectivity and system stability. In order to reduce computation time, a number of methods were proposed based on shorter or adaptive data window length, for instance the method based on the Phase-Modified Fourier Transform [4]. In this case time computation was drastically reduced but frequency resolution worsened slightly. Alternative methods based on deterministic and random input signals are also presented with an aim to improve the accuracy of parameter estimation while minimizing the error in the quadratic sense, which leads to the least-square solutions in the non-recursive or recursive forms [2], [3]. Techniques based on Kalman filtering and parameter estimation [5], [10], [11], are mainly used to improve the algorithm results in the presence of high signal distortion and measurement noise [5], but also when random parameters variations could occur, e.g. in case of variant arc resistance [12], [13].

The method here proposed aims at achieving good performance in presence of both noise and uncertainties on line parameters. The adopted line model is the well known and most frequently used R_d and L_d equivalent circuit [5], [14]. In this case, the following differential equation can be written:

$$R_d \cdot i(t) + L_d \cdot \frac{di(t)}{dt} = v(t) \tag{1}$$

where the unknowns can be expressed as:

$$\vartheta = [R_d \quad L_d].$$

The symmetrical component approach, which was proposed in the past and is still now used widely, operates by extracting medium physical effects through signal filtering. In this case the use of the $R_d L_d$ circuit and an analysis in the complex space allow to achieve the fault parameter estimation.

The most recent digital procedures have greatly improved this approach, allowing better precision and reliability in the estimated results [4], [5], but the need for a phasor representation has always prevented some of the capabilities offered by digital technology, for instance the possibility to operate directly on the samples of the acquired quantities instead of on medium-filtered, pre-processed signals.

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In order to exploit the latter possibility, in the following the combined use of the Kalman filter and Weighted Recursive Least-Square (WRLS) approach is proposed to estimate fault line parameters.

II Application of the Kalman filter

Let us suppose that a ϑ (deterministic and unknown) vector must be assessed starting from m independent measurements. Each y_i measurement is supposedly affected by an added n_i noise, which can be represented with a Gaussian distribution with zero mean and known σ_{ni}^2 variance:

$$\begin{aligned} & y_i = a_i \cdot \vartheta + n_i \\ & E\{n_i\} = 0 \qquad i = 1,\dots,m \\ & E\{n_i^2\} = \sigma_{ni}^2 \end{aligned} \tag{2}$$

If the y_i measurements and n_i noises are reported in two different vectors, named Y and N respectively, equation (3) can be written in a matrix form as follows:

$Y = A \cdot \vartheta + N$ where:

- A is the coefficient matrix.
- N is a random Gussian vector representing an additive noise with zero mean and σ_N^2 variance.
- ϑ is a deterministic, unknown vector.
- Y is a random Gussian vector with zero mean and $m_Y = A \cdot \vartheta$ variance.

With regards to the above described notations, the observed X random variable can be written as follows:

$X = A \cdot \vartheta + N$ where:

- $A = [x(k) \quad u(k) \quad u(k+1)]$ is the coefficient matrix; $x(k)$, $u(k)$ and $u(k+1)$ are the known quantities.
- N is a random Gaussian vector with zero mean.
- $\vartheta = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$ The quantities α, β and γ depend on the system under examination.

The problem to be solved involves an assessment of the maximum verisimilitude for ϑ parameters starting from x measured values.

If the known quantities of the X vector are acquired at different, subsequent instants of time, the following dynamic formulation can be given:

$$X(k) = A(k) \cdot \vartheta(k) + N(k)$$

where:

$$X(k) = \begin{bmatrix} X(k-1) \\ --- \\ x(k) \end{bmatrix} = \begin{bmatrix} x(1) \\ x(2) \\ x(3) \\ . \\ . \\ x(k) \end{bmatrix}$$

$$A(k) = \begin{bmatrix} A(k-1) \\ --- \\ a(k) \end{bmatrix} = \begin{bmatrix} x(0) & u(0) & u(1) \\ x(1) & u(1) & u(2) \\ x(2) & u(2) & u(3) \\ . & . & . \\ . & . & . \\ x(k) & u(k) & u(k+1) \end{bmatrix}$$

$$N(k) = \begin{bmatrix} N(k-1) \\ --- \\ n(k) \end{bmatrix} = \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ \varepsilon_k \end{bmatrix}$$

where ε represents the error affecting the performed measurements.

The algorithm of the recursive minimum mean-square estimation allows a computation of the optimum assessment of $\hat{\vartheta}(k+1)$ using the knowledge of the $\hat{\vartheta}(k)$ assessment of the previous time instant.

