Eigendecomposition-Principal Components Technique for Resolution Enhancement of Acoustical Holographic Source Localization

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Abstract—A large number of signal processing problems are concerned with estimating unknown signal parameters from noisy measurements. This area has drawn much interest and many methods for parameter estimation have appeared in the literature. The problem of spatially detection and imaging of closely separated objects is investigated. This paper presents eigendecomposition-principal components- method, a high resolution technique, for ultrasonic holographic imaging problem. The in-line holography is employed. The performance of the method is compared with the traditional method, Fourier transform method . The technique is investigated for different values of signal to noise ratio (SNR) and synthetic aperture length.

Keywords—Spectral estimation, High resolution, Parameters estimation, Principal components method, Acoustical imaging, Signal reconstruction.

I. Introduction

THE estimation of the number of harmonics, and their amplitudes and frequencies, from the noisy measurements, is a problem frequently encountered in several signal processing applications, such as in estimating the direction of arrival (DOA) of narrow-band source signals, radio and microwave communication, underwater acoustics, etc.

The problem under consideration concerns information extraction from measurements using spatially synthetic aperture scanning of ultrasonic transmit/receive transducer. Given observations of the scanning outputs , the objective is to estimate unknown parameters associated with the wavefronts. These parameters can include the sources localizations, amplitudes, etc. An important goal for source localization methods is to be able to locate closely spaced sources in presence of considerable noise.

The interest in acoustic waves for detection and imaging stems from its properties as highly coherent waves. The ability of ultrasound waves to penetrate many materials that are optically opaque makes them very important for detecting and imaging targets that cannot be imaged by light waves [1].

A Concealed object is illuminated by acoustic (ultrasound) waves. A transmit/receive ultrasonic transducers are scanning over a synthetic aperture. The idea of in-line holography is employed, where the reflected(backscattered) wave from the object is added to a plane wave, at the receiver, to record both the amplitude and phase of the reflected wave. The addition is performed electronically. The received signal is added to a reference signal, of the same frequency, to generate the in-line holography. A recording of this interference pattern is known as hologram. Holography has received considerable attention since more than three decades. It has been applied in the fields of optical, acoustical, and microwave radiations [2],[3],[4].The use of in-line holography [5] enables improvement of the signal-to-noise ratio by coherently cumulating the acoustic field on the ultrasonic transducers when scanning the field.

The classical spectral estimation approach (the fast Fourier transform (FFT)) is computationally efficient but it suffers from limited resolution (minimum separation to be detected between the parts of the object) when the data length (aperture length) is short, and a leakage in the spectral domain due to the windowing of the data [6]. Therefore it is required to look for another method that has no sidelobes and better resolution . A modern , also known as eigendecomposition methods, contribute in solving the limited resolution. A principal component-minimum variance technique is one of these.

II. Principle of Holography

A. Backscattered Field Analysis

The object ,under imaging process, is assumed to have a field distribution *D(p)*. The distribution is caused by reflection of the incident waves. This distribution propagates to the recording axis *X* where it produces the field distribution S(x) [7], given by:

$$S(x) = \frac{B}{Z_o \lambda} \int_p D(p) \exp(jkr(p,x))dp \qquad (1)$$

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which is the paraxial approximation to the Huygens-Fresnel principles[7]. B is a complex constant, Zo is the distance between the object and recording(observation) axes(Fig.1), λ is the wavelength of the received wave , $k=2\pi/\lambda$ is a propagation constant (wave number),and r is the distance from a typical point on the object to a typical point on the recording axis X . r is given as:

$$r(p,x)=\sqrt{Z_o^2+(x-p)^2} \tag{2}$$

According to paraxial approximation [7],

$$\left((x-p)^2/Z_o^2\right)\langle\langle 1, \tag{3}$$

Equation 2 is written as:

$$r(p,x)=Z_o+\frac{(x-p)^2}{2Z_o}-\frac{(x-p)^4}{8Z_o^3}+----\tag{4}$$

In Fresnel region, r can be approximated by the first two terms of (4) [7] , hence:

$$r(p,x)=Z_o+\frac{p^2}{2Z_o}+\frac{x^2}{2Z_o}-\frac{px}{Z_o}\tag{5}$$

Substituting (5) into (1) yields

$$S(x)=B_1\exp(\frac{jkx^2}{2Z_o})\int_p D(p)\exp\left(\frac{jkp^2}{2Z_o}\right)\exp\left(\frac{-jkxp}{Z_o}\right)dp\tag{6}$$

where B_1 is a complex constant resulting from (1)&(5).

