Abstract—A novel estimated method for the speed and mechanical torque of the AC Asynchronous Electrical Dynamometer is proposed in this paper. The method use the voltage, the current and the frequency of the stator of the AC Asynchronous Electrical Dynamometer to estimated the speed and mechanical torque where the wavelet least squared support vector regressions (WLSVR) is used to regression the nonlinear relationship between the variables of the dynamometer. Experiment shows that it is a new way for the measurement of the speed and mechanical torque of the AC Asynchronous Electrical Dynamometer in the power- testing system and can simplify the structure of the currently used measurement system.

Keywords—Measure for the speed and the mechanical torque, WLSVR, the AC Asynchronous Electrical Dynamometer, Power-testing

I. INTRODUCTION

Due to the rapidly development of automobile industry, the severe restriction of the emission of the engine for the environment protection and the problem of energy shortage, high performance of the engine is required in recent years. To get a good engine, one first needs an instrument to judge how good the considered engine is. As a result, much attention has been paid to the engine testing system.

Power testing is the main part of the engine testing system. Normally, the torque and the speed of the engine are measured by the dynamometer, and then the output power of the engine can be calculated. As the load of the engine, the dynamometer can be used not only to absorb the output power of the engine, but also can be used to adjust load torque and speed at different prescribed value which means that the engine can be tested under the different conditions when needed.

At the present time, the hydraulic dynamometer and the eddy current dynamometer are widely used in the testing system. The hydraulic dynamometer has good stability but the precision is not good enough and its operation range is small. Moreover, the hydraulic dynamometer is a complicated system because it needs a water cycle system in order to absorb the output power of the engine. Eddy current dynamometer has a simple structure but has less ability to absorb the output power of the engine. It also has a unacceptable precision when working at low speed. When the hydraulic dynamometer or the eddy current dynamometer is used in the testing system, the output power of engine has been wasted since it is absorbed by the dynamometer and be converted to heat. Furthermore, the system can not drag the engine when the cold running-in is needed.

Since the AC asynchronous electrical dynamometer has a simple structure, good precision and widely working range, it is used widely in the engine testing system recently [1]. In this system, the AC asynchronous electrical dynamometer can drag the engine when it need cold running-in and the output power of the engine can be feedback to the power grid through inverter feedback. The basic structure of the testing system of the engine with the AC asynchronous electrical dynamometer is illustrated in Fig.1.

As show in Fig.1, the mechanic torque and the speed of the AC asynchronous electrical dynamometer are, respectively, the load torque and speed of engine, which are to be measured. In order to measure the torque and the speed, there are two method widely used at the present. One is to change the structure of the stator of the AC asynchronous electrical dynamometer that is called as the suspended electrical dynamometer. This method has the especially requirement to the motor’s structure and increase the cost and causes the difficulties of dynamometer production. The other one is to use the high precision torque
meter or the torque flange but it has especially requirement to the installation and working environment. Also the cost of the torque flange is fairly high.

As it is known, the electric torque of the AC asynchronous motor can be calculated by the voltage and the current of the stator which can be easily measured [2]. If the losses of the motor are estimated, the mechanic torque can be calculated form electric torque. In this paper, the AC asynchronous electrical dynamometer is regarded as a nonlinear system and is modeled by the wavelet least squared support regressions. Based on this model, an estimated method is put forward to estimate the mechanic torque and the speed of the asynchronous electrical dynamometer. By using the proposed method, the AC asynchronous electrical dynamometer need not to have especially structure, because it can be a normal asynchronous motor such as a squirrel cage induction motor, and the testing system also need not to have a torque meter nor a torque flange. As a result, the cost and the complexity of the testing system are greatly reduced and the reliability of the system is enhanced.

II. WAVELET LEAST SQUARED SUPPORT VECTOR REGRESSIONS (WLSVR)

A. The Least Squared Support Regressions (LSVR)

The support vector machines (SVM) use kernel function to map the data in input space to a high dimensional feature space where the problem can be treated in linear form. LSVR is a method of SVM that can be used to identification and control a nonlinear system [3-7].