In order to use the Kalman filter formulation, the problem must be rewritten in a dynamic form [5], [10], [11]. Since the ϑ vector parameters are constant, the state equation can be simply written as:

$$\begin{cases} \vartheta(k+1) = \vartheta(k) \\ x(k) = a(k) \cdot \vartheta(k) + \varepsilon_k \end{cases}$$

III. A combined use of the Kalman filter and WRLS approach

Once the type of a short circuit is identified, each relay must estimate the distance between its own position and the fault in order to achieve the required selectivity for a correct line trip either during instantaneous operation or in reserve (second or third step).

The problem is introduced by assuming the validity of the (1) differential equation and the availability from A/D conversion devices of sampled and digitalized signals.

In addition, the following assumptions are established:

- $T_s = \dfrac{1}{f_s}$: sampling step (f_s: sampling frequency).
- $t = k \cdot T_s$: the discretized real time, where $k \in N^+$. For simplicity reasons in the following this time value will be indicated as: $t = k$.
- The derivative operation is approximated using the centered Euler method.

According to the above assumption, the following relation can be written:

$$\frac{di(t)}{dt} \approx \frac{i(k+1) - i(k-1)}{2T_S} = D\, i(k).$$

As a consequence, the (1) differential equation can be rewritten as:

$$\tilde{R}_d \cdot i(k) + \tilde{L}_d \cdot \frac{i(k+1) - i(k-1)}{2T_s} = v(k).$$

Finally, by adopting the matrix notation, the same relation takes the following form:

$$\begin{bmatrix} i(k) & D\, i(k) \end{bmatrix} \begin{bmatrix} \tilde{R}_d(k) \\ \tilde{L}_d(k) \end{bmatrix} = \begin{bmatrix} v(k) \end{bmatrix} \qquad (3)$$

A heuristic solution can be obtained by writing the (3) relation for two subsequent instants and solving a system with two equations and two unknowns $\tilde{\vartheta} = [\tilde{R}_d \;\; \tilde{L}_d]$. Unfortunately, this method supplies solutions oscillating around the right value. For this reason, by assuming that the samples of voltages and current must always satisfy the discrete (2) relation, the same equation can be written *m* times so as to obtain the following equation system:

$$\begin{bmatrix} i(k-m+1) & D\, i(k-m+1) \\ \cdots & \cdots \\ i(k-h) & D\, i(k-h) \\ \cdots & \cdots \\ i(k) & D\, i(k) \end{bmatrix} \cdot \begin{bmatrix} \tilde{R}_d(k) \\ \tilde{L}_d(k) \end{bmatrix} = \begin{bmatrix} v(k-m+1) \\ \cdots \\ v(k-h) \\ \cdots \\ v(k) \end{bmatrix} \qquad (4)$$

In a compact form, the same system can be written as:

$A(k) \cdot \tilde{\vartheta}(k) = Y(k)$, where $A(k) \in R^{(m \times 2)}$, $\vartheta(k) \in R^2$ and $Y(k) \in R^m$.

The problem is here to solve the (4) redundant system defined inside the *k* time interval. In order to obtain an optimal solution, objective criteria must be established to evaluate the reliability of the estimate. As a matter of fact, this means to establish an objective, either cost, weight or merit function. It is evident that for each objective criterion established, a different optimal solution will be obtained. Since a continuous evaluation of the goodness of the estimate can be obtained from the computation error, assuming the error as $\varepsilon = Y - A\tilde{\vartheta}$, the most frequently adopted criterion refers to the norm of the ε error. In this case, the objective function refers to the minimum value of the norm of ε, which is defined as:

$$\|\varepsilon(k)\| \overset{\Delta}{=} \sqrt{\varepsilon(k)^T W(k) \varepsilon(k)}$$

where $W(k)$ is named the weight matrix, that is defined as symmetrical and positive. The associated optimal $\hat{\vartheta}(k)$ estimate is obtained by solving the following optimization problem:

$$\min_{\tilde{\vartheta} \in R^2} [(Y(k) - A(k)\tilde{\vartheta}(k))^T W(k)(Y(k) - A(k)\tilde{\vartheta}(k))]$$

The procedure exhibits a single solution only when the rank of the $A(k)$ matrix has the same dimension as the unknown vector. This solution is obtained by setting the gradient of the previous relation as equal to zero:

$$-2A(k)^T W(k)(Y(k) - A(k)\tilde{\vartheta}(k))\Big|_{\hat{\vartheta}} = 0$$

The Weighted Least-Square estimate can be written as:

$$\hat{\vartheta}(k) = A_W^{\dagger}(k) \cdot Y(k) \qquad (5)$$

where $A_W^{\dagger}(k) = (A(k)^T W(k) A(k))^{-1} A(k)^T W(k))$; $A_W^{\dagger}(k)$ is a pseudo-matrix.