B. In-Line holography

Figure.1 shows the geometry of recording the in-line hologram with a plane-wave reference The plane-wave reference can be easily synthesized in an experimental recording system by simply introducing a constant reference signal in the receiver. Assuming that the synthesized plane-wave reference is $A_r\exp(j\Phi)$ where A_r and Φ are constants, the in-line hologram $h(x)$ is given by

$$h(x)=\left|S(x)+A_r\exp(j\phi)\right|^2=A_r^2+\left|S(x)\right|^2+A_r\exp(j\phi)S^*(x)+A_r\exp(-j\phi)S(x)\tag{7}$$

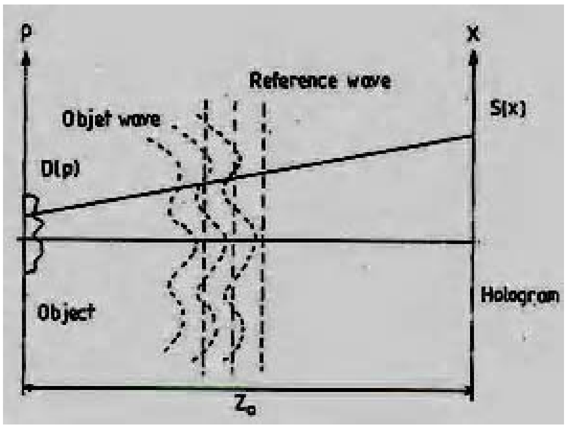

Fig.1 Recording hologram geometry

An image can be extracted from the recorded hologram $h(x)$ through its multiplication by a phase factor $\exp(\frac{-jkx^2}{2Z_o})$ and subsequent use of the classical(FT) [8] or a principal component –based method, as in this paper. The phase factor produces an in-focus image from the fourth term of $h(x)$.The first term, A_r^2 , of $h(x)$ is constant and it can be subtracted from the recorded hologram prior to reconstruction [5,9]. The second term produces a defocused autocorrelation function of $D(p)$, and its effect can be reduced by setting A_r few times larger than the maximum of $|S(x)|$.

III. PRINCIPLE COMPONENT SPECTRUM ESTIMATION

The orthogonality of the signal and noise subspaces could be used to estimate the parameters magnitudes and location) of L complex exponentials in white noise. The algorithm uses vectors that lie in the signal subspace. This method is based on a principal components analysis of the autocorrelation matrix and is referred to as signal subspace method. The basic idea of these methods is as follows: Let $\mathbf{R}_s$ be an $M\times M$ autocorrelation matrix of a process that consists of L complex exponentials in white noise. With an eigendecomposition of $\mathbf{R}_s$ we have

$$\mathbf{R}_s=\sum_{i=1}^{M}\lambda_i\mathbf{v}_i\mathbf{v}_i^H=\sum_{i=1}^{L}\lambda_i\mathbf{v}_i\mathbf{v}_i^H+\sum_{i=L+1}^{M}\lambda_i\mathbf{v}_i\mathbf{v}_i^H\tag{8}$$

Where H denotes the conjugate transpose , $\mathbf{v_i}$ is the *ith* eigenvector of the autocorrelation matrix, and λ_i is its *ith* eigenvalue. It is assumed that the eigenvalues have been arranged in decreasing order, $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_M$. Since the second term in (8) is due only to the noise, we may form a *reduced rank* approximation to the signal autocorrelation matrix , $\mathbf{R}_{sg}$, by retaining only the principal eigenvectors of $\mathbf{R}_s$,

$$\hat{\mathbf{R}}_{sg} = \sum_{i=1}^{L} \lambda_i \mathbf{v}_i \mathbf{v}_i^H \tag{9}$$

This principles components approximation may then be used in the place of $\mathbf{R}_s$ in a spectral estimator such as the minimum variance method. The net effect of this approach is to *filter out* a portion of the noise, thereby enhancing the estimate of the spectral component due to the signal alone, i.e., the complex exponentials. Another way to view this approach is in terms of a constraint that is being placed on the autocorrelation matrix. Specifically, given that a process consists of L complex exponentials in noise, since the rank of the autocorrelation matrix due to the signal, $\mathbf{R}_{sg}$, is L , then a principal components representation simply imposes this rank-L constraint on $\mathbf{R}_s$. In the following subsection we discuss how a principal components analysis of the autocorrelation matrix may be used in conjunction with the minimum variance method.