Let \( T = \{ (x_1, y_1), \cdots, (x_l, y_l) \} \), be the sample data set, where \( x_i \) are the input of system and \( y_i \) are the output of system. By some suitable nonlinear mapping \( \psi(x_i) \), the sample data set \( T \) is mapped into a high dimensional feature space where a optimization problem is defined:

\[
\begin{align*}
\min & \quad \frac{1}{2} \| w \|^2 + c \cdot R_{emp} \\
\text{s.t.} & \quad \left| y_i - w^T \psi(x_i) - b \right| \leq \varepsilon_i
\end{align*}
\]

(1)

Where \( w \) is the distance of \( T \), \( R_{emp} \) is the experiential risk, \( C > 0 \) is a constant \( h \) is called punish factor, \( \varepsilon \) is a small positive number which is the permission error in regression and \( b \) is the offset.

After the solving of the optimization problem (1), the decision function (2) can be expressed as:

\[
f(x) = w^T \psi(x) + b
\]

(2)

Different experiential risk function \( R_{emp} \) will lead to different support vector machines, and LSVR can be expressed as (3) where \( R_{emp} \) is defined as the quadratic norm of \( \varepsilon \) :

\[
\begin{align*}
\min & \quad \frac{1}{2} \| w \|^2 + c \cdot \sum_{i=1}^{l} \varepsilon_i^2 \\
\text{s.t.} & \quad \left| y_i - w^T \psi(x_i) - b \right| \leq \varepsilon_i
\end{align*}
\]

(3)

By using the lagrangier multiplier and the \( \psi(x_i) \) is replaced by the kernel function

\[
K(x_i, x_j) = \langle \psi(x_i), \psi(x_j) \rangle
\]

(4)

The optimization problem (3) resort to the solution of the following linear equations (5)

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_l
\end{bmatrix}
= K(x_1, x_1)+\cdots+K(x_l, x_l)
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_l
\end{bmatrix}
\begin{bmatrix}
0 \\
1 \\
\vdots \\
1
\end{bmatrix}
\]

(5)

Where \( \alpha_i, \alpha_j \) is the lagrangier multiplier. After obtaining the optimization resolution \( \overline{\alpha} = (\overline{\alpha_1}, \cdots, \overline{\alpha_l}) \) and \( \overline{b} \) by solving (5), the decision function (2) can be rewritten as

\[
f(x) = \sum_{i=1}^{l} (\overline{\alpha_i} - \overline{\alpha_i})K(x_i, x) + \overline{b}
\]

(6)

B. Wavelet Kernel

It can be seen form (6) that the support vector kernel \( K(x_i, x_j) \) influences the regression performance of LSVR. Normally, the kernel \( K(x_i, x_j) \) can be chosen as radial basis function, polynomial function etc. However, although by choosing these kernels, LSVR might have good mapping performance, it can not ensure a good approximation of arbitrary function in \( L^2(R^d) \) space, since the bases constructed by these kernel functions through dilations or translations are not complete. It is well known that the bases built up by a family of wavelet functions are complete, and can be used as kernel function in LSVR that called as wavelet LSVR (WLSVR). The main advantage of WLSVR is that it can approximate arbitrary functions in \( L^2(R^d) \) space.

The idea of the wavelet analysis is that any function \( f(x) \) can be expressed or approximated by a family of functions \( h_{a,c}(x) \) generated by dilations and translations of a function \( h(x) \) called mother wavelet [7]:

\[
h_{a,c}(x) = \left| a \right|^{-1/2} h\left( \frac{x-c}{a} \right)
\]

(7)

\[
f(x) = \sum_{i=1}^{l} W_i h_{a_i,c_i}(x)
\]

(8)

Where \( x, a, c \in R \), \( a \) is the dilations factor, \( c \) is the translations factor and \( h_{a,c}(x) \) constitutes a complete base of \( L^2(R^d) \) space.

It can be written as the product of one- dimensional wavelet
functions for a common multidimensional wavelet function:

\[ h_d(x) = \prod_{i=1}^{d} h(x_i) \quad (9) \]

A wavelet support kernel can be defined as [10]

\[ K(x_i, x_j) = \prod_{i=1}^{d} h(\frac{x_i - x_j}{a}) \quad (10) \]

It can be deduced from (10) that the wavelet support kernel is a function with the character of translation invariant which means \( K(x_i, x_j) = K(x_i - x_j) \).