This formulation involves a static problem since the $A(k)$ matrix is built until the *m* dimension is reached; once the estimate is computed, in case a new estimate is required further samples must be acquired and more equations must be added to the $A(k)$ matrix.

The dynamic or recursive formulation is a technique that uses a reduced numerical complexity and minor data storage capabilities, allowing to calculate an optimal estimate from the knowledge only of both the estimate computed in the previous instant and the equation in the current time instant. To this purpose, it is useful to define the following matrix:

$$S(k) \overset{\Delta}{=} (A(k)^T W(k) A(k))^{-1} \qquad (6)$$

In this case, taking into account equation (5), the optimal estimate at instant *k* is:

$$\hat{\vartheta}(k) = S(k) A(k)^T W(k) Y(k).$$

The problem can be solved by establishing the following positions:

$$A(k+1)=\begin{bmatrix} A(k) \\ a(k+1) \end{bmatrix},$$

$$W(k+1)=\begin{bmatrix} W(k) & 0 \\ 0 & w(k+1) \end{bmatrix} \qquad (7)$$

and $Y(k+1)=\begin{bmatrix} Y(k) \\ y(k+1) \end{bmatrix}$

At this point, specific relations must be searched for the computation of the following two quantities:

- $S(k+1)=(A(k+1)^T \cdot W(k+1)\cdot A(k+1))^{-1}$ (when $S(k)$ is known).
- $\hat{\vartheta}(k+1)=S(k+1)\cdot A(k+1)^T \cdot W(k+1)\cdot Y(k+1)$ (when $\hat{\vartheta}(k)$ is known).

The recursive relation between $S(k+1)$ and $S(k)$ can be found by developing the following linked block matrices:

$$S(k+1)=(A(k+1)^T \cdot W(k+1)\cdot A(k+1))^{-1}=$$

$$=\left(\begin{bmatrix} A(k)^T & a(k+1)^T \end{bmatrix}\begin{bmatrix} W(k) & 0 \\ 0 & w(k+1) \end{bmatrix}\begin{bmatrix} A(k) \\ a(k+1) \end{bmatrix}\right)^{-1}=$$

$$=\left(\begin{bmatrix} A(k)^T \cdot W(k)\cdot A(k)+a(k+1)^T \cdot w(k+1)\cdot a(k+1) \end{bmatrix}\right)^{-1}=$$

$$=\left(S(k)^{-1}+a(k+1)^T \cdot w(k+1)\cdot a(k+1)\right)^{-1}=$$

$$=\left(I+S(k)\cdot a(k+1)^T \cdot w(k+1)\cdot a(k+1)\right)^{-1}S(k)$$

$$S(k+1)=\left(I+S(k)\cdot a(k+1)^T \cdot w(k+1)\cdot a(k+1)\right)^{-1}S(k) \qquad (8)$$

The computation of the second quantity requires a derivative operation of the recursive relation between $\hat{x}(k+1)$ and $\hat{x}(k)$:

$$\hat{\vartheta}(k+1)=S(k+1)\begin{bmatrix} A(k)^T & a(k+1)^T \end{bmatrix}\begin{bmatrix} W(k) & 0 \\ 0 & w(k+1) \end{bmatrix}\begin{bmatrix} Y(k) \\ y(k+1) \end{bmatrix}=$$

$$=\left(I+S(k)\cdot a(k+1)^T \cdot w(k+1)\cdot a(k+1)\right)^{-1}\cdot\hat{\vartheta}(k)+ \qquad (9)$$

$$+\left(I+S(k)\cdot a(k+1)^T \cdot w(k+1)\cdot a(k+1)\right)^{-1}\cdot S(k)\cdot a(k+1)^T w(k+1)\cdot y(k+1)$$

Finally, the fault distance can be accurately estimated by means of relations (4), (5), (7), (8) and (9).

IV THE PROPOSED ESTIMATE ALGORITHM

Based on the above demonstrations, the computation algorithm shown in Fig. 1 was implemented.