A. Minimum-Variance Power Spectrum Estimation

Given the autocorrelation sequence $r_s(k)$ of the process $s(n)$ for lags $|k| \leq M$, the *Mth*-order minimum variance spectrum estimate is :

$$\hat{P}_{MV}(e^{j\omega}) = \frac{M}{\mathbf{e}^H \mathbf{R}_s^{-1} \mathbf{e}} \tag{10}$$

where $\mathbf{e}$ is given as

$$\mathbf{e} = \left[1, e^{j\omega}, e^{j2\omega}, ..., e^{j(M-1)\omega}\right].$$

With an eigendecomposition of the autocorrelation matrix, the inverse of $\mathbf{R}_s$ is

$$\mathbf{R}_s^{-1} = \sum_{i=1}^{L} \frac{1}{\lambda_i} \mathbf{v}_i \mathbf{v}_i^H + \sum_{i=L+1}^{M} \frac{1}{\lambda_i} \mathbf{v}_i \mathbf{v}_i^H \tag{11}$$

where L is the number of complex exponentials. Retaining only the first L principal components of $\mathbf{R}_s^{-1}$ leads to the principal components minimum variance estimate

$$\hat{P}_{PC-MV}(e^{j\omega}) = \frac{M}{\sum_{i=1}^{L} \frac{1}{\lambda_i} \left|\mathbf{e}^H \mathbf{v_i}\right|^2} \tag{12}$$

IV. Experimental Results

A test object consisting of two steel rods of 2.5 cm diameter separated by 7.5 cm was used to represent the 2-point object. The other experiment parameters were $Z_o = 50cm, \lambda = 0.4cm$ and sampling interval $\Delta x = 1.0cm$. A Styrofoam plate of thickness 3cm was used as a concealing material. The object was illuminated by ultrasonic waves using ultrasonic transmitting transducer. The reflected wave from the object that impinging the receiving transducer is added electronically to a reference signal to form the in-line hologram. The transmit/receive transducers were scanned across the hologram aperture to record the received signal at uniformly spaced positions by Δx. The total number of recorded samples was N. It is worthwhile to mention that the resolution formula for FT method is given as [8]

$$\sigma = \frac{\lambda Z_o}{a} \tag{13}$$

where a represents half of the aperture length, and given as

$$a = ((N-1)\Delta x)/2 \tag{14}$$

. The minimum number of samples that was used, to resolve the two rods was $N = 20$, (aperture length=20 cm). This represents more than twice the minimum number of samples required as given by (13). The degrading effect of the concealing plate results in the high level of the sidelobes, and the need for large number of samples. When the principal components -minimum variance method was used, the number of samples was less($N = 13$), and the level of sidelobes was low enough, so the two peaks that correspond to the two rods were noticed well. This means that principal components -minimum variance method reduces the degrading effect of the concealing material,. When N was decreased, the similar results were obtained to clearly define the two peaks. The performance of the principal components -minimum variance method is studied by varying the number of samples. Figure 2 shows the percentage error in absolute difference between the two peaks(amplitude) intensity as a function of the number of samples. The maximum error is less than 5% for N less than 13. The percent error in separation between the rods is shown in Fig.3. It is less than

10%, $N > 9$, and the breakpoint(> 10%) is when $N = 8$ .The error in separation for FFT is 36%. The only disadvantage of principal components -minimum variance method, compared with FFT method, is that it computationally less efficient than FFT.

To study the problem more deeply, a white noise is added. Figure4 shows the percentage error in difference in peak intensity as a function of signal to noise ratio SNR in the recorded signal, with $N = 13$ for principal components -minimum variance method and $N = 20$ for FFT method. The maximum error is less than 10% for principal components -minimum variance method , It is clear that the performance of principal components -minimum variance method is noticeably better than FFT method. Figure5 shows the performance for the case of error in separation between the rods. Also $N = 13$ for principal components -minimum variance method and $N = 20$ for FFT method.