Following the theory of kernel, a symmetry function in \( L^2(R^d) \) space is the dot-product in the feature space should satisfied Mercer condition, i.e.

\[ \int_{R^d} \int_{R^d} K(x_i, x_j) g(x_i) g(x_j) dx_i dx_j \geq 0 \]

\[ \forall g(x) \in L_2(R^d) \quad (11) \]

It is very difficult to decompose the wavelet kernel (10) into dot-product form denote by (4), the Fourier transform of (10) should be considered.

From this point, we have the following statement: A translation invariant kernel \( K(x_i, x_j) = K(x_i - x_j) \) is an admissible SV kernel if and only if the Fourier transform

\[ F[K](\omega) = (2\pi)^{-N/2} \int_{R^d} K(x) e^{-j\omega x} dx \geq 0 \quad (12) \]

It has been proved that (11) can deduce from (12) [3, 9].

The wavelet kernel used in this paper expressed as [10]

\[ K(x_i, x_j) = \prod_{i=1}^{d} h(\frac{x_i - x_j}{a}) \]

\[ = \prod_{i=1}^{d} \cos(\omega \frac{x_i - x_j}{a}) e^{-/(x_i - x_j)^2/2a^2} \quad (13) \]

III. THE ESTIMATED METHOD FOR THE SPEED AND MECHANICAL TORQUE

In the testing system showed in Fig.1, the AC asynchronous electrical dynamometer works as a generator to absorb the output power of the engine and its mechanic torque and speed are the load torque and speed of engine. Normally, the AC asynchronous electrical dynamometer is working in constant torque area showed in Fig.2.

![Fig.2 the working area of the AC asynchronous electrical dynamometer](attachment:image.png)

Form the theory of electric machinery; it is well known that the AC asynchronous electrical dynamometer is a multivariable, nonlinear, strongly coupled and time-varying system. In \( d - q \) axes components in a synchronous rotating frame, the model of the AC asynchronous electrical dynamometer can be expressed as

\[ \begin{bmatrix}
    v_{dq} \\
    v_{qr} \\
\end{bmatrix} =
\begin{bmatrix}
    R_s L_s & -L_m & L_m & -L_m \\
    L_m & R_r L_r & -L_m & -L_m \\
    L_m & L_m & R_r L_r & -L_m \\
    -L_m & -L_m & L_m & R_r L_r \\
\end{bmatrix}
\begin{bmatrix}
    i_{dq} \\
    i_{qr} \\
\end{bmatrix} \]

Where

- \( R_s \) Stator resistance per phase;
- \( R_r \) Rotor resistance per phase;
- \( L_s \) Stator inductance per phase;
- \( L_r \) Rotor inductance per phase referred to stator;
- \( L_m \) Mutual inductance per phase;
- \( p \) Differential operator;
- \( \omega_e \) Synchronous rotating frame angular speed;
- \( \omega_s \) Slip angular speed;
- \( v_{ds} (v_{qs}) \) d-axis (q-axis) stator voltage;
- \( i_{ds} (i_{qs}) \) d-axis (q-axis) stator current;
- \( \lambda_{dr} (\lambda_{qr}) \) d-axis (q-axis) rotor flux linkage;
- \( L_\sigma \) linkage inductance factor, \( L_\sigma = \frac{L_s L_r - L_m^2}{L_r} \);

Resolving (14) with the \( v_{ds} (v_{qs}) \) and the \( i_{ds} (i_{qs}) \) can be computed from stator voltage \( u_s \) and current \( i_s \), the \( \omega_e \), \( \omega_s \) and the \( \lambda_{dr} (\lambda_{qr}) \) can be calculated. Then, the rotor speed \( n \) and the electric torque \( T_e \) of the AC asynchronous electrical dynamometer can be get respectively form (15) and (16)

\[ n = \frac{30}{P} (\omega_e - \omega_s) \quad (15) \]

\[ T_e = \frac{3P}{4} L_m (i_{qs} \lambda_{dr} - i_{ds} \lambda_{qr}) \quad (16) \]

Where \( P \) is the number of poles.
Hence, the speed $n$ and the electric torque $T_e$ can be respectively expressed as

$$n = f_n(u_s, i_s) \quad (17)$$

$$T_e = f_T(u_s, i_s) \quad (18)$$

Where $f$ means nonlinear relationship.