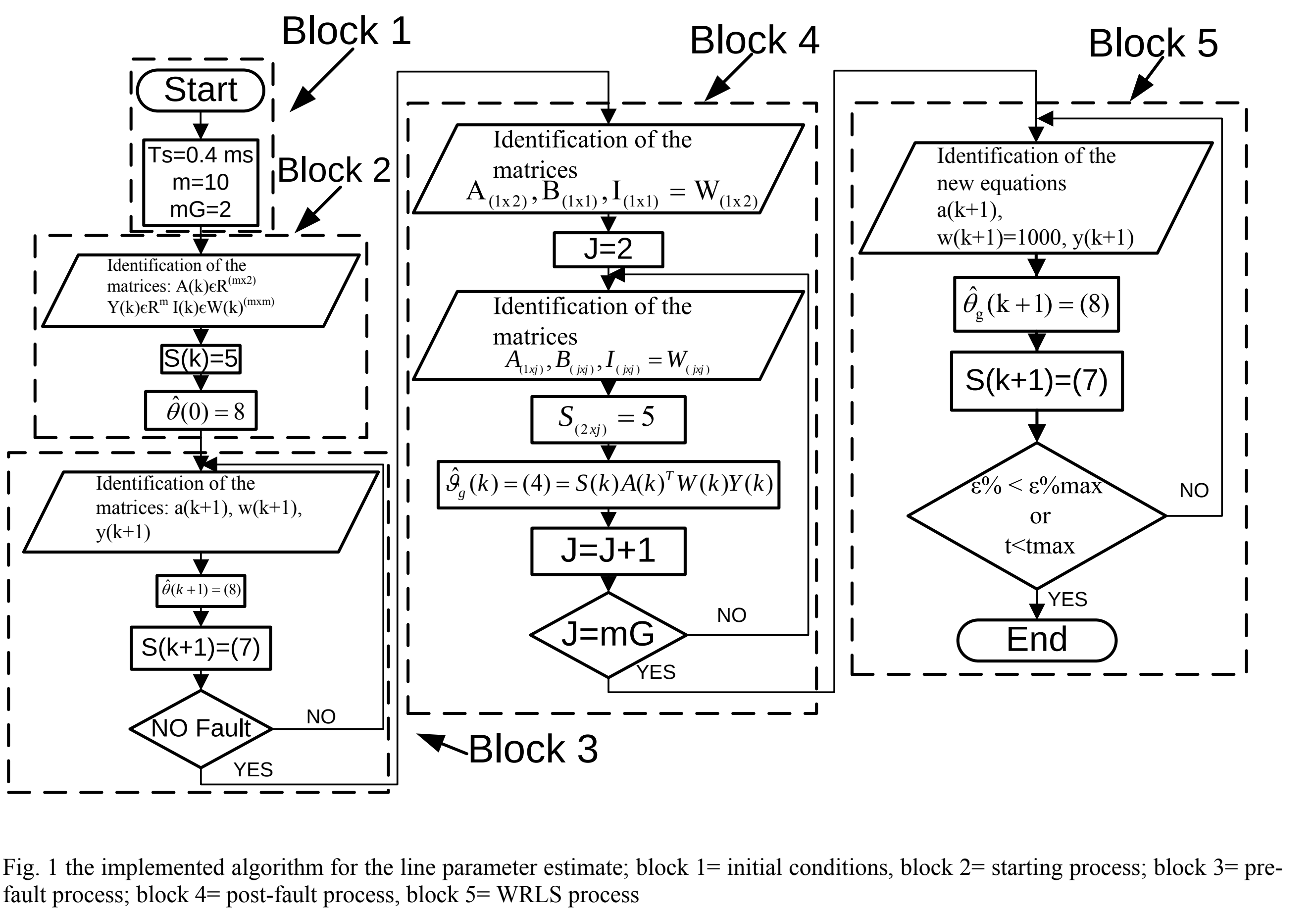

Fig. 1 the implemented algorithm for the line parameter estimate; block 1= initial conditions, block 2= starting process; block 3= pre-fault process; block 4= post-fault process, block 5= WRLS process

The proposed algorithm was tested through simulations performed on a *150 kV* sub-transmission system whose characteristics were reported in previous papers [4], [5]. The validation of the algorithm requires the availability of current and voltage signals reaching the relay terminations when a fault occurs. This information is obtained from simulations performed with a MATLAB code. In order to account for the random disturbances caused by non-linear loads, two harmonic generators were inserted in the electric system. The simulations that were performed on the behavior of the electrical system regarded many significant fault transients [8]. Data obtained from the MATLAB simulator are subsequently processed by the proposed algorithm in order to evaluate fault distances. As an example, Fig. 2 shows signals observed by R_3 and R_4 relays placed at the faulted line terminations as provided by the MATLAB simulation. The consequent fault distances computed by the proposed algorithm (described in Fig. 1) are shown in Fig. 3. Through the L estimate, after only *4ms* all involved relays exhibit error percentages lower than *2%* (Fig. 3b). As to the R estimate, errors are below *2%* after *8ms* (Fig. 3 a).