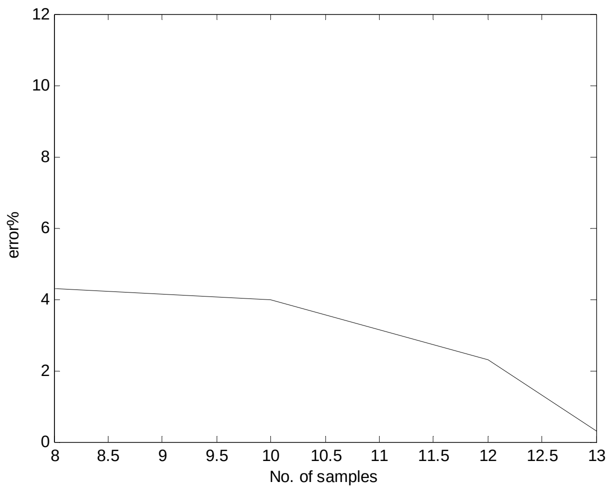

Fig.2 percentage error in absolute difference between the two peaks(amplitude) intensity as a function of number of samples.

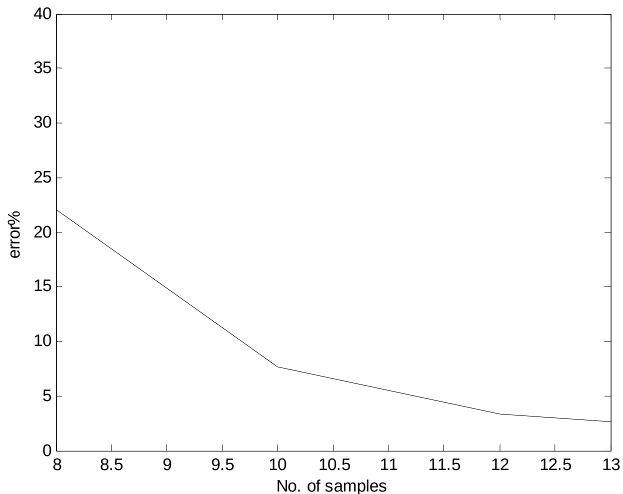

Fig.3 percentage error in separation between the two rods as a function of the number of samples.

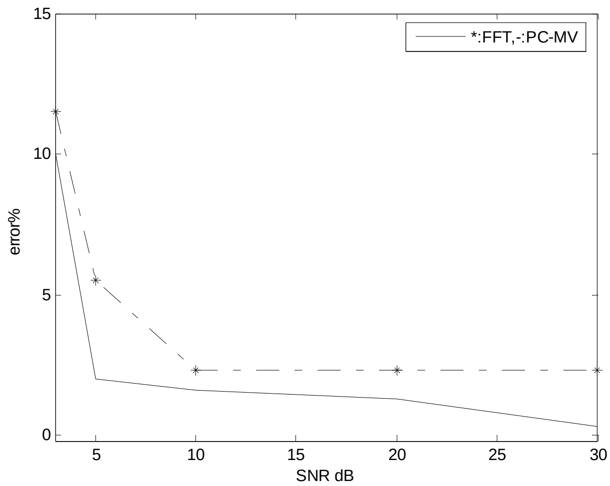

Fig. 4 percentage error in absolute difference between the two peaks(amplitude) intensity as a function of signal to noise ratio, SNR.

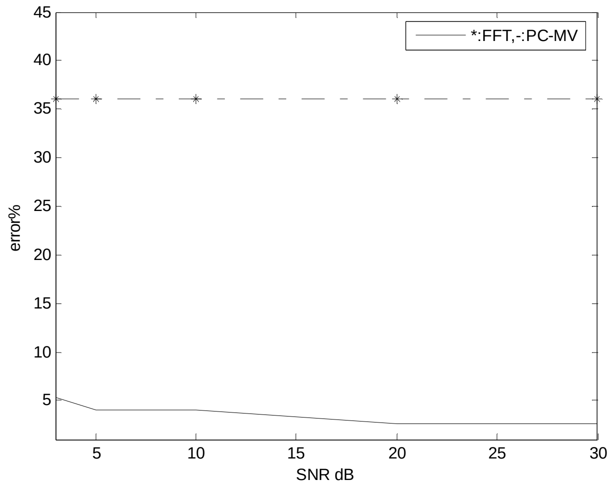

Fig. 5 percentage error in separation between the two rods as a function of signal to noise ratio, SNR.

V. Conclusion

The application of principal components-minimum variance method to the problem of detection of closely separated parts of concealed target has been demonstrated. Also it is found that principal components -minimum variance method is sensitive to the wave features and hence removing , or at least highly decreasing, the effect of the concealing material.

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