To get the mechanic torque $T$ of the AC asynchronous electrical dynamometer, the losses should be considered. However, the calculation of the losses of the AC asynchronous electrical dynamometer is not an easy job, for losses depending on a lot of factors, such as the voltage, current, temperature, the synchronous frequency etc., and there is no specific formula to estimate the losses, except some general ideas about the loss distribution.

For instance, regardless the motor temperature variation, it is mentioned that the losses of the AC asynchronous electrical dynamometer $\Delta p$ has nonlinear relations to $n$, the speed of rotor; $u_s$, $i_s$, the voltage and the current of stator; $f_s$, the frequency of stator and can be expressed as [11]

$$\Delta p = f_p(f_s, n, u_s, i_s) \quad (19)$$

Then, the mechanic torque $T$ can be expressed as

$$T = T_e + \frac{\Delta p}{n} \times 9550 \quad (20)$$

Consider (18), the mechanic torque $T$ also can be expressed as

$$T = f_T(f_s, u_s, i_s, n) \quad (21)$$

Based on (17) and (21), an estimated method for the speed and the mechanic torque is proposed in Fig.3

$$\begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} = \sqrt{2} \begin{bmatrix} \frac{1}{3} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix} \quad (23)$$

$$\begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} = \sqrt{2} \begin{bmatrix} \frac{3}{2} \\ 0 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} \quad (24)$$

Where $u_{sa}(u_{sb})$ $\alpha$ -axis ($\beta$ -axis) stator voltage; $i_{sa}(i_{sb})$ $\alpha$ -axis ($\beta$ -axis) stator current; $u_A(u_B, u_C)$ Measured stator voltage of A (B, C) phase; $i_A(i_B)$ Measured stator current of A (B) phase.

Then, the $U_s$, $I_s$ and $\theta_{wi}$ can be expressed as

$$U_s = \sqrt{u_{sa}^2 + u_{sb}^2} \quad (25)$$

$$I_s = \sqrt{i_{sa}^2 + i_{sb}^2} \quad (26)$$

$$\theta_{wi} = \theta_u - \theta_i \quad (27)$$

Where

$\theta_u$ The phase of stator voltage, $\theta_u = \arctan\left(\frac{u_{sb}}{u_{sa}}\right)$;

$\theta_i$ The phase of stator current, $\theta_i = \arctan\left(\frac{i_{sb}}{i_{sa}}\right)$;

It is easily to deduce that in the steady state the stator frequency $f_s$ can be derived form $\theta_u$ or $\theta_i$

$$f_s = \frac{1}{2\pi} \frac{d\theta_u}{dt} \quad (28)$$

$$f_s = \frac{1}{2\pi} \frac{d\theta_i}{dt} \quad (29)$$

As discussed above, equation (17) and (21) can be rewritten as

$$n = f_n(f_s, U_s, I_s, \theta_{wi}) \quad (30)$$

$$T = f_T(f_s, U_s, I_s, \theta_{wi}, n) \quad (31)$$

Fig.4 the vector of stator voltage and current in the $\alpha - \beta$ axis

In the $\alpha - \beta$ axis, the stator voltage and current can be expressed as
Based on the (30) and (31), the structure of the method is described in Fig. 5

![Fig. 5: Estimated method based on WLSVR](image)

In the Fig. 5, the two nonlinear relationships are modeled by the speed WLSVR and the torque WLSVR, and the outputs of them are the speed and the mechanic torque of the AC asynchronous electrical dynamometer respectively. $U_s, I_s, \theta_{mi}$ and $f_s$ are computed in calculation portion. The function of training portion is to training WLSVR with training samples to get the decision function (6).

**IV. EXPERIMENT RESULTS**

The experiment equipment is showed in Fig. 6. The drive motor imitates the engine that running in the prescribed speed. A hydraulic dynamometer is used here to measure speed and mechanic torque. Its parameters are 11KW, 50Hz, 55N⋅m and 1490rpm. The drive motor is directly connected to the hydraulic dynamometer and both of them are fed by the inverter with variable voltage and variable frequency ability. The speed, mechanic torque and the stator voltage and current are collected and calculated by the testing equipment.

![Fig. 6: The experiment equipment](image)

Training samples obtained from the experiment are collected to training two WLSVRs to get the decision function (6). Its input portion includes stator voltage $u_s$ and current $i_s$; the outputs of them are speed and mechanic torque respectively. The $T_i$ can be expressed as

$$
T_i = f_T(x_i, y_i) = f_T((u_s, i_s), (n, T))
$$

(32)

To enhance the training effect, training sample should be laid over the working area of the dynamometer. In order to reduce the influence of the temperature rising, all the samples are measured after the dynamometer has reached the heat balance state. The distribution of the training samples in terms of the torque and frequency are showed in fig. 7.

Testing samples are used to verify if the output of the WLSVR satisfies the relationship denoted in equation (32). Fifty-five group testing samples are employed and showed in fig. 8.

![Fig. 7: The distribute of the training samples](image)

![Fig. 8: The distribute of the testing samples](image)

According to the (6) and (13), the accuracy of WLSVR output is influenced largely by the constant $C$ and kernel parameters $a$. However, there is no good solution to choose them so far. In this paper, the try and error method is used to determine $C$ and $a$, and $\varepsilon$ is set to be 0.1. The average relative error $\varepsilon_r$ between the torque values estimated the torque WLSVR by using different $C$ and $a$, and their real value are shown in table 1.

| Table 1: The average relative error $\varepsilon_r$ between the mechanic torque estimated value and their real value |

---

*Note: The table content is not provided in the image.*
From table 1, the best results of mechanic torque estimation is arrived at the point when \( C = 50 \) and \( a = 20 \). The average relative error \( \varepsilon_r \) is 2.60\% which might be acceptable in engineering testing.

The try and error method also is used in speed MLSVR, from the similar step we concluded that \( C = 30 \) and \( a = 20 \) might produce the best results. Fig.8 illustrates the estimated values (by “+”) of the method depicted in Fig.4. In order to compare, the associated testing samples are also showed in fig.9 (by black dot).

Fig.9 the estimated values for the testing samples of the method

Finally, a LSVR with radial basis function [8] showed in (33) is used in the mechanic torque observer.

\[
K(x_i, x_j) = \exp(-\|x_i - x_j\|^2 / p^2) \quad (33)
\]

The parameter \( p = 21 \) and \( C = 300 \) are determined by try and error method. The compare of relative error to the testing samples between it and the observer with WLSVR is showed in fig.10.

As showed in fig.10, the output of WLSVR is better than LSVR. The reason is that WLSVR has not only the advantage of the support regression but also has good partial time-frequency merit inhabited from the characteristics of the wavelet function, which make it can regresses the detail of the nonlinear system better.

V. CONCLUSION

Using WLSVR to model the nonlinear relationship between the stator voltage, current and the speed, mechanic torque of the AC asynchronous electrical dynamometer, a novel estimated method for the speed and the mechanic torque observer is proposed in this paper. Experiment proved the method can work effectively to estimate the speed and the mechanic torque under the steady state without speed and torque sensors or torque flange. This should greatly reduce the cost of testing system and simplify the structure of testing system. It is expected to be availably in industry testing after further improvement of its accuracy.

In the future research, more attentions will be paid on the temperature variation of the AC asynchronous electrical dynamometer. The aim of this research is to improve the accuracy of observer in order to satisfy the requirement of the automobile testing system.
REFERENCES


Zhang Guixiang was born in Yuyang, Hunan, China, Aug, 28th, 1954. She is graduated from Hunan University, China, in 1982, and received M.S. from Hunan University in 1986. She is now a professor in College of Mechanical and Automobile Engineering, Hunan University. Her research interests are in power electrics, motor control and engineer’s testing.

Chen Hongwei is graduated from Xiangtan University, China, in 1998, and now is pursuing the Ph.D degree at the Hunan University, China.

Zhou Cong is graduated from Hunan University, China, in 2003, and now is pursuing the Ph.D degree at the Southwest JIAOTONG University, China.