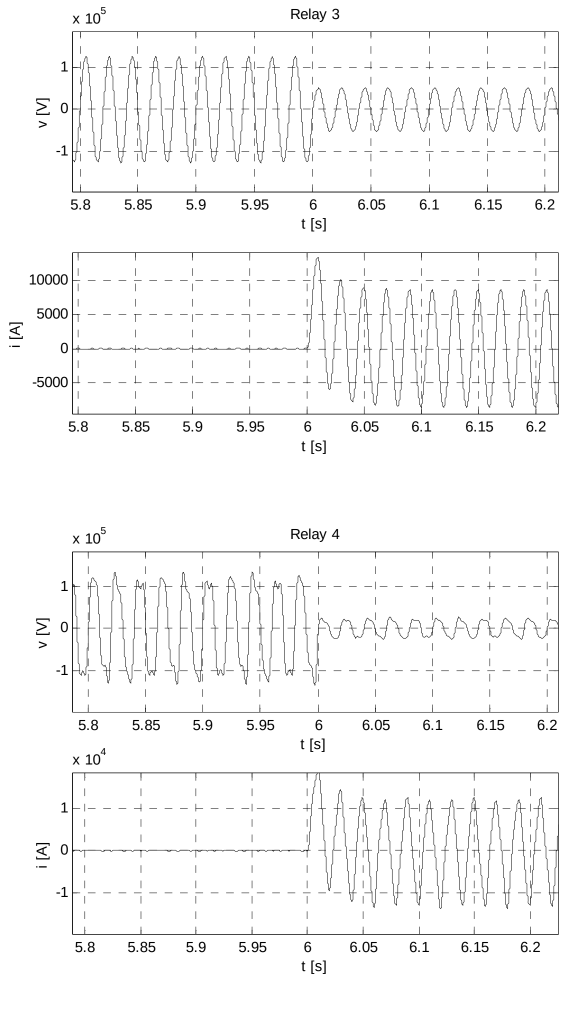

Fig. 2 voltage and current signals as observed by the R_3 and R_4 relays involved in the line trip

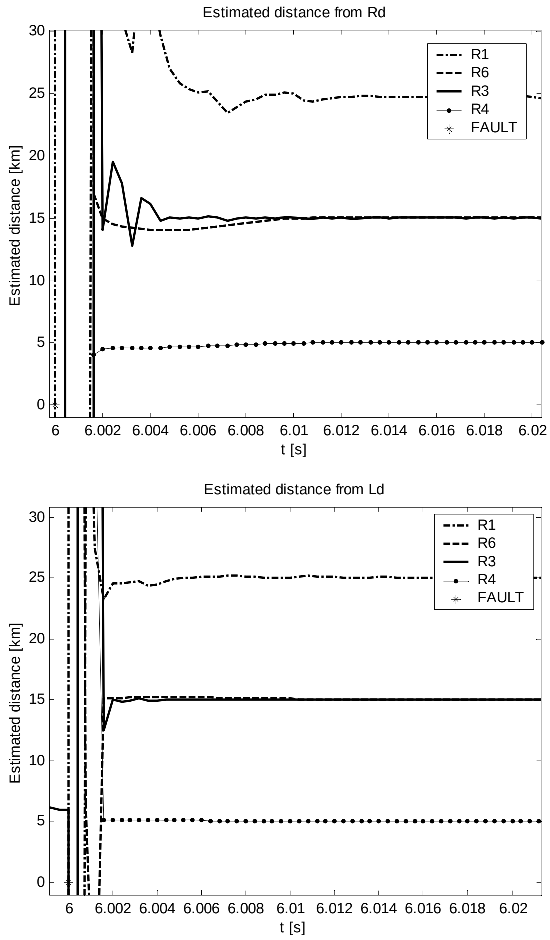

Fig. 3 estimated fault distances by R_1, R_3, R_4 and R_6 relays; a) estimate from R_d; b) estimate from L_d

V CONCLUSIONS

The contribution of this paper concerns an improvement of parameter estimation in a faulted line. The proposed digital procedure can be easily and usefully implemented as a part of the present signal processing of the most used distance protection algorithms. The approach, which is based on the commonly adopted simplified line model, combines the use of the recursive least-square method and Kalman filter estimator, providing good performances during either transient or fault conditions.

Simulation results indicate that the estimate procedure is fast and accurate and particularly suitable in case of signal distortion, measurement noise, and parameter uncertainties.